A THEORY OF READINGS

Draft

This lengthy draft was written and assembled over a period of years, fragmentary aspects of which have been subsequently refined and updated in other of my papers that appear on this website. The reason for including this draft among them in its present form is to indicate the general orientation of natural language out of which these subsequent papers have issued and which motivated them.

ABSTRACT

A reading of a natural language word string, when fully specified, is intended to provide a complete formal description of a particular way of understanding that word string, by which I mean a formal description of a body of inter-related information that could reasonably be regarded as comprising all that is conveyable to a particular language user by that word string relative to a given context of utterance, and which would, generally speaking, vary as language users and contexts of utterance varied.

This study is limited to consideration of those aspects of understanding a word string that can influence a language user's comprehension of its *entailment relationships to other word strings*. These aspects, which I call the entailment relevant aspects of understanding, are to be considered as only one of the various kinds of elements entering into a particular way of understanding a word string. Accordingly, the notion of reading forwarded here is limited to the formulation of a *partial* rather than complete formal description of a particular way of understanding a word string.

My primary purpose in this paper is to provide a precise notion of reading in terms of which one can formalize the entailment relevant aspects of understanding. My secondary purpose is to formulate a sufficiently general theoretical base capable of accommodating the formalization of further aspects of understanding as well.

The reasonableness of regarding the comprehension of entailment relationships among word strings as a legitimate component of one's understanding of those word strings is based on the following assumptions: (1) A way of understanding the sentences in a collection of sentences, each in a particular way, determines - i.e., induces - a specific pattern of entailments among those sentences, and (2) A person who understood the sentences in a given collection of simple sentences, each in a particular way, would be expected, at least in principle, (that is, ideally) to recognize which of the sentences of that collection entailed which others under that way of understanding them; that is, he should, in principle, be able to recognize the pattern of entailments that held among the sentences of that collection under that way of understanding them. For more complex sentences, Assumption 2 would not ordinarily be expected to hold.

C O N T E N T S

CONTENTS
(Continued)

CHAPTER 1

Background Concepts of the Theory of Readings

In this paper I attempt to provide an integrated account of intuitively perceived natural language entailment in terms of the relationship between the surface forms of natural language word-strings[1] and their underlying modes of syntactic and semantic organization, called readings.

While the examples used herein are almost exclusively of English word-strings, I will argue that the account forwarded here applies to many of the world's languages, and, in particular, that it applies to all natural languages satisfying certain intuitive semantic conditions. For reasons that will become apparent in the sequel, I call these conditions thing-relation (TR) conditions and call languages satisfying them thing-relation (TR) languages. In particular, I attempt to make a case, through examples, that English is a TR-language.

1.1 Readings as Models of Understanding

A reading of a natural language word-string, when fully specified, is intended to provide a complete formal description of a particular way of understanding[2] that word-string, by which I mean, very roughly, a formal description of a body of

---

Note 1. I use the term "word-string" in a pre-critical and thoroughly general way, intended to comprehend any of the usual ways of partitioning sequences of graphic or phonetic elements into sequences of written or spoken words. The usefulness of the notion of "word" derives from its already having a fairly well established common usage, thus affording a reasonably secure pre-analytic base for subsequent analysis.

Note 2. I intend this term to be neutral with respect to speaker and hearer.

interrelated information which could reasonably be regarded as comprising all that is conveyable[2] to a particular language-user by that word-string with respect to a given context-of-utterance,[2.1] and which would, generally speaking, vary as language-users and contexts-of-utterance varied.

This study is limited to consideration of those aspects of understanding a word-string that can influence a language-user's comprehension[2] of its entailment relationships to other word-strings. These aspects, which I call <u>the entailment-relevant aspects of understanding</u>, are to be considered as only one of various kinds of elements entering into a particular way of understanding a word-string. Accordingly, the notion of reading forwarded here must be regarded as addressed to the formulation of a <u>partial</u> rather than complete formal description of a particular way of understanding a word-string.

My primary purpose in the present study is to provide a precise notion of reading in terms of which one can formalize the entailment-relevant aspects of understanding. My secondary purpose is to formulate a sufficiently general theoretical base capable of accommodating the formalization of further aspects of understanding as well, such as those pertaining to intuitively

<u>Note 2.1</u>. We understand the notion of context-of-utterance as an "interpreted" physical situation rather than as the physical situation itself. Thus a given language user may well interpret one and the same physical situation in different ways, each comprising a different context-of-utterance, hence allowing the possibility that a given individual can understand a given word-string in different ways within the same physical situation. In this regard, see Section 1.7.2, also Note 95 on page 326.

perceived focal and presuppositional relationships among word-strings.

The reasonableness of regarding the comprehension of entailment relationships among word-strings as a legitimate component of one's understanding of those word-strings is based on the following Assumption A:

Assumption A:

(1) A way of understanding the sentences in a collection of sentences, each in a particular way, determines or, as we shall say, induces, a specific pattern of entailments among those sentences, which we then say "holds" among the sentences of that collection under that way of understanding them, and

(2) A person who understands the sentences in a given collection of sentences, each in a particular way, would be

expected, at least in principle, (that is, ideally) to recognize which of the sentences of that collection entailed which others under that way of understanding them. That is to say, he should, in principle, be able to recognize the pattern of entailments that held among the sentences of that collection under that way of understanding them.

For example, a person who purported to understand the English sentences "John loves Mary," "Mary is loved by John," and "Something loves Mary", under the most usual way of understanding these sentences, would be expected, at least in principle, (an ideal which could be approximated in practice by determining that such a person understood what "entailment" meant, and had no unusual deficiencies in memory, powers of concentration, interest, eyesight/hearing, etc.) to recognize that the first two inter-entailed each other, and that the third was entailed by each of the first two but did not entail either of the first two. If he persistently failed to recognize any part of this pattern of entailments as, for example, if he were to accept the truth of "John loves Mary" but not accept the truth of "Mary is loved by John," we could not, it seems to me, regard him as properly understanding both of these sentences, at least in any usual or "normal" way.[3]

Note 3. We do allow, of course, that there may be "non-normal," i.e., idiosyncractic ways, in which a person might persistently understand a sentence, which are either wholly unique to him or which he may share with very few other speakers of the language. Psychologicallly, these would, when precisely identified, possibly be taken as indicative of some language-related cognitive dysfunction.

In the sequel I will attempt to show how given patterns of entailments (such as the immediately preceding ones) among given sentences are fully determined by particular ways of understanding the sentences that enter into those patterns, by showing how given patterns of entailments among given sentences are fully determined or, as we shall say, induced, in a precise and explicit sense, by or "under" given readings of those sentences that formalize those particular ways of understanding them.

Let us examine the import of Assumption A further: we extend our earlier examples by adding some further simple sentences to them and considering additional "possible" ways of understanding these sentences.

Consider, then, the following sentences of English:

(1) John loves Mary

(2) Mary is a person

(3) John loves a person

(4) John does not love Mary

(5) Something loves Mary

(6) Mary is loved

(7) John knows Mary

(8) Mary is loved by John

There is one possible way of understanding each of (1) - (8) which can be partially (and very roughly) characterized by the assumptions that (i) each of the sentences (1) - (8), when considered independently of the others, is to be taken in its dominant sense, e.g., that in (1), John is an entity that bears the relation of loving to the entity Mary; in (2), that Mary is an entity that has the characteristic of being a person, and so on, for (3) through (8), (ii) all occurrences of the same word (or meaningful word-part, e.g., "love __" throughout (1) - (8) have the same meaning, (iii) Mary may or may not be a person, and (iv) loving a person means in part that one knows that person.[4] This way of understanding (1) - (8) would induce a pattern of entailments among them that would include the following entries: (1) and (2) together entail (3), but (1) alone does not entail (3); (1) entails each of (5), (6), and (7); (1) and (8) entail each other, hence (8) entails each of (5), (6), and (7); (1) does not entail (4), nor does (4) entail (1); and so on. There are numerous other ways of understanding each of (1) - (8). For example, a second way can be partially characterized by the assumptions (i), (ii) and (iv) above together with the assumption (iii') Mary <u>is</u> a person. This way of understanding (1) - (8) induces a pattern of entailments which includes, in addition to the above, also the following entries: (1) (alone) entails (3), and (8) (alone) entails (3). A third way of understanding (1) - (8) can be partially characterized by

---

Note 4. These assumptions are intended only to be suggestive. The intended ways of understanding (1) - (8) are by no means fully determined by such fragmentary assumptions, which are forwarded here only to incline the reader's existing intuitions regarding how to "read" these sentences into directions that help distinguish between the various intended ways of understanding these sentences. A full determination is possible only within a theory of readings, of the kind forwarded in this study. Moreover, the notion of "inducing" is obviously used here in an intuitive and suggestive sense rather than in a precise one. This notion will be precisely defined in Chapter 2.

retaining the assumptions (i), (ii) and (iii) and dropping assumption (iv) altogether. This way of understanding (1) - (8) induces a pattern of entailments which is like that induced by the first way of understanding indicated above, except that, now, (7) is no longer entailed by any combination of the remaining sentences among (1) - (8). There are also some less usual ways of understanding each of (1) - (8); one such can be partially characterized by retaining assumptions (i) and (iii), but replacing assumption (ii) by two assumptions (ii') and (ii''); namely (ii'): "John" and "loves" may or may not have the same meaning in (1) as in (3), and (ii''): "loves" in (1) and (8) means the opposite of "loves" in (4), (5), or (6), say that "loves" in the first two instances means the usual thing, but, in the last three instances "loves" means what "hates" usually means. This way of understanding (1) - (8) induces a pattern of entailments which is very different from any of the above three patterns; in particular, (1) and (2) together no longer entail (3), (1) no longer entails either (5) or (6), and moreover, (1) now entails (4), and (4) entails (1).

As the last example above shows, not all possible ways of understanding (1) - (8) induce entailments which would ordinarily be considered consistent with English speakers' intuitions regarding correct entailments among (1) - (8). In particular, it cannot be considered consistent with such intuitions that (1) and (2) together do not entail (3), or that (1) entails (4).

Thus, in our above example, under each of the first three ways of understanding sentences (1) - (8), those of their entailments that we had identified there would (by my ear) ordinarily be considered as consistent with English speakers' intuitions regarding correct entailments while those entailments identified as following under the fourth way of understanding those sentences would not ordinarily be considered as consistent with such intuitions.

If a way of understanding each sentence in a collection of sentences induced entailments among the sentences of that collection that would ordinarily be considered to be consistent with the intuitions of language users regarding entailment, then that way of understanding which induced those entailments would be said to be normal relative to entailment; if that way of understanding them induced entailments that would not ordinarily be considered to be consistent with language users' intuitions regarding entailment, then it would be said to be non-normal relative to entailment. Accordingly, by this usage, each of the first three ways of understanding sentences (1) - (8) would be considered normal relative to entailment, while the fourth way of understanding sentences (1) - (8) would be considered non-normal relative to entailment.

Since the normality of word-strings relative to entailment is characterized in terms of consistency with language users' intuitions regarding correct entailment, and since the precise character or inclination of those intuitions is strongly conditioned by the context-of-utterance in which those word-strings are produced, the normality of word-strings relative to entailment needs also to be relativized to that context-of-utterance. By an entailment relation induced on a set of sentences by a given way of understanding the sentences of that set being "consistent with language users' intuitions regarding entailment with respect to a given context-of-utterance," I mean, very roughly, that those entailments among the sentences of that set that a typical language user would accept as holding in that context-of-utterance would, generally speaking, be included among the entailments induced on that set of sentences by that way of understanding them.

Thus when we say that each of the first three ways of understanding sentences (1) - (8) would be considered normal relative to entailment, we mean that each of these ways of understanding sentences (1) - (8) would be considered normal relative to entailment with respect to some usual context-of-utterance; and, when we say that the fourth way of understanding sentences (1) - (8) would be considered non-normal relative to entailment we mean that it would not be considered normal relative to entailment with respect to any usual context-of-utterance.

Also, we need to allow that different induced entailment relations on a given set of sentences may be to different degrees consistent with language users' intuitions regarding entailment with respect to a given context-of-utterance in which those sentences are produced. Thus we need ultimately to speak of the <u>degree of normality</u> of a given way of understanding each sentence in a collection of sentences relative to entailment and with respect to a given context-of-utterance as the degree to which the entailment relation it induces on that set is consistent with language users' intuitions regarding entailment with respect to that context-of-utterance. We will also refer to a way of understanding a natural language sentence e as <u>dominant</u> relative to entailment with respect to a given context-of-utterance C if it is normal relative to entailment with respect to C and if there is no other way of understanding e that has a higher degree of normality relative to entailment with respect to C. If, moreover, there is only one dominant way of understanding e relative to entailment with respect to C, we will refer to it as _the_ dominant way of understanding e relative to entailment with respect to C.

I can now sharpen the earlier statement of purpose:

My primary concern in the present paper is to develop a notion of reading in terms of which one can account for intuitively perceived natural language entailment in the sense that, for any usual context-of-utterance, and for any given way of understanding sentences in any given collection of sentences of any thing-relation language, which way of understanding is normal to some degree relative to entailment with respect to that context-of-utterance, one can specify readings for the sentences of that collection which induce, in a precise and explicit sense, just that pattern of entailments among those sentences which is intuitively induced by that way of understanding them with respect to that context-of-utterance. It is also intended that the notion of reading we develop, while sufficiently flexible to meet the above requirement, should, in addition, be required to permit the specification of readings that are at once "simple" and also bear a relatively transparent relationship to the word-strings they represent. (See Sections 2.1, 2.2; especially Note 38.1). Satisfaction of this latter requirement is made possible by the restriction of the domain of application to thing-relation languages, a restriction which imposes certain intuitive semantic uniformities on word-strings that our notion of reading is designed to exploit.

My secondary concern is to provide a notion of reading which could be readily extended to accommodate the formalization of ways of understanding word-strings that are "normal" relative to further aspects of understanding that pertained to word-strings, that is, to aspects of understanding other than entailment, where, by normality relative to an aspect of understanding other than entailment I would mean, in strict analogy with the case of entailment, "consistency with language users' intuitions" regarding that aspect.

1.2 Reading Assignments on Sets of Sentences

In order to formalize those ways of understanding word-strings that are normal relative to entailment, the notion of a reading will be defined in such a way that the assignment of a reading to each sentence in a set of sentences (such as the set comprised by (1) - (8) above), called a reading assignment on that set, induces, in a precise and fully explicit sense, a specific "pattern" of entailments among its member sentences, where a pattern of entailments is characterized as an entailment relation on the set of those sentences, that is, as a binary relation between the subsets of sentences of that set and the individual sentences of that set, which those subsets are said to "entail" and which fulfills the following condition: every subset of sentences of that set entails (a) each of its member sentences and (b) the sentences entailed by any subset of the set of its entailed sentences.[5]

Note 5. We can formulate this notion more compactly in mathematical English as follows: an entailment relation R on a set S of sentences is a subset $K \subseteq \overline{PS \ \ X \ \ S}$ (where PS is the set of all subsets of S and X is the Cartesian product operation) such that (a) if $J \subseteq S$ and $j \in J$, then $\langle J, j \rangle \in K$; and (b) if $\langle J, j_1 \rangle, \ldots, \langle J, j_n \rangle \in K$ and $\langle \{j_1, \ldots, j_n\}, j_{n+1} \rangle \in K$, then $\langle J, j_{n+1} \rangle \in K$.

A reading assignment on a set of sentences is then defined as normal to a given degree[6] relative to entailment with respect to a context-of-utterance C if the entailment relation it induces among the sentences of that set is to that degree consistent with language users' intuitions regarding entailment with respect to context C.[6.1] Furthermore, a reading of a single sentence is defined as normal to at least a given degree relative to entailment with respect to a context-of-utterance C if there is a reading assignment on some nontrivial[7] set of sentences containing the sentence in question which is itself normal to that degree relative to entailment with respect to C. Finally, a reading of a subsentential word-string, such as a phrase or clause, will be defined as normal to at least a given degree relative to entailment with respect to a context-of-utterance C if it is a subreading (in an obvious sense) of a reading of a sentence in which that subsentential word-string occurs, which sentence is itself normal to at least that degree relative to entailment with respect to C.

Note 6. The notion of degree of normality is formalized in Chapter 2 as a relative notion rather than an absolute one; that is, a given reading of a given word-string e has a greater-or-lesser degree of normality than another reading relative to entailment with respect to a given context-of-utterance C, rather than having a particular degree, such as a specific real number. See also Notes 26 and 30 for further remarks on this notion.

Note 6.1. No attempt is made here to treat this notion of "consistency" in an exact way: it appears, however, to have sufficient intuitive force to justify our anchoring the notion of "normality" to it. Thus, whereas the notions of "reading," "reading assignment," "inducing," and "entailment relation" are independent of context, the notion of "normality" is dependent on context. See Sections 1.7, 2.3.3 for a further discussion of the notion of context.

Note 7. By "nontrivial" I mean that the set contains, in addition to the given sentence, at least one other sentence.

In the sequel, if e is a natural language sentence or a meaningful part of a natural language sentence, and if C is a context-of-utterance in which e is produced, we will use the phrase "a (the)(dominant) normal reading of e (with respect to C)" to refer to a (the) reading of e that appears best to express a (the)(dominant) normal way of understanding e (with respect to C), when the assumption that there is indeed a (the)(dominant) normal way of understanding e (with respect to C) seems warranted.

Finally, a reading assignment on a set of sentences, on a single sentence, or on a subsentential word-string is defined to be normal relative to entailment if there is some usual context-of-utterance C such that that reading assignment is to some degree normal relative to entailment with respect to C. This last definition enables us to speak of normality of a reading assignment (or of a reading) relative to entailment without reference to degree or context.

1.3 Readings: Internal Structure

A reading of a natural language word-string is a pair consisting of two components: (1) a *syntactic representation* of that word-string and (2) a *semantic theory* that interprets that syntactic representation. Different readings of a given word-string are obtained by varying either the syntactic representation, ~~or~~ the semantic theory that interprets it, or both. In accord with our limited objective of formalizing only the entailment-relevant aspects of understanding, I will, in this paper, describe only those parts of the syntactic representation of a word-string and of the semantic theory that interprets it that affect the way that that word-string enters into entailments; I will refer to these as the *entailment relevant parts of the specification of a reading*. In accord with Assumption A, these can be regarded as just those parts of the syntactic and semantic components of a reading required to account for a language-user's ability to recognize patterns of entailment among the word-strings of the language.

The entailment-relevant part of the syntactic representation component of a reading of a word-string describes the underlying entailment-relevant syntactic structure of that word-string, which is a network of syntactically interconnected interpretable parts.

The entailment-relevant part of the semantic theory component of a reading of a word-string assigns a set-theoretic meaning to every interpretable part identified in the syntactic representation of that word-string, and thereby interprets that syntactic representation, proceeding from its (syntactically) smaller meaning-bearing parts and, by a recursive process, ultimately to the full network comprising that syntactic representation as a whole.[8,9]

The minimal meaning-bearing parts of a syntactic representation of a word-string are called <u>representational morphemes</u>.

Thus, the entailment-relevant part of a reading of a word-string is a system of representational morphemes that are syntactically interconnected within the syntactic representation of the word-string, and are semantically interconnected by the interpretations of that syntactic representation within the semantic theory under which each constituent representational

---

Note 8. The system of set-theoretic semantics presented here is essentially novel, though it has points of contact with other work. In particular, the general orientation and technical style of the present system closely follows that of other set-theoretic approaches to natural language semantics, based on the general techniques of model-theoretic semantics for formal languages, deriving ultimately from the work of Tarski [1]. The substance of our approach differs markedly, however, from the usual sorts of set theoretic approaches and, of the literature with which I am familiar, has specific points of contact only with some concepts in Altham [2] though I was not aware of his work at the time I developed my own formulations. In any case, the points of contact are slight, for Altham assimilates the concepts in question within a predicate logic framework rather than generalizing them as a basis for a logic of a very different sort, as we do.

Note 9. The essential difference between set-theoretic approaches to natural language semantics and traditional linguistic approaches is that, while both accord some sort of syntactic representations to natural language word-strings, only the former ~~endows those representations~~ with set-theoretic interpretations, from which one can derive, in a wholly general yet precise way, the common deductive properties of those representations, hence derivatively of the natural language word-strings they represent.

morpheme of the word-string is assigned a set-theoretic meaning - which is recursively propagated to assign set-theoretic meanings to successively larger syntactic compounds into whose construction those representational morphemes enter and, ultimately, to assign a set-theoretic meaning to the syntactic representation of the full word-string itself.

For expository simplicity, since this paper is limited to a discussion of the entailment-relevant aspects of understanding word-strings, _I will, for the remainder of this paper, use the word "reading" to refer to the entailment-relevant part of a reading_, unless explicitly remarked otherwise, as in Section 1.10 below, and, accordingly, will regard a reading as specified when the entailment relevant parts of its syntactic and semantic components are specified. It is to be kept in mind, then, that this usage does not preclude the subsequent enrichment of this narrower sense of "reading" adopted in this paper by appropriate elaborations of the syntactic and semantic components for the formalization of aspects of understanding other than those that are relevant to entailment, such as those relevant to focus, presuppositions, irony, humor, or whatever. Parallel with this usage, I will also use the word "normal" to mean "normal relative to entailment."

1.4 Parts: of Word-Strings and of Syntactic Representations

In the sequel I distinguish between "parts" of word-strings and "parts" of syntactic representations of word-strings. The notion of "part," as it applies to word-strings, is to be understood in the sense that the sequence of letters and blanks comprising the part in question is a subsequence of the sequence of letters and blanks comprising the containing word-string, and is not intended to be semantically interpretable. On the other hand, the notion of "part," as it applies to syntactic representations of word-strings, _is_ intended to be semantically interpretable; that is, "part" in this latter sense means "interpretable part," whereas in the case of word-strings, it does not.

There is, however, an _indirect_ sense in which parts of word-strings are interpretable for, as will be seen later, we can describe a systematic, albeit loose, association between parts of word-strings and parts of the syntactic representations of those word-strings, which association is such that any semantic interpretation of a part of a syntactic representation can be _indirectly_ regarded as a semantic interpretation of that part of the word-string with which it is associated.[10]

Note 10. On the other hand, we note that it is possible to specify certain highly regular fragments [3] of a given natural language, for which one can describe this association in a rigorous way through suitable "syntactic rules" that construct (i.e., generate) word-strings from selected syntactic representations, and thereby to regard the semantic interpretations of those selected syntactic representations as _direct_ semantic interpretations of the word-strings themselves. However, when one attempts to extend such fragments to include even the most ordinary sorts of word-strings that occur in everyday speech, the apparently simple "syntactic rules" inevitably require burdensome ad hoc specifications of further rules to handle even minor deviations from the regularities characterizing that fragment. Approaches that attempt to carry out such a program adopt a view of natural language which appears to me to be fundamentally mistaken; rather, word-strings of a natural language are specific instances of language behavior issuing as a product of a social and historical process, and, as such can have only their underlying structures "cleanly" formalized, not their specific surface string structures as it were. That is to say, the syntactic and semantic rules embodied in formal methods are appropriate only for the characterization of relatively regular underlying structures, that is, readings, from which widely varied sorts of surface word-strings might issue, and are wholly inappropriate for the characterization of the specific structures of the word-strings themselves that is adequate to identify those word-strings that are "grammatical" or "acceptable." This is not to say that a characterization of "grammatical" or "acceptable" word-strings of everyday speech as word-strings having a certain structure in terms of their constituent word-string parts cannot be <u>approximated</u>, but only that that characterization cannot be carried out in any precise and determinate way, as it would be by syntactic rules. Indeed, such an approximation can be realized within the present framework by <u>reading rules</u>, which associate readings with word-strings, and by <u>generation rules</u>, which associate word-strings with readings. I regard both sorts of rules as part of the <u>pragmatics</u> of natural language, rather than as part of the syntax or semantics of natural language. Reading and generation rules are not like syntactic rules; they inter-relate word-strings and readings, that is, they inter-relate word-strings to underlying structures. The distinction between grammatical and ungrammatical word-strings can be made in terms of "general principles of grammar" that describe the way that a word-string functions as a signal system which, in conjunction with a given context-of-utterance in which it is produced, signals one or more possible readings for that word-string that are normal with respect to that context-of-utterance. These principles are, again, part of pragmatics, not of syntax or semantics. Such principles are not rules in the sense of determinate functions; rather, they are <u>conventions</u> for extracting underlying structures, that is, readings, from word-strings. This view of the nature of natural language, which regards the study of natural language as a branch of sociology rather than as a branch of mathematics, underlies our approach, and will be articulated in numerous ways in the course of this study.

For ease of expression, I will occasionally refer to "interpretable parts" of word-strings in this indirect sense, namely, as those parts of word-strings which are associated with interpretable parts of their syntactic representations within given readings, and will also speak of "interpretations" of parts of word-strings to mean interpretations of parts of those syntactic representations within given readings that are associated with them.

1.5 Morphemes: Logical and Lexical Natural Language Morphemes; Logical and Lexical Representational Morphemes

As remarked in Section 1.3, a reading of a word-string specifies a system of syntactically and semantically interrelated representational morphemes.

Consistent with (some versions of) standard linguistic usage, I regard the notion of a natural language morpheme as a theoretical construct, i.e., as an abstract entity that is "realized" in a given word-string in either of two ways: (a) explicitly, indicated in part by and corresponding to a specific part of that word-string called a "morph" or (b) implicitly, indicated solely by global relations among the parts of that word-string, involving factors like order of occurrence, juxtaposition, intonation patterns (if oral), perceived grammatical and semantic relationships among word-string-parts, etc. A natural language morpheme that is explicitly realized in a part of (i.e., as a morph occurring in) a given word-string is also said to be explicitly marked (in that word-string by that part) (i.e., by that morph); a natural language morpheme that is implicitly realized in a given word-string by certain global relations among its parts is said to be implicitly marked (in that word-string by those relations).[11]

Note 11. We can extend the use of the notions of explicitly realized and implicitly realized to apply also to abstract linguistic constructs which, unlike natural language morphemes, are "compounded" out of such morphemes. These constructs, and the compounding operations which articulate their structure, are properly formalized as expressions of the syntactic representation language to be described in Chapter 2. Indeed, our abiding intent is that this syntactic representation language provide a concrete embodiment for these abstract linguistic constructs: its expressions are precisely the formal analogues of these constructs.

In the direction of explicit avowal of such constructs, we might postulate the existence of <u>abstract expressions</u> (AEs) of a given natural language as abstract entities "obtained" as combinations (in some sense) of morphemes of that natural language. Natural language AEs are to be conceived as having the same metaphysical status as natural language morphemes, comprising thereby that generalization of them that includes also all "larger" meaningful compounds. That is, the AEs of a given natural language are to be regarded as those entities that are "realized" in actual word-strings of the language. For example, one might postulate as such an abstraction compound an entity that is realized in the English word-string "three men" in the same sense that the natural language morpheme customarily regarded to be realized in the English word-string "man" is realized in the latter.

Consistent with this postulation, we could regard the set of AEs of a given natural language as the primary or underlying language and the set of word-strings in which those AEs are realized as a manifestation (phonological or orthographic) of that primary language. We could thus characterise the expressions of a formal representation language for the given natural language as comprising explicit orthographic embodiments of the set of AEs of the natural language that that representation language formalizes. In this sense the representation language together with the semantic theory that interprets it can be regarded as affording a precise explication of that primary language (comprised of AEs) that underlies the given natural language.

In accord with the preceding discussion of "part," we distinguish the notion of a natural language morpheme in the above sense from that of a representational morpheme; the latter is an actual expression of a syntactic representation of a word-string in the sense that it occurs as an explicit part of that syntactic representation. As will be detailed in the sequel, representational morphemes are expressions of the formal language in which syntactic representations of word-strings are written, which are semantically minimal in the sense that their meanings are not determined by the meanings of their proper syntactic parts, if any.

In this sense, representational morphemes provide an explicit orthographic embodiment of (abstract) natural language morphemes within a special formal language (to be described), and can be regarded as the formal counterparts of natural language morphemes.

For simplicity of exposition, I shall adopt the usual custom of not always distinguishing between natural language morphemes that are realized in morphs and the morphs in which they are realized. For example, I will sometimes speak of the morphs "boy," "walk," "all," "s," and so on, as natural language morphemes rather than merely as the morphs in which those morphemes are realized.

I will later describe a framework within which readings that are normal relative to entailment can be specified. The mechanism for constructing them utilizes representational morphemes as basic building-blocks and combines these into readings that induce the right sort of entailment relations, namely, those that are consistent with the intuitions of language users regarding entailment.

For this purpose, it is useful to subclassify natural language morphemes into _lexical_ natural language morphemes and _logical_ natural language morphemes, (and to subsequently reflect that distinction among representational morphemes). The distinction[12] between them is an intuitive semantic one: roughly speaking, a _lexical_ natural language morpheme is one that intuitively denotes some entity, relation, or characteristic of an entity or relation, such as "boy," "walks," "hits," "tall," "slowly," etc.; whereas a _logical_ natural language morpheme is one that intuitively denotes some way of operating on denotations, such as "all," "and," "not," "many," "after," etc.

Note 12. As it applies to natural languages, this distinction parallels the traditional one between lexical morphemes on the one hand, and "grammatical" or "functional" morphemes on the other. As it applies to artificial languages, this distinction parallels that between "non-logical" and "logical" constants.

The usefulness of this distinction (for the purpose of specifying readings that are normal relative to entailment) is based on the following methodological assumption:

Assumption B. We assume that a native speaker's intuitive judgments regarding entailments among given sentences derive from his intuitive judgments regarding semantic interconnections among the logical natural language morphemes realized in those sentences, and from his intuitive judgments regarding semantic interconnections among the lexical natural language morphemes realized in those sentences.

Let us examine the import of Assumption B by reconsidering the English sentences (1) - (8) of Section 1.1, which we repeat here for convenient reference:

(1) John loves Mary

(2) Mary is a person

(3) John loves a person

(4) John does not love Mary

(5) Something loves Mary

(6) Mary is loved

(7) John knows Mary

(8) Mary is loved by John

Assumption B, particularized to these examples, and particularized further to the first way of understanding these sentences as indicated on pages 4,5, means in part that: the intuitive judgments that (1) and (2) together entail (3), that (1) entails each of (5), (6), and (8), and that (1) does not entail (4), derive ultimately from intuitive judgments regarding semantic interconnections among the logical natural language morphemes realized in those sentences, which include, in part, the explicit logical natural language morphemes "not" and "is," and various implicit logical natural language morphemes (to be indicated below). We are suggesting, then, that the typical English speaker who understood the meanings of these logical natural language morphemes would assent to the above entailments even if he did _not_ understand the meanings of the lexical natural language morphemes "John," "Mary," "love," and "person" occurring there. On the other hand, under the same particularization to the first way of understanding each of the sentences (1) - (8), an English speaker's intuitive judgment that (1) entails (7) would derive _both_ from his intuitive judgments regarding semantic interconnections among the _logical_ natural language morphemes occurring in sentences (1) and (7), _and_ from his intuitive judgments regarding the semantic interconnections between the _lexical_ natural language morphemes "loves" and "knows" that hold under the same particularization to the first way of understanding (1) - (8), which we had earlier (page 5) indicated by the statement that "loving a person means, in part, that one knows that person." We thus regard a language users' intuitive judgments pertaining to lexical natural language morphemes as dependent in part on his judgments pertaining to logical natural language morphemes. While the nature of this dependency can properly be described only in the context of the formal development of Chapter 2, we can at this point at least roughly indicate what is involved: a language user's intuitions regarding semantic interconnections among the lexical natural language morphemes of (1) and (7) which would underly his intuitive judgment that (1) entails (7), would require, for the consummation of that judgment, also an intuitive comprehension of the semantic interconnections among the logical natural language morphemes of (1) and (7), for these latter would provide logical structure to these sentences relative to which the interconnections among the lexical natural language morphemes occurring in (1) and (7) are framed.

Logical and Lexical Normality. Furthermore, we can speak of entailments as being consistent with language-users' intuitions regarding semantic interconnections among logical natural language morphemes and as being consistent with language users' intuitions regarding semantic interconnections among lexical natural language morphemes. A reading assignment on a set of sentences that induces entailments that are consistent with intuitions of the former kind will be said to be *logically normal*, that is, normal with respect to logical structure though not necessarily with respect to lexical structure. A reading assignment that induces entailments that are consistent with intuitions of the latter kind will be said to be *lexically normal*, that is, normal with respect to lexical structure. *Thus the methodological significance of Assumption B is that a reading assignment on a set of sentences is normal relative to entailment if and only if it is both logically and lexically normal.*

Assumption B can, in a certain sense, be viewed as a *factoring* assumption, asserting that a native speaker's intuitive judgments regarding entailment can be factored--i.e., divided, as it were--into his intuitive judgments regarding interconnections among logical natural language morphemes and his intuitive judgments regarding interconnections among lexical natural language morphemes.

The adoption of Assumption B strongly conditions the structure of the formalism to be described in Chapter 2. In order to appreciate the import of Assumption B in this regard, we need to make a further distinction: A meaning-bearing part of a syntactic representation that contains no proper (i.e., smaller) syntactic parts is called a *representational morpheme in the narrow sense*. A representational morpheme in the narrow sense then is a *syntactically* minimal meaning-bearing part of a syntactic representation, and is intended to be the formal counterpart of the notion of natural language morpheme understood in the usual linguistic sense. A meaning-bearing part of a syntactic representation that *does* contain a proper syntactic part but whose meaning is not determined by the meanings of its proper syntactic parts is called a *representational morpheme in the wider sense*. A meaning-bearing part of a syntactic representation that does contain a proper syntactic part and whose meaning _is_ determined by the meanings of its proper syntactic parts is called a _representational compound_. The semantic interconnections among given representational morphemes are then provided by the semantic interpretations of the representational compounds in which they occur. Among representational compounds, we include representations of sentences, relative clauses, noun phrases, and verb phrases. A representational morpheme in the wider sense, then, is a meaning-bearing part of a syntactic representation that is neither syntactically minimal nor has its meaning determined by the meanings of its proper syntactic parts. Put differently, if we understand a meaningful part of a syntactic representation to be _syntactically minimal_ if it contains no meaningful proper syntactic parts and to be _semantically minimal_ if its meaning is not determined by the meanings of its proper syntactic parts, then representational morphemes in the narrow sense are both syntactically and semantically minimal, whereas representational morphemes in the wider sense are semantically but not syntactically minimal, and representational compounds are neither syntactically nor semantically minimal.

In particular, Assumption B justifies the following procedure regarding the treatment of representational morphemes in Chapter 2 to follow: first, we separate the representational morphemes entering into the syntactic representations of natural language word-strings into two types: _logical representational morphemes_, which formalize the intuitive notion of logical natural language morpheme, and _lexical representational_ morphemes, which formalize the intuitive notion of lexical natural language morpheme. Second, we separate the semantic axioms which define the semantic theory of a reading into two types: logical semantic axioms, which specify the set-theoretic structures of the interpretations of the logical representational morphemes (and, recursively, of representational compounds,) and lexical semantic axioms, which specify the set-theoretic structures of the interpretations of the lexical representational morphemes. A consequence of this treatment of representational morphemes (and one which it shares with other formal semantic approaches to natural language) is the following: only the logical semantic axioms need to be specified in set-theoretic terms; the lexical semantic axioms can be specified simply as syntactic representations of natural language sentences. This strongly parallels procedures already customary in formalizations of branches of mathematics, where the semantic theory of the underlying logic, through suitable "interpretation rules" specifies the meanings of so-called "logical constants" and where the particular mathematical axioms of interest (e.g. for arithmetic) specify the meanings of the so-called "non-logical constants", i.e., the terms of the mathematical theory being formalized, simply by forwarding so-called "non-logical postulates" formulated in the language of the logic with no additional direct specification of how the mathematical (as opposed to logical) terms are to be interpreted, beyond that imposed by the mathematical axioms themselves. Logical constants are analogous to our logical representational morphemes, and the interpretation rules of the semantic theory of the underlying logic are analogous to our logical semantic axioms. Non-logical constants are analogous to our lexical representational morphemes, and non-logical postulates are analogous to our lexical semantic axioms.[13]

Thus, our semantic theory needs to specify directly only the set-theoretic meanings of its logical representational morphemes: the meanings of the lexical representational morphemes will be specified indirectly by simply formulating suitable syntactic representations into which those lexical representational morphemes enter.[14]

Note 13. The distinction between logical and lexical (or, equivalently, between logical and non-logical) representational morphemes, is clear in certain cases, such as those alluded to in the present discussion, but is not so clear in others, such as regards the lexical status we accord to representational "case morphemes" in the sequel. See note 59 below.

Note 14. This fact has considerable significance for applications to automatic natural language inferencing (see Chapter 4) for, once the meanings of the logical representational morphemes are suitably specified (see Chapter 4) then the domain-specific meanings of lexical representational morphemes can be specified simply by stating suitable syntactic representations of natural language sentences or, if an automatic parsing algorithm were also available to convert natural language sentences into their "syntactic" representations, then the domain-specific meanings of lexical representational morphemes could be specified even more simply by stating suitable natural language sentences, i.e., ordinary natural language sentences that expressed the intended relationships among the natural language lexical morphemes. That is to say, given an adequate automatic parsing algorithm, the domain-specific meanings of lexical representational morphemes could be specified simply by stating those natural language sentences that expressed those meanings.

Logical and Lexical Morphemes More Finely Drawn: Examples

We now consider two of the above sentences, namely (1) and (5), in more detail, each under one of its readings that is normal relative to entailment, and which can be regarded as comprehended under what we had above called "the first way of understanding sentences (1) - (8)."

Let us repeat sentences (1) and (5) for convenient reference:

(1) John loves Mary,

(5) Something loves Mary

My purpose in the remainder of this section is (i) to identify some of the natural language morphemes realized in (1) and (5), and (ii) to introduce those representational morphemes that are to formalize them in the sequel. The morphemes identified in these sample sentences reflect the general characteristics of what I call "thing-relation" languages, to be described later.[15] Our discussion should still be considered as informal insofar as we defer (until Chapter 2) both of the following: (a) a precise description of the way that the respective syntactic representations of (1) and (5) are to be built out of the representational morphemes we introduce in this section, and (b) the (set-theoretic) semantic interpretations of these representational morphemes and, recursively, the respective interpretations of the syntactic representations of (1) and (5).

Note 15. This description occurs in Section 2.1, from pages 81 through 90. The reader might find it useful to refer to those pages in the course of the present discussion, insofar as the intuitive description of thing-relation languages given there provides some of the motivation underlying the concepts and notational devices to be introduced here.

Natural Language Morphemes Realized in (1):[16]

(a) The logical natural language thing-morpheme, whose associated logical representational morpheme we write as "T" (for Thing), and which, when attached to the syntactic representation of a word-string, (such as to the syntactic representation of "John" or "Mary" in (1)) or when standing alone, indicates that the syntactic representation resulting from such attachment, (and, derivatively, the word-string it represents), designates a thing of some kind as opposed to, say, a relation.

In more detail: The logical natural language thing-morpheme is an implicit rather than an explicit morpheme of English, being indicated dominantly by various global properties of the word-string in which it is (implicitly) realized.[17] This thing-morpheme signals that any word-string part with which it is associated is to be interpreted[18] as a "thing" relative to other word-string parts which occur with it within a containing word-string. For

---

Note 16. The discussion that follows in this and in the following subsection culminates on page 40 in syntactic representations (1') and (5') of the word-strings (1) and (5), respectively. The reader might refer to (1') and (5') in the course of the following discussion in order to orient the separate items discussed below to their more global culmination. We note that (1') is the syntactic component of the dominant normal reading of (1), while (5') is the syntactic component of the dominant normal reading of (5).

Note 17. This notion is introduced in Section 1.5.

Note 18. The mechanism for interpretation is spelled out in Chapter 2 by Semantic Axioms. At this introductory point, the notion of "interpret" is intended in an informal and suggestive sense.

example, in the word-string (1), the lexical natural language morpheme (explicitly) realized in English in the word-string "John," by virtue of having the natural language thing-morpheme associated with it in a normal reading, is to be interpreted as a "thing." As indicated, we use the letter "T" as the associated logical representational morpheme associated with the natural language thing-morpheme. Now, the syntactic representation of the lexical natural language morpheme "John" (in a normal reading of (1)) is a lexical representational morpheme which we write as "JOHN." When the logical representational morpheme "T" is attached to the lexical representational morpheme "JOHN," to form "$\underset{T}{\text{JOHN}}$", the presence of "T" indicates that "$\underset{T}{\text{JOHN}}$" designates a "thing." The mode of attachment is, as indicated, to have the symbol "T" placed immediately below the symbol "JOHN," to form the (compound) symbol

$$\underset{T}{\text{JOHN}}$$

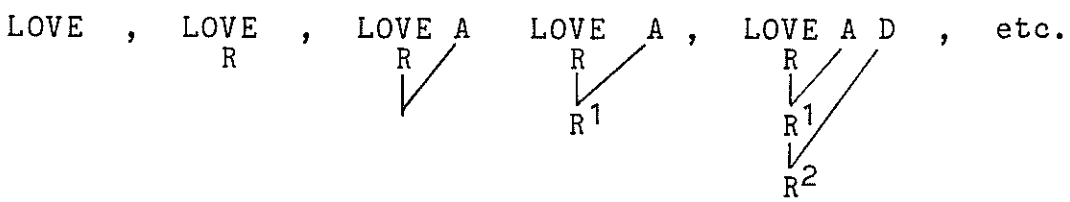
(b) The logical natural language <u>relation-morphemes</u>, whose associated logical representational morphemes we will write as "R", "R^1", "R^2", etc. ("R" for base-relation, "R^1" for one-place relation, "R^2" for two-place relation, etc.), and which, when attached to the syntactic representation of a word-string, (such as to the syntactic representation of "loves" in (1), indicates that the syntactic representation resulting from such attachment, (and, derivatively, the word-string it represents), designates a relation among things, a one-place relation among things, a two-place relation among things, etc. As with the preceding case involving the representational thing-morpheme "T," the symbols R, R^1, R^2, etc. are placed immediately below the symbol to which they are attached.

In more detail: The logical natural language relation morphemes are implicit rather than explicit morphemes of English, being indicated dominantly by global properties of the word-string in which they are (implicitly) realized. These relation-morphemes signal that any word-string parts with which they are associated are to be interpreted variously as a base-relation, a one-place relation, a two-place relation, etc. For example, the lexical natural language morpheme (explicitly realized in English in the word-string) "loves," by virtue of having the natural language two-place relation-morpheme associated with it in a normal reading, is interpreted as a two-place relation relative to the two word-strings "John" and "Mary." As indicated, we use the symbols "R," "R^1," "R^2," etc. as the associated logical representational morphemes associated respectively with the natural language relation morphemes of base-relation, one-place relation, two-place relation, etc. Now the syntactic representation of the word-string "loves" (in a normal reading of (1)) is a lexical representational morpheme in the wider sense,[19] which we build up out of a lexical representational morpheme that we write as LOVE together with the successively attached logical representational morphemes R, R^1, and R^2, and three further representational morphemes A, D, and PRESENT introduced under

---

Note 19. See pages 25, 26 for this notion.

(d), (e), (f) below, as shown on page 40. While generally, when any of the logical representational morphemes "R," "R^1," "R^2," etc., is attached to any syntactic representation the resulting syntactic representation designates, respectively, a relation, a one-place relation, a two-place relation, and so on, for readings of word-strings (such as "love"), that are to designate relations, we construct their syntactic representations by a succession of constructions like the following:



where A and D are special lexical representational morphemes associated with lexical natural language morphemes called "case morphemes." This mode of composition involving case morphemes will be partly described in the present series of examples and detailed much more fully later in Chapter 2. (See (e) and (f) below for a discussion of the specific case morphemes that enter into the syntactic representation of "love" in (1); also see (1') on page 40 to see these examples in context.)

(c) The logical natural language <u>individuator-morpheme</u>, whose associated logical representational morpheme we write as "IND" (for <u>Ind</u>ividuator), and which, when attached to the syntactic representation of a word-string, indicates that the syntactic representation resulting from such attachment (and, derivatively, the word-string it represents) designates an individual (rather than, say, a class.)

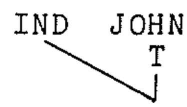
For example, by attaching the symbol

IND

to the compound symbol

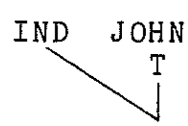
JOHN
T

to form



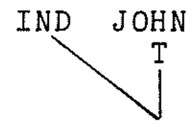
we thereby (i.e., by attaching "IND" in this way) indicate that a (compound) symbol designates an individual.

Note here that the attached symbol is placed horizontally adjacent to the symbol to which it is attached whereas, when attaching the symbols "T," "R," "R^1," "R^2," etc., to other symbols, the attached symbol is placed directly below the symbol to which it is attached, which we can regard as a vertical attachment. All attachments are of these two types, i.e., <u>horizontal attachment</u>, wherein the attached symbol is placed either to the immediate left or right of the symbol to which it is attached, and the symbols are joined by downward sloping lines that meet below them, viz:

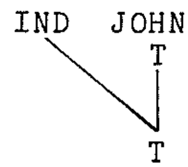


or <u>vertical attachments</u>, wherein the attached symbol is placed immediately below the symbol to which it is attached, with no intervening or auxiliary markings. The only symbols that are vertically attached are "T," "R," "R^1," "R^2," etc.; <u>all other symbols are horizontally attached</u>. (In particular, the case symbols mentioned under (b) above are horizontally attached, as shown on page 33.)

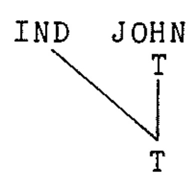
Continuing with our example, if we were, further, to attach "T" to



to form



we thereby indicate that the thus resulting (compound) symbol (and, derivatively, the word-string "John" that it represents) designates an individual thing rather than, say, an individual relation. Note here that a given natural language word-string, e.g., "John," occurring as a part of a containing natural language word-string, e.g., "John loves Mary," can be syntactically represented by at most one[20] sub-expression of the syntactic representation of the containing expression. Thus "John" in the English sentence "John loves Mary" under a normal reading of that sentence is syntactically represented by only the sub-expression



Note 20. Not all word-string parts occurring in a containing word-string need be syntactically represented in a syntactic representation of the latter.

of the syntactic representation (1') (see page 40) of "John loves Mary," and not by any of its constituent sub-expressions:

$$\text{JOHN}\ ,\ \text{T},\ \underset{\text{T}}{\text{JOHN}}\ ,\ \text{IND}\ ,\ \text{IND}\ \underset{\text{T}}{\text{JOHN}}$$

which have entered into its construction.

(d) The logical natural language *present-tense morpheme* (explicitly realized in (1) by the terminal "s" in "loves"), whose associated logical representational morpheme we write as "PRESENT" and which, when attached to the syntactic representation of a word-string to which the relation-morpheme "R" has already been attached (as it would be to the syntactic representation of "loves"), indicates that the syntactic representation resulting from such attachment (and, derivatively, the word-string it represents), designates a relation that occurs at the *present* time. (See (1') and (5') on page 40.)

(e) The lexical natural language *agentive-case-morpheme*, whose associated representational morpheme we write as "A" (for *A*gent), and which, when attached to the syntactic representation to which the logical representational relation-morpheme "R^n," for some non-negative integer n, has already been attached (such as the syntactic representation of "loves") indicates that the n+1st place of the relation is to correspond to the *agent* of the relation. (See (1') and (5') on page 40.)

(f) The lexical natural language *direct-object-case morpheme*, whose associated logical representational morpheme we write as "D" (for *D*irect-object) and which, when attached to the syntactic representation to which the representational relation morpheme "R^n", for some non-negative integer n, has already been attached (such as the syntactic representation of "loves" in (1)) indicates that the n+1st place of the relation is to correspond to the <u>direct object</u> of the relation. (See (1') and (5') on page 40.)

(g) The lexical natural language morpheme explicitly realized in the morph "John," which we write as the representational morpheme "JOHN."

(h) The lexical natural language morpheme explicitly realized in the morph "love," the initial part of the word "loves" in (1), which we write as the representational morpheme "LOVE."

(i) The lexical natural language morpheme explicitly realized in the morph "Mary," which we write as the representational morpheme "MARY."

(j) A final application of the logical representational thing-morpheme "T" to the syntactic representation of the entire word-string (1), as amplified by the attached morphemes identified in (a) - (f) above, indicates that the syntactic representation of (1) resulting from such attachment (and, derivatively, (1) itself) designates a thing and, by virtue of the fact that (1) is a sentence, it will turn out that that syntactic representation designates a special kind of thing called an <u>event</u> (to be defined); relative to (1), this can be described as the event of John loving Mary. (See (1') and (2') on page 40.)

The logical natural language morphemes identified under (a) - (c) are not standard in the literature. That is, what I have identified here as the thing-morpheme, the relation-morphemes, and individuator-morpheme have not, to my knowledge, been isolated and treated as such.

The logical natural language morphemes we distinguish here and in the sequel are those associated with logical representational morphemes in our formal theory of readings. By assigning suitable denotations to the latter within the framework of a semantic theory, logically normal readings can be exactly specified for natural language sentences (like (1)), in which those natural language morphemes are realized.

As indicated at the beginning of this section, our remarks on the "meanings" of the various logical natural language morphemes identified here are intended only to be suggestive: their precise semantic content will be spelled out in the semantic clauses that specify the set-theoretic denotations of the associated logical representational morphemes which are their formal analogues in the theory of readings.

Natural Language Morphemes Realized in (5)

We turn now to the natural language morphemes realized in (5) under the indicated normal reading of (5), that is, that normal reading which can be regarded as comprehended under what we have been calling "the first way of understanding sentences (1) - (8)." All but one of the natural language morphemes realized in (5) are also realized in (1), hence have already been covered in (a) - (j) above. The one natural language morpheme realized in (5) that is not among the morphemes identified under (a) - (j) is the following:

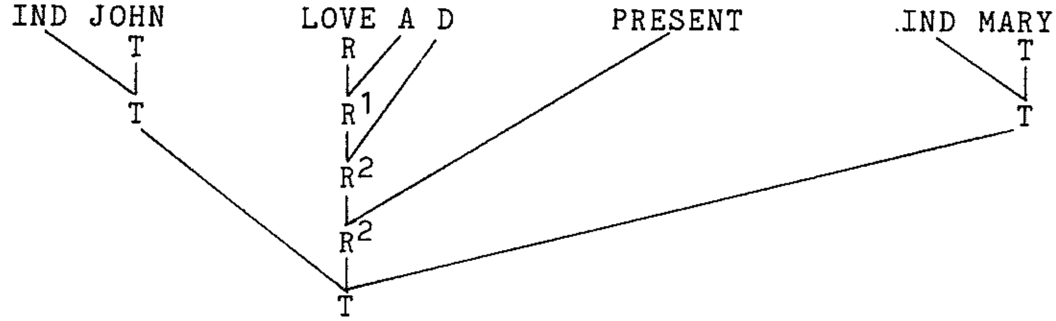
(k) The logical natural language indefinite morpheme (realized in (5) by the morph "some," which is the initial part of the word "something" in (5)), whose associated logical representational morpheme we write as "INDEF" (for Indefinite), and which, when attached to the syntactic representation of a word-string, indicates that the syntactic representation resulting from such attachment (and, derivatively, the word-string it represents) designates an indefinite entity (rather than, say, an individual, a specific class, or the null entity). (See (5') on page 40.)

We note finally:

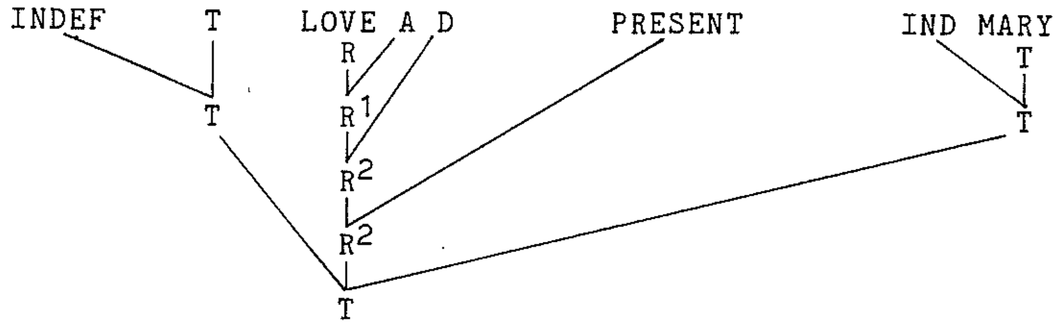
(l) The logical natural language thing-morpheme (explicitly realized in (5) by the morph "thing" which is the terminal part of the word "something" in (5) but only implicitly realized in (1), as indicated earlier in (a)), whose associated logical representational morpheme is "T" (for Thing).

For the sake of definiteness, as well as to motivate the discussion of Chapter 2, we exhibit below syntactic representations (1') and (5') of (1) and (5), respectively, which are built up out of the representational morphemes discussed above under (a) - (l), and which would comprise the respective syntactic components of the dominant normal readings (1*) and (5*) of (1) and (5).

(1')

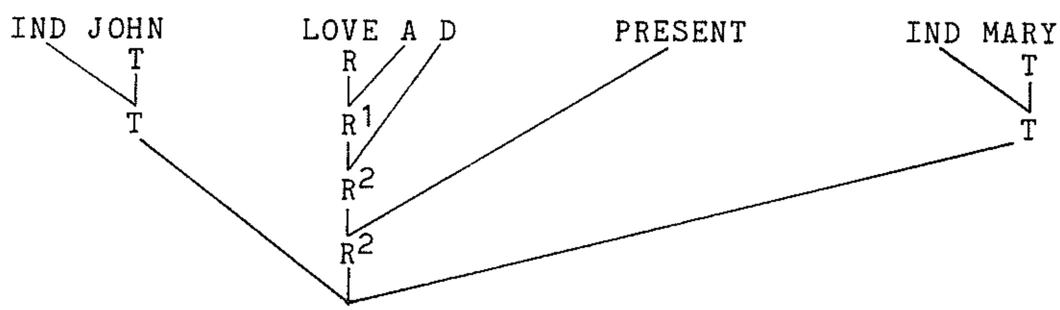


(5')

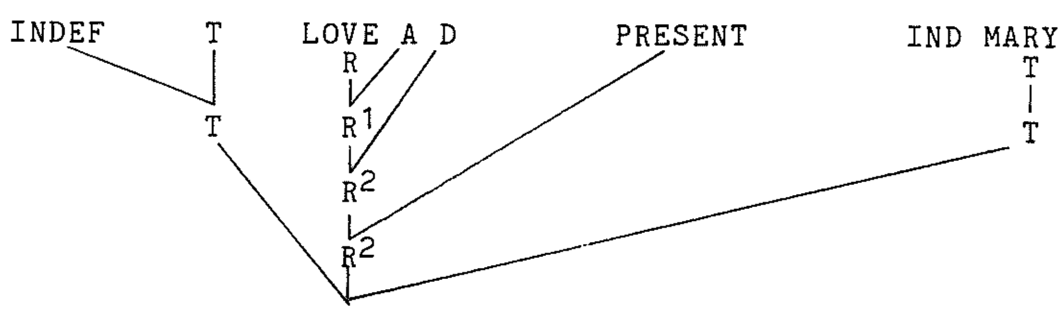


We refer to each of the expressions (1') and (5') as being in <u>sentence-as-assertion form</u> (SAA-form), to be distinguished from corresponding forms ($1'_o$) and ($5'_o$) below, which are obtained by deleting the application of the bottom-most T in (1') and (5') respectively:

($1'_o$)



($5'_o$)



The forms ($1'_o$) and ($5'_o$) are referred to as being in sentence-as-modifier form (SAM-form). A sentence given in SAM-form is interpreted as a modifier, that is, as a function from things and relations to things and relations, whereas a sentence given in SAA-form is interpreted as a "thing" with a special set-theoretic structure, called an event[21].

We refer to a syntactic representation simply as a sentence (of the syntactic representation language, to be specified later) if it is either in SAA or SAM-form[22]. Intuitively, then, a word-string in L is syntactically represented by a sentence in SAA-form if it is to be interpreted as an assertion, and is syntactically represented by a sentence in SAM-form if it is to be interpreted as a modifier.

While somewhat misleading, it may be useful to forward crude paraphrases (1'') and (5'') of the meanings of (1) and (5) expressed by the intended readings whose syntactic representation components are the SAA-forms (1') and (5') respectively:

(1") the individual thing John stands as an agent of the two-place relation present-love relative to the individual thing Mary that stands as a direct object of that relation.

(5") some thing stands as an agent of the two-place relation present-love relative to the individual thing Mary that stands as a direct object of that relation.

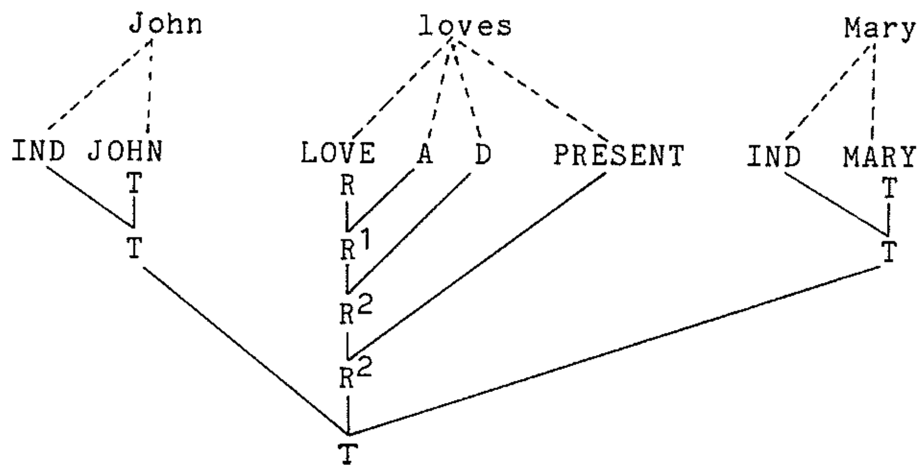
Note 21. See (j), given earlier. Also see pages 165 and 166.

Note 22. See pages

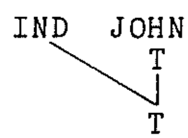
We introduce the following convention: when we wish to explicitly exhibit the relationship between natural language word-strings and their syntactic representations we join those word-strings to their associated syntactic representations by dashed lines as in:

(1)

(1')

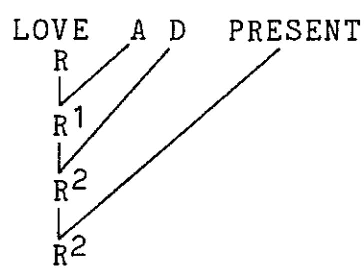


More generally: the associated syntactic representation of a given sub-word-string w of (1) is defined as the largest sub-expression (if any) of (1') that contains all and only those representational morphemes of (1') that are connected by dashed lines to the words (or word-string parts) occurring in w. Thus the word-string "John" in (1) has the syntactic representation:



;

the word-string "loves" in (1) has the syntactic representation:



;

the word-string "Mary" in (1) has the syntactic representation

;

and the entire word-string (1) has the syntactic representation (1').

One of the purposes of this study is to develop semantic theories s relative to which one can, as a particular application, specify normal readings (1*), (5*) of sentences such as (1) and (5), respectively, as pairs <(1'),s>, <(5'), s>, and rigorously prove that (1*) and (5*) induce, among other entailments, the entailment of (5) from (1): that is to say, we wish to be able to specify a reading assignment A on the set {(1),(5)} that assigns to each of the English sentences (1) and (5) readings (1*) and (5*) such that A induces precisely the following entailment relation among the sentences of that set:

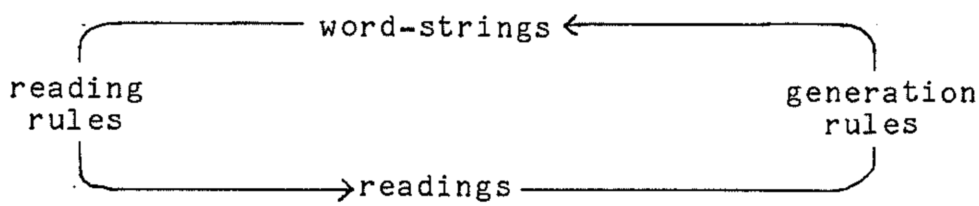
{<{(1)}(1)>,<{(5)}(5)>,<{(1)}(5)>},

which contains the nontrivial entailment of (5) from (1) as well as the two trivial entailments of the sentences (1) and (5) from the sets {(1)} and {(5)}, respectively.

1.6 Reading Rules and Generation Rules

The grammer of a natural language can be conceived in either of two related ways: first, as an interpretation grammar, described by a set of rules called reading rules, which associate readings with (grammatical) word-strings that are normal relative to entailment; second, as a generation grammar, described by a set of rules called generation rules that associate grammatical word-strings with such readings. Thus reading rules and generation rules can be considered as functional inverses of each other. The formal treatment of interpretation and generation grammars can be undertaken independently of each other. From the point of view of the psychological modeling of language behavior, however, interpretation grammars need to be incorporated within generation grammars, for the following reason: interpretation grammars provide explanatory models of human competency in understanding natural language word-strings, while generation grammars provide explanatory models of human competency in producing natural language word-strings. Hence, allowing the intuitively plausible assumption that a person ordinarily understands that which he produces though he may well not produce, or even be able to produce, that which he understands, it would follow, still from a psychological modeling perspective, that interpretation grammars should be incorporated within generation grammars, while generation grammars need not be incorporated within interpretation grammars. Thus it would appear that an adequately specified interpretation grammar would not only provide a model for competency in understanding natural language word-strings, but would also be required as a component in an adequate specification of a generation grammar insofar as the latter were intended to provide a model for competency in <u>producing</u> natural language word-strings. The relationship between interpretation grammars and generation grammars has not (to my knowledge) been addressed as such in the literature. In my judgment, the study of this relationship can be meaningfully undertaken only by adopting the broadest sort of perspective on the fundamental relationship between natural languages and their readings, and utilizing a sufficiently precise notion of reading to permit the extraction of determinate consequences concerning those relationships.

It was remarked above that reading rules can be regarded as inverses of generation rules in the sense that they account for the way that underlying readings are assigned to grammatical strings of words, while generation rules account for the way that grammatical strings of words are constructed from readings. We can schematically depict this relationship as follows:



1.7 As Context Pertains to Word-Strings

1.7.1 Word-Strings: Tokens and Types

When one speaks of a word-string or word-string-part w being produced in a given context-of-utterance C, that which is produced is, properly speaking, a token w# of w, rather than w itself. That is, the token w# of w that a language-user produces is a physical entity, identified visually, aurally, or tactually, as a pattern of some sort, such as a mark or marks on paper like those appearing on this page, which form a visual pattern of printing-ink particles distributed onto a paper surface.

The traditional terminology used to describe the relationship between w and w# is that the word-string w is the type of the token w#. The type itself is usually regarded as an abstract (i.e., non-physical) entity of some sort having, so to speak, a metaphysical status of being that underlying form which has the given tokens as its physical instances. Thus one produces tokens in a context-of-utterance, not types, i.e., not the word-strings themselves. However, for ease of expression, we shall continue to speak of the word-string w, i.e., the type, as being itself produced in a context-of-utterance C, and by this mode of speaking, intend to be understood as meaning that some particular token w# of w has been produced in C. We also need to update another mode of expression that we use: namely that of an occurrence of a word-string. Accordingly, we will understand an occurrence of a word-string w as a particular token w# of w, and we will say that a given word-string w_1 has an occurrence within a given word-string w_2 to mean that some token $w_1\#$ of w_1 is an actual physical part of a token $w_2\#$ of w_2. In this sense, we will speak of an occurrence of w_1 preceding or following another occurrence of w_1 or of an occurrence of w_1, preceding or following an occurrence of another word-string w_2.

Since we characterize a natural language L as a set of word-strings, from the perspective of the present discussion, a natural language L is a set of types. We will occasionally, as in the following paragraphs, use the expression "L-token" to mean a token of a type of the natural language L.

1.7.2 Contexts-of-Utterance

A context-of-utterance can be regarded in various ways: first, it (like a word-string-token) can be regarded as a physical entity, that is, as a physical configuration[23] comprised in part of verbal (i.e., linguistic) entities of a physical sort, i.e., word-string tokens (written or spoken), and comprised in part of non-verbal (i.e., non-linguistic) entities, also of a physical sort; second, it can be regarded as a way of understanding or interpreting a context-of-utterance-as-a-physical-entity. Thus we distinguish between *context as a physical entity* and *context as an interpreted physical entity*. It is context as an

---

Note 23. By a "physical configuration" I mean a physical entity that has parts and can be spoken of as a "physical configuration comprised of" those parts. It is to be distinguished from the set having those parts as elements in the same sense that a brick wall can be regarded as a physical configuration of its constituent bricks, and is to be distinguished from the set of those same bricks.

interpreted physical entity that best fits our present use of this notion, namely as something with respect to which a reading of a word-string is or is not normal, for whether or not a reading of a given word-string induces entailments that are consistent with language users' intuitions relative to the context-of-utterance in which that word-string is produced means that that context must be "interpreted," i.e., understood in a certain way by the language user if his intuition is being exercised with respect to it.

However, the sense of context as a physical (rather than interpreted) entity also has theoretical utility, and should be preserved along with context-as-interpreted, for, when taken as a physical entity, the context-of-utterance C of a word-string token e can be regarded as a <u>physical extension</u> of e; that is, C, like e, can be regarded as a (sign) token that itself can be understood in various ways, thus affording an interesting parallelism between word-strings and contexts-of-utterance: A word-string token is a physical entity that can be understood in various ways, and each such way is formally represented by a reading of that word-string. That is, we formalize the notion of a "way of understanding" a word-string in the exact notion of a "reading" of that word-string. We could attempt an analogous formalization of the notion of a "way of understanding" of a context-of-utterance, as, for example, by developing an extended notion of reading which applies, not only to word-strings, but also to their contexts-of-utterance. Such a pursuit, however, would take us beyond the more limited goals of this study. A context-of-utterance C of a word-string token e, regarded as a physical entity as indicated above, is thus a physical configuration comprised of a _verbal part_ and a _non-verbal part_. The verbal part of C is comprised of word-string tokens which, in turn, can be broadly separated into those which are spatially/temporally close to the token e, called _immediate verbal parts of C relative to e_, and those which are spatially/temporally more removed, called the _remote verbal parts of C relative to e_. The non-verbal part of C is comprised of non-linguistic elements which can also be separated into _immediate and remote non-verbal parts of C relative to e_, the former containing those physical entities that are spatially/temporally close to e, such as the speaker and other persons in the immediate spatial/temporal environment in which e is produced, their gestures, postures, physical relationships to each other, etc., the latter containing configurations of physical entities which are more spatially/temporally remote from the production of e, such as those generally belonging to what might be regarded as the general cultural and historical setting surrounding the production of e.

A given context-of-utterance C#,[24] considered as physical, of a word-string token e, when interpreted by a given language user, yields an _interpreted_ context-of-utterance, which we might

---

Note 24. We use the symbol # to indicate that the entity in question is a token, i.e., a physical entity that could be understood, interpreted, etc., in some way so that it functioned as a "sign." In analogy to the type/token distinction for word-strings, we can also consider the notion of _context type_, designate by C#', is assumed to provide a body of <u>information</u>, varying somewhat from user to user but largely shared by different users, that wholly determines a <u>reading hierarchy</u> of e, which is the partially ordered set of readings of e that are normal with respect to C#', where one normal reading r_1 of e precedes-with-respect-to-C#' another normal reading r_2 of e if and only if the degree of normality of r_1 with respect to C#' is greater than or equal to the degree of normality of r_2 with respect to C#'.[26]

We also distinguish between <u>complete</u> and <u>partial</u> contexts-of-utterance C# of a given word-string e, considered as a physical context. The former is comprised of the entire verbal and non-verbal environment that surrounds the production of (a token e# of) e, including all immediate and remote elements that could possibly be used by any language user in understanding e.[25] We contrast the complete context-of-utterance of e with what we refer to as <u>partial</u> contexts-of-utterance of e, (also considered as physical contexts,) each being a particular physical subconfiguration of the

---

provided that we allow that two context tokens can differ inessentially in their physical character in the sense that they functioned identically (or nearly-so) as signs, hence can be considered as instances of the same context "type", wholly analogous to the way that two word-string tokens can differ inessentially in their physical character in the sense that they function identically (or nearly-so) as signs, hence are considered instances of the same word-string type.

Note 25. We do not address here any questions relating to the extent of such a complete context-of-utterance of a given word-string, leaving open metaphysical issues such as whether the complete contexts of utterance of distinct word-strings are themselves distinct, or if they are distinct, precisely where the difference lies.

Note 26. A given reading assignment A on a set K of natural language sentences e can be of varying degrees of normality with respect to a given context-of-utterance C. A given degree of normality of A with respect to C, which can be thought of either quantitatively, as say, a number, or descriptively, as say, "slight," "moderate," "high," etc., is determined by the extent to which the entailment relation on K induced by A is consistent with the intuitions of language users with respect to the context C, that is, consistent with the intuitions of language users regarding intuitive inter-entailments among the sentences of K relative to the context C in which they are produced (putting aside questions regarding precisely what such consistency means or how this might be measured). The degree of normality of a given reading r of a natural language sentence e with respect to C can then be characterized as follows: Let V(C) be the verbal part of C, understood as including the sentence e; then every reading assignment A on V(C) can be regarded as having a degree of normality with respect to C, and we can define _the degree of normality of the reading r of e with respect to C_ to be the maximal degree of normality of any reading assignment A' on V(C) with respect to C such that A'(e)=r. Finally, the degree of normality of a given reading r of a natural language word-string e (not necessarily a sentence) with respect to C can be defined as the maximal degree of normality of any reading r^o of any sentence a in which e occurs (as an interpretable part) with respect to C such that r^o is a subreading of r in the sense that, letting $r^o = \langle a', s \rangle$, and letting $r = \langle e', s \rangle$, then e' is a part of a'.

The notion of "degree of normality" suggests a way to approach the characterization of the notion of "degree of sameness of meaning." Since normality in this study is relativized to entailment, perhaps we should speak of defining "degree of sameness of entailment-relevant meaning." Accordingly, we might regard two natural language word-strings e_1, e_2 to have a given degree of sameness of entailment-relevant meaning with respect to a given context-of-utterance C as a function of the amount of overlap (putting aside questions regarding the measurement of "amount of overlap") the degrees of normality of normal readings with respect to C of e_1 and of corresponding normal readings with respect to C of e_2, and regard e_1, e_2 to have a given degree of sameness of entailment-relevant meaning as a function of the degrees to which they have sameness of entailment-relevant meaning with respect to all or, possibly, at least to "usual" contexts-of-utterance C. The case of entailment-relevant synonymy between two word-strings e_1 and e_2 would require the highest possible degree of sameness of entailment-relevant meaning, in the sense that the normal readings $r_1, \ldots, r_m$ of e_1 could be mapped onto the normal readings of e_2 under a 1-1 mapping J such that, for all contexts-of-utterance C, and for all $1 \leq i \leq m$, r_i had the same degree of normality for e_1 with respect to C that $J(r_i)$ had for e_2 with respect to C and, furthermore, r_i and $J(r_i)$ were equivalent in the sense that they had the same semantic theory (as second components) and the syntactic representations comprising their complete context-of-utterance of e. Generally speaking, there are a multiplicity of possible partial contexts-of-utterance of a given word-string token e, each comprising a context-token C# which, under some way of understanding C#, say C#', provides a particular body of information that wholly determines a reading hierarchy of e. Two different individuals might attend to different aspects of the complete context-of-utterance of e, hence have different partial contexts-of-utterance of e and, even if attending to precisely the same aspects of the complete context, that is, even if attending to precisely the same partial context-of-utterance of e, the two individuals may yet interpret them differently, or one and the same user may interpret given elements of a partial context of e differently as it becomes augmented with further elements in terms of which he can re-interpret the previous ones, etc.

We note that a given reading of a given word-string e can be normal or non-normal only with respect to interpreted contexts-of-utterance of e. Accordingly, we will continue to use the simple phrase "context-of-utterance" rather than the more cumbrous phrase "interpreted partial context-of-utterance," as well as abandoning the "#" notation for contexts except where it is essential to make the distinction, and can thus speak of a multiplicity of contexts-of-utterance of a given word-string (token) e. We also note that that reading may be normal with respect to one context-of-utterance C, but not with respect to another.

first components had the same interpretations under their common semantic theory.

Generally speaking, lexical normality is more context-dependent than is logical normality, although logical normality is also context-dependent to some degree, as when the English word-string "a" is variously syntactically represented by the logical representational morphemes "DEF," "UN," and "INDEF," according to the context in which "a" occurs.

There are readings for some word-strings that are normal with respect to most usual contexts-of-utterance, so that the word-string itself can be considered as (i.e., taken as identical to) its context-of-utterance.

I will refer to one reading a of a word-string w as being *more normal* than another reading b of w if a has a higher degree of normality with respect to more of the usual sorts of contexts-of-utterance of w than b, and will refer to a as *dominant* if there is no reading b of w that is more normal than a.

Pragmatics. Pragmatics generally concerns the relationship between word-strings and their possible normal readings with respect to given contexts-of-utterance, and, primarily, the nature of the reading rules governing the identification of the possible normal readings of given word-strings with respect to given contexts-of-utterance. (Recall: reading rules associate readings with word-strings with respect to contexts-of-utterance, while generation rules associate word-strings with readings with respect to contexts-of-utterance.)

The identification of a normal reading for a given word-string with respect to a given context-of-utterance includes both the identification of a suitable syntactic representation for that word-string as well as the identification of a suitable semantic theory that interprets that syntactic representation. This identification is an "inductive inference" of sorts from the word-string and the context in which it is produced to that reading. That is, it is an inference from the word-string and context-of-utterance to a syntactic representation of that word-string and a semantic theory to interpret that syntactic representation. Selection of a reading commits one to a way of understanding that word-string that is, generally speaking, underdetermined by the word-string considered in isolation from context, but can be regarded as reasonably determined, at least for practical communication purposes, by the joint consideration of the word-string and the context-of- utterance in which it is produced. Reading rules are intended to codify this inference into a set of rules that are formulated in terms of word-strings and contexts-of-utterance. We can also regard the reading rules (and the inference rules they codify) as rules for disambiguating word-strings, since the ambiguity of a word-string is simply the (inevitable) presence of multiple possible normal readings of that word-string, hence disambiguation of a word-string can be viewed simply as the process of choosing one of these possible normal readings.[27]

Note 27. There are various signals which help to disambiguate word-strings which are, properly speaking, not part of the context of utterance of that word-string, so need to be considered as part of that word-string. Such signals are very pervasive in oral speech, provided by intonation and stress patterns, which can be considered part of the word-string. Such signals are much less pervasive in written speech, as in the examples of this study, but have some written analogues, indicated, variously by bold-faced type, italics, underlining, or the use of special spacing or punctuation. Such marks, like those of stress or intonation in oral speech, seem to be best considered as part of the word-string. On the other hand, a case can possibly also be made that they are not part of the word-string at all, but rather are a part of the context of utterance. For consider: stress could be indicated in oral speech by a physical gesture attending the production of the word-string, such as raising one's hand(s) or extending one's index finger. Such gestures would have to be considered as part of the context of utterance rather than of the word-string itself. Their function would be the same as that provided by actual stress on the intended word-strings.

1.8 Ambiguity in Natural Language

It has often been pointed out that natural language is inherently ambiguous. Some have suggested that this is a deficiency as compared to artificial languages, others have suggested that this is a virtue of natural language.

In order to motivate our general program, I will discuss in this and the following two sections the notion of ambiguity and the relationship between natural and artificial languages regarding ambiguity.

A natural language word-string (considered in isolation from any context of utterance) can be regarded as ambiguously expressing its various normal readings. If it has only one logically normal reading with respect to ordinary contexts-of-utterance or if all of its readings that are logically normal with respect to some ordinary context-of-utterance are equivalent, then we say that it is logically unambiguous and hold that those equivalent logically normal readings can be indifferently indicated simply by exhibiting the word-string itself[28]. For example, the English word-string "John loves Mary" can be regarded as having a single logically normal reading in this sense. Most natural language word-strings beyond those of this simple sort, however, have multiple inequivalent logically normal readings. This is not to say that all logically normal readings of a word-string are equally likely, but only that they

---

Note 28. Any natural language word-string typically has multiple possible lexically normal readings, so that analogous notions relating to lexical ambiguity are uninteresting.

are each possible under some ordinary condition of use of that word-string, i.e., with respect to some ordinary context of utterance. When a word-string has multiple inequivalent logically normal readings, it is no longer possible to indicate an intended reading simply by exhibiting the word-string itself. Nor is it in general possible to indicate an intended logically normal reading of a given word-string by exhibiting a _second_ word-string that is itself logically unambiguous and whose single logically normal reading is precisely the intended reading of the original ambiguous word-string. Let us call a second word-string introduced for this purpose a _canonical alternate_ of the given word-string, where a canonical alternate of a given natural language word-string can be a word-string of the same language or of another language, _natural or artificial_. As already indicated, it is unlikely that, given a logically ambiguous word-string, a canonical alternate of it can be found which is itself logically unambiguous.

For example, the English sentence

(1) Every man loves some woman

is generally appreciated as being logically ambiguous. One might attempt to indicate two of its possible logically normal readings by using as respective canonical alternates the English sentences:

(2) Every man is such that he loves some woman

and

(3) Some woman is such that every man loves her

But the suitability of (2) and (3) for this purpose depends, in part, on (2) and (3) being themselves logically unambiguous. This may not hold: for consider, first, that the pronoun "he" in (2) and the pronoun "her" in (3) may or may not refer back to the denotations of "every man" and "some woman" in (2) and (3) respectively, though such latter readings may be less normal than the dominant logically normal readings of (2) and (3). For example,[29] regarding (2), a particular man, say Jones, may be such that he loves some woman only on the condition that every man have some particular property which he shares with every

---

Note 29. There are two ways of understanding the phrase "such that" in (2), which we can distinguish by reference to two further canonical alternates (2.1) and (2.2) of (2).

(2.1) There is a property possessed by every man by virtue of which he loves some woman;

(2.2) There is a property possessed by every man and that property is that he loves some woman.

If in (2), "he" is regarded as refering back to "man," then (2.2) expresses the more correct reading; if in (2), "he" is regarded as refering to a particular man, say Jones, then (2.1) expresses the more correct reading, for compare:

(2^{o}) Every man is such that Jones loves some woman

and the possible canonical alternates:

(2.1^{o}) There is a property possessed by every man by virtue of which Jones loves some woman.

(2.2^{o}) There is a property possessed by every man and that property is that Jones loves some woman.

Here (2.1^{o}) expresses by far the more correct reading of (2.1), while the reading expressed by (2.2^{o}) is extremely unlikely. In the ensuing example(2a), we understand the phrase "such that" in the sense expressed by (2.1).

other man, e.g., being mortal: then (2) asserts that, indeed, every man is such (i.e., has the property of being mortal) and, because it is the case that every man is such, the particular man Jones (i.e., the denotation of "he") loves some woman; note further, that this way of construing (2) re-opens the possibility of reading "some woman" as a particular woman, a reading which (2) was designed to eliminate. To convince oneself of this consider the following sentence--exclusive of its parenthesized parts--with a structure somewhat akin to that of (2), the situational context of which is already known, and whose "most normal" reading would be wholly analogous to the first of the two latter readings for (2) which were possible but not "most normal" for (2).

(2a) Every man is such that He sent some man (Jesus)(to save him).

Because of the known context there is no temptation to have "He" refer back to "every man," as one would have had with (2).

There are also normal readings of (2), albeit more marginal ones, in which "some woman" is understood as a particular woman, yet in which "he" refers back to "every man," in a <u>collective</u> sense, in which "he" means "every man," and also in a <u>distributive</u> sense, in which "he" applies to each man singly. We can indicate the intended readings of (2) as follows: let the particular woman be Mary, and let the particular common property shared by every man be (a): the property that no man has ever seduced Mary, if "he" refers to "every man" in the collective sense; and (b): the property that he has not ever seduced Mary, if "he" refers to "every man" in the distributive sense; (2) then asserts respectively that (a) every man is such (i.e., has the property of never having seduced Mary) and, because it is the case that every man is such, every man indeed does love Mary, and (b) every man is such (i.e., has the property of never having seduced Mary) and, because it is the case that he (that man) is such, he (that man) indeed does love Mary.

Sentence (3) is not quite so logically ambiguous as sentence (2); in particular, there would not appear to be any reasonable context-of-utterance with respect to which some reading of (3) in which "some woman" was not understood as a particular woman would be logically normal. However, with a little ingenuity some possible context-of-utterance could be described with respect to which a reading in which "some woman" was understood in this way could be considered logically normal. We will not forward any such possibilities here; for the above examination of sentence (2) already strongly suggests that since the possible logically normal readings of sentence (1) are not adequately separated out by the sentences (2) and (3), consequently (2) and (3) cannot be considered satisfactory canonical alternates of (1); and that, more generally, natural language sentences are not suitable for this purpose.

By the above considerations, I of course do not intend to suggest that (2) and (3) have the same meaning, for they do not: for even if they had precisely the same set of logically normal readings (which is doubtful) they would at the very least differ in the degree of normality of those readings, that is, they might have different "reading hierarchies" (as described in Section 1.72 earlier). A case in point is to be found in each of the above readings of (2) wherein "some woman" was understood as a particular woman, for even though these readings were arguably normal readings of (2), they were each of low degree of normality relative to (2), whereas they were of a high degree of normality relative to (3); indeed, one of those readings is clearly the dominant reading for (3).

We hypothesize that language users distinguish the intended meanings of sentences like (2), (3), or even (1) by (i) attending to the context-of-utterance of a word-string in the sense of becoming aware of the information the context conveys, and (ii) identifying the intended reading of the word-string as that reading that has the highest degree of normality with respect to that context.[30] The intended reading will in general vary as the context-of-utterance varies. Readings of (2) that I referred to as "marginal" are "possible" in the sense that contexts of utterance are possible with respect to which such readings would have the highest degree of normality with respect to that context. Moreover, in the course of this study we will urge that the resolution of natural language ambiguity is not to be effected at the level of syntax or semantics, but rather at the level of pragmatics.

Our foregoing discussion concerning sentences (1), (2), and (3) highlights a certain aspect of natural language ambiguity, namely, that logical ambiguity in natural language is pervasive, since it can be found even with fairly simple word-strings (like

(1)), and that such ambiguity does not appear readily resolvable within natural language itself, insofar as reasonable canonical alternates for natural language word-strings that are themselves logically unambiguous cannot be easily found among other natural language word-strings (like (2) and (3)).

One might perhaps be tempted to indicate an intended reading of a word-string by exhibiting _multiple_, i.e., one or more, further word-strings which possess a common single normal reading that is precisely the intended reading of the original word-

---

Note 30. In practice, language users customarily invoke the context-of-utterance in which a given word-string is produced in order to identify those normal readings of that word-string which have the highest degree of normality with respect to that context-of-utterance. If the context-of-utterance were sufficient to identify a unique normal reading of highest degree with respect to that context, we would regard that word-string as _unambiguous_ with respect to it; if that context-of-utterance were not sufficient, we would regard that word-string as _ambiguous_ with respect to it. We can describe the situation as follows in terms of the notion of "reading hierarchy": We assume that, for any word-string e and for any context-of-utterance of e, the reading hierarchy of e relative to C contains certain maximal elements, which follows from the assumption that the notion of degree of normality is meaningful. If two readings of e are maximal then, by the meaning of maximality, those two readings are incomparable relative to degree of normality with respect to C (though they may well be so comparable with respect to other contexts-of-utterance), which means that there is no way that the given context C can incline the language user's intuitive judgments pertaining to the entailment relations into which e enters which would enable him to distinguish between the two maximal readings of e. Accordingly under these conditions we would regard e to be ambiguous with respect to C, and to be unambiguous with respect to C otherwise. We can embody this usage in a definition as follows: A word-string e is _unambiguous with respect to any given context-of-utterance C_ if and only if the reading hierarchy of e relative to C has a _unique_ maximal reading (i.e., a reading of e that has a higher degree of normality with respect to C than any other reading of e that is logically normal with respect to C). In the event that that maximal reading is not unique, then e is _ambiguous with respect to the given context-of-utterance C_.

string one wishes to indicate. Thus, for example, one might try to indicate the intended meaning of (1) by exhibiting (2) together with further sentences such as (2'):

(2') There exist at least two men who do not love the same woman

(2') has a logically normal reading that is consistent[31] with apparently only the intended logically normal reading of (2). If this is indeed the case, then that logically normal reading is precisely the intended reading of (1), and (2) and (2') could indeed be jointly used to indicate that intended reading. Note that this does not suggest that the conjunction of (2) and (2') would be logically unambiguous, but only that there is a logically normal reading of (2') that is consistent[31] with only one certain logically normal reading of (2). This approach to disambiguation is, however, pragmatic rather than semantic or syntactic, for the introduction of (2') consists, essentially, of a further articulation of the context-of-utterance by introducing (2') as part of the verbal part VC of the context-of-utterance with respect to which the intended reading of (2) has the highest degree of normality. The context-of-utterance in which a word-string is produced is a filter for selecting the intended reading of that word-string; the introduction of further word-strings, like (2') above, for the purpose of elucidating that intended

---

Note 31. "Consistency" has a well-defined meaning here, namely, that there exists an interpretation within the semantic theory that is common to both logically normal readings such that the syntactic representations of (2) and (2') respectively drawn from those logically normal readings are both true under that interpretation.

reading, function also to further delineate that context-of-utterance, refining it as a filter, as it were.

1.9 Formal Languages and Logics

A formal language is a recursive set of expressions, each built up in exactly one way from component expressions. A logic is a formal language together with a semantic theory for interpreting it, formulated within a suitable metalanguage for the logic. Thus if we regard an expression of a logic as a symbol-string, in analogy with the word-strings of natural languages, we can apply the concept of a "logically normal reading" to the expressions of a logic. The difference is that the expressions of a logic, unlike the expressions of a natural language, can (i) be regarded as syntactic representations of themselves, and (ii) have a specific associated semantic theory. Thus a logically normal reading of an expression of a logic would be one that induces entailments that are consistent with the logic users' intuitions regarding the interconnections among the logical representational morphemes of the logic. If the logic is a first-order predicate logic, then those morphemes will consist of the two quantifiers: "(A)," "(E)," and the five sentential connectives: "&," "v," "¬," "⟶," and "⟷." If a logic user's intuitions were not consistent with the induced entailments, he would probably cease to use that logic, or modify it accordingly, as has historically occurred, for example, with so-called "intuitionistic logic" variants of the predicate calculus. The question of lexically normal readings of an expression of a logic might be pursued in conjunction with the formalization of concepts that are accorded lexical inter-connections that are unusual or nonstandard relative to the standard theory of the discipline being formalized, as, for example, (i) in a formalization of non-Euclidean geometry which replaces the axiom of the parallels by, say, its negation, thereby inter-relating lines and points in ways that are counter-intuitive, i.e., non-normal relative to ordinary (Euclidean) geometric intuition, and (ii), in so-called "nonstandard analysis" which formalizes the Newtonian concept of "infinitesimal" and inter-relates it to customary real numbers in a way which is unusual hence "nonstandard" in the theory of real numbers. However, the question of lexical normality in a logic intended to formalize some mathematical system is not ordinarily of great interest insofar as there is less of an issue about the real-world inter-connections among concepts than there is for logics intended to formalize some system of "real-world" intuitions to the effect, say, in part, that cats are animals.

The main virtue of a logic is that its semantic theory assigns essentially one logically normal reading to each of its expressions.[32,33] That is to say, each expression of the formal language is ordinarily intended to be essentially logically unambiguous, where by "essentially" I mean that it is subject

---

Note 32. I use "expressions" rather than "word-strings" here because it applies to precisely interpretable linguistic entities.

Note 33. If the semantic theory also assigns exactly one <u>lexically</u> normal reading to each of its expressions, then the theory is said to be <u>complete</u>, by which is meant that every sentence of the formal language is either true in every interpretation of the semantic theory or false in every interpretation of that theory.

only to the residual logical ambiguities in the informal mathematical metalanguage within which that semantic theory is formulated. As has been frequently noted in the literature on formal languages, one can really do no better: for if one elected to formalize the metalanguage, hence the semantic theory formulated within it, the residual logical ambiguities would simply be deferred to the semantic metametalanguage within which that formalization was specified. Thus the residual ambiguity would still be propagated downward to the object language expressions--i.e., the expressions of the logic itself--with undiminished force. Thus, one can either attempt to indicate the intended readings of

(1) Every man loves some woman

(discussed in the preceding section) by using, as canonical alternates to (1), word-strings of a natural language, such as

(2) Every man is such that he loves some woman

(3) Some woman is such that every man loves her

(of the preceding section) which, as noted earlier, retain considerable logical ambiguity, or expressions of a logic, such as those of predicate logic, which as just remarked, suffer only the residual ambiguity inherent in the semantic meta-language of that logic. If we were to choose predicate logic for this purpose, we could consider (4) below as a canonical alternate of (2) (hence of (1), above),and we could consider (5) below as a canonical alternate of (3) (hence also of (1)):

(4) (x) (man x $\longrightarrow$ (Ey) (woman y & loves xy))

(5) (Ex) (woman x & (y) (man y $\longrightarrow$ loves yx).

That is, (4) could be taken as expressing the intended reading of (1) which we had attempted to express by (2) above, and (5) could be taken as expressing the intended reading of (1) which we had attempted to express above by (3).

Two questions arise: (a) in what sense are (4) and (5) logically unambiguous, i.e., in what sense do (4) and (5) have only one logically normal reading, and (b) in what sense do (4) and (5) "express" logically normal readings of (2) and (3) respectively; more precisely, in what sense can (4) and (5), taken together with the semantic theory of the predicate calculus, be regarded as logically normal readings of (2) and (3), respectively? The answer to question (a) is that (4) and (5) are interpreted by the semantic theory of the predicate calculus, which provides a precise model-theoretic description of the conditions under which (4) and (5) are true, hence provides each of (4) and (5) with exactly one reading, modulo, of course, the residual ambiguity of the semantic theory for the predicate calculus which is formulated in mathematical English. The answer to question (b) is somewhat less straightforward: for it asks, essentially, why the semantic interpretations which the semantic theory of the predicate calculus assigns to (4) and (5) should be regarded as representing the respective meanings of the canonical alternates (2) and (3) of (1). That is, it asks why (4) and (5) together with the semantic theory of the predicate calculus should be regarded as logically normal readings of (2) and (3) respectively.

The answer to question (b), I believe, lies partly in the fact that the translation of (4) (the case for (5) is wholly analogous) into the semantic metalanguage of the predicate calculus, which translation we might express in mathematical English[34] as "tr(4)" below, under the most usual ways of understanding tr(4) and (2).

tr(4) For every individual thing x, if x belongs to the class of individual things designated by "man," then there is an individual thing y such that y belongs to the class of individual things designated by "woman" and such that the pair ⟨xy⟩ belongs to the class of pairs of individual things designated by "loves."

The above considerations only partially answer question (b), which asks in what sense do (4) and (5) "express" the possible logically normal readings of (1). A second sort of consideration provides a fuller answer: the predicate calculus (and any legitimate extension thereof) has a precisely defined notion of semantic entailment. Now by translating the semantic entailments of (4), say, as spelled out by the semantic theory of the predicate calculus, back into English, we find that they are largely[35] consistent with

Note 34. tr(4) could have been written here with a heavier infusion of mathematical symbolism by expressing its content wholly within the language of informal set theory. But keeping tr(4) English-like makes the intended point somewhat clearer.

Note 35. I say "largely" rather than "wholly" because there are cases such as those relating to the material conditional where users' logical intuitions are apparently violated.

the logical intuitions of English speakers regarding what is intuitively entailed by those English translations. For example, one semantic entailment of (4) is:

(4) (x) (man x $\rightarrow$ (Ey)(x loves y))

A translation of (4') back into a sentence of English that would share its most likely reading would be:

(2') Every man is such that he loves some thing which, indeed, intuitively follows from (2). Thus the fact that semantic entailments of the predicate calculus are consistent with the logical intuitions of English speakers constitutes a second reason for accepting (4) as a possible reading of (1).[36]

The predicate calculus has, however, certain limitations as a meaning representation language for natural languages insofar as natural language word-strings, beyond those of a somewhat stilted sort such as (2) and (2') above, require suitable extensions of the predicate calculus in order to assure that two natural language sentences with non-identical readings be translated by distinct predicate calculus expressions. Such extensions of the predicate calculus have been attempted in order to handle constructions such as passives, (selected) determiners, modals, intensions, cases, etc., in order to enhance the capability of the predicate calculus to provide suitable meaning representations for a wider class of natural language expressions. The price paid for such extensions has traditionally been very high: for it has produced an excessive

---

Note 36. The reason for this is ultimately grounded in assumption A (page 2), insofar as that assumption can be reasonably construed as implying that the way that given sentences are understood is evidenced partly by patterns of entailments that language users regard as holding among them; then, under this construal, the isomorphism between the respective patterns of entailments of English sentences and their translations into the metalanguage of the predicate calculus that are regarded by language users (who understand those metalanguage translations) as holding among them evidences (at least in part) that those English sentences and their respective translations into the metalanguage of the predicate calculus are "understood" in the same way, which is to say, they "express" the same normal readings.

notational encumbering of the predicate calculus, requiring logical expressions of great complexity to represent the meanings of even fairly simple natural language word-strings. Even more importantly, the resultant logical expressions do not bear a simple and coherent relationship to the intuitive syntactic and semantic structure of the natural language word-strings whose readings they are intended to express, and, consequently, cannot be considered to provide an optimal model of the syntactic and semantic structure of the language itself. For these reasons, I do not regard predicate-logic-based meaning representation languages as offering an adequate technical base for a theory of readings. Our approach is to offer an alternative logic which does not appear to suffer the above-mentioned difficulties. We call this alternative logic a thing-relation (TR) logic. Conceptually, TR-logics appear to provide the right sort of intuitive base for the study of natural languages in the sense that they provide what appears to be an optimal model of the syntactic and semantic structure of a very extensive family of natural languages (called TR-languages in this paper, and described below in Section 2.1). Notationally, TR-logics are simple, flexible, and widely applicable, enabling the treatment of a very wide class of diverse phrasing constructions across many different languages. Moreover, the approach adopted here is intended to provide a more global characterization of some interconnections between linguistic and logical notions, affording, in particular, the possibility of examining the nature of natural language understandability and grammaticality from a different and, in my judgment, more illuminating, conceptual perspective.

1.10 On the Optimality of Natural Languages [37]

It seems reasonable to suppose that natural languages evolve to the simplest possible forms sufficient to convey meanings, that is, to forms which can easily be handled by language users of greatly varying symbolic manipulation skills and which can yet convey such complex interconnections among concepts as are of interest to those users. We can formulate this supposition in terms of the relationship between word-strings and their normal readings as an hypothesis:

The Optimality Hypothesis

The forms to which a given natural language evolves are the resultant of the action of two simultaneous and complementary forces: (1) a force directed to simplifying the word-strings of that language, hence, derivatively, directed to simplifying the generation rules for that language, that is, the rules that associate word-strings with readings and (2) a counteracting force directed to simplifying the reading rules of that language, that is, the rules that associate readings with those word-strings. The reason these two forces are complementary is that the simpler the word-strings, the more ambiguous they are, hence the greater is the burden on the reading rules which disambiguate them.

Note 37. In this section the notion of "reading" is to be taken in its widest sense, that is, that sense in which all formalized aspects of understanding have been incorporated within it, and not simply the entailment-relevant aspects of understanding which the body of this study attempts to treat.

The first force (1) is exerted by the press for maximal economy of expression under which a word-string will tend to assume the simplest possible form, that is, that form that is easiest for humans to produce, both conceptually and physically. The second force (2) is exerted by the press for accuracy in communication, whereby that reading of any given word-string that is intended by a speaker can be identified by the typical hearer on the basis of that word-string in conjunction with the auxiliary information provided by the context-of-utterance in which that word-string is produced. Thus the first force acts to minimize complexity, while the second force acts to minimize word-string ambiguity.

The limits to word-string complexity are limits in human capacity or willingness to generate or understand very complex word-strings. The limits to word-string ambiguity are limits in the amount and kind of information that the hearer can retrieve from the context of utterance and apply to the identification of readings of word-strings that are intended by the speaker.

Under the interaction of these two forces, any given word-string of the language will tend to assume the simplest possible form commensurate with its capacity to have its intended logically and lexically normal readings identified by a typical hearer through his reference to usual contexts of utterance.

These two forces can resolve in a variety of possible ways: every natural language (dialect) (at a given point in time) embodies a particular resolution of them and, like any equilibrium reached in nature through the interaction of complementary forces, each resolution, that is, natural language, must be regarded as optimal. The evolutionary path of a particular natural language is guided by the interaction of these two forces. Natural languages are very different from artificial languages in this respect, and should not be regarded as deficient in any sense owing to their ambiguity since this very ambiguity makes them appropriate vehicles for communication--indeed, optimal vehicles, insofar as they have evolved as optimal resolutions of the forces mentioned. Indeed, one could well expect that, if a group of persons were initially to adopt an artificial language as their only language of communication, thereby subjecting it to the action of these two forces, that language would be subject to a social process under which it would eventually evolve towards an optimal resolution wherein an acceptable balance would be struck between the simplicity of its word-strings, hence of the generation rules required to associate word-strings with readings, and the simplicity of the reading rules required to associate readings with word-strings. That is to say, it would evolve to a natural language.

The mechanism of interaction of these forces would properly be described under pragmatics wherein the interplay of word-strings with contexts is taken into account.

Any given natural language provides a sort of loose connection between its (spoken or written) word-strings and the broad range of possible meanings of those word-strings. As indicated, we represent those possible meanings by the structures we have called readings, and we represent the loose connection by reading rules and generation rules.

Intrinsic Limitations of Reading Rules

A word-string of a given natural language, being essentially ambiguous, by definition cannot, ordinarily, of itself, that is, independently of any considerations of the context-of-utterance in which it is produced, provide sufficient signals to identify that one of its possible normal readings which would be appropriate to or intended in that context. That is to say, the reading rules of the language cannot be sufficiently refined so as to identify any specific normal reading, but can only identify the <u>class</u> of normal readings of that word-string. The requisite additional information needed to identify a specific normal reading must be supplied by elements <u>external</u> to the word-string, that is, by elements in the context-of-utterance which comprise both the verbal and nonverbal (i.e., psychological, social, and historical) setting within which the word-string (occurrence) in question has been produced. The reading rules of the language explain only the capacity of the word-strings of that language to express their normal readings.

A Consequence of the Optimality Hypothesis

According to the optimality hypothesis, a given natural language word-string w evolves to an equilibrium form between the simplicity of the reading rules that assign readings to w and the simplicity of the generation rules that generate w. Thus, if a certain word-string or word-string-part w^o occurs within a containing word-string w, then w^o must be regarded as essential in the sense that it has survived the press for maximizing the simplicity of the generation rules, hence must be considered as providing an essential signal to be employed by the reading rules. Thus, if we do not use it to signal at least one normal reading of w, then we must regard ourselves as not having identified all of the normal readings of that word-string, and, thereby, as not having adequately explained the function of w^o in w. That is to say, if w is a word-string that has at least one normal reading, then, for every word-string or word-string-part that occurs within w, there is at least one normal reading r(w) of w that is signalled, in part, by the presence of w^o.[38]

Note 38. It must, of course, be understood that the mechanism for signalling readings may have global as well as local aspects. That is, whether the presence of a given word w^o in a containing word-string w acts as a part of a signal for some normal reading r(w) of w, depends in part on the way that w^o functions as a signal when occurring within other word-strings of the language. For example, different natural languages signal the definite article DEF in different ways. We can divide these broadly into two general kinds of signals: first, explicit signals, implemented by the occurrence of explicit natural language morphemes--e.g. "the," or even "a", in some occurrences--in American English; second, implicit signals, implemented through some means involving higher-order properties of word-strings, such as order, or even the absence of any determiner--e.g., as in the phrases "in hospital" or "in office" of English English. The mechanism of operation of both kinds of signals, and particularly of the latter kind, i.e., implicit signals, involves global relationships across the word-strings of the language. These global relationships go beyond the verbal part of the context-of-utterance, and involve the way that all of the signals interrelate across all the word-strings of the language. For example, the fact that the phrase "in hospital" in English English can signal at least one reading whose syntactic representation would include the insertion of the definite determiner DEF immediately before the lexical morpheme HOSPITAL (of which "hospital" is the analogue), depends, in part, on the fact that the definite article "the" is indeed often used before common nouns to signal the same sort of reading, and partly on the fact that other determiners could also have occurred before "hospital," yet did not. In particular, if there were no explicit analogue of DEF in English English, or if no other determiners could have occurred before "hospital," then the mere absence of "the" before "hospital" would not have been adequate to signal the intended definite determiner reading of "in hospital." (There are, of course, other possible normal readings of "in hospital" besides the definite determiner reading.)

CHAPTER 2

A Theory of Readings for TR-Languages: Basic Concepts

The informal notion of "reading" used throughout Chapter 1 is not sufficiently precise to be useful in accounting for intuitively perceived entailment relationships among natural language sentences.

Indeed, in our preceding examples, we referred to specific readings of word-strings only *indirectly*, variously (i) by employing fragmentary assumptions to suggest intended readings, (ii) by exhibiting canonical alternates of those word-strings, that more clearly indicated one or another possible intended reading, or (iii) by exhibiting certain intuitive entailment relations in which those word-strings entered that held under (i.e., were induced by) the unexhibited reading in question, and thereby suggested the intended readings of those word-strings. In none of these cases, however, did we *intrinsically* characterize the intended readings of those word-strings. The purpose of this chapter is to provide an intrinsic characterization of readings of word-strings, first, in a general sense for arbitrary languages (Section 2.2), and then in a specific sense for thing-relation (TR) languages (Section 2.3). These will be preceded by an account of what I mean by a "TR-language" (Section 2.1).

2.1 Thing-Relation (TR) Conditions and Thing-Relation (TR) Languages

The TR reading framework to be described below in Section 2.3 comprises the formal theory of readings as it pertains to entailment, While this theory is intended to be applicable to arbitrary natural languages, it is more appropriate for the analysis of certain languages than it is for the analysis of others. In this subsection I attempt to provide an informal characterization of those languages to which the formal theory of readings would, on intuitive grounds, appear to apply most aptly, in the sense of allowing for relatively simple[38.1] specifications of normal readings for all possible understandable word-strings of L. I characterize these languages as natural languages fulfilling various conditions which I call <u>thing-relation</u> (TR) <u>conditions</u>, and refer to languages satisfying them as <u>thing-relation (TR) languages</u>. The reason for regarding these conditions as typifying those languages to which the theory could be best applied resides essentially in the fact that these conditions describe the intuitive semantic structure of natural language word-strings that the theory is intended to formalize. At the very least, these conditions display the motivation underlying the construction of the theory.

Note 38.1. "Simplicity" here is to be understood in both a notational sense, whereby syntactic representations of word-strings are notationally simple, and in a structural sense, whereby those syntactic representations are also "homologous" in relation to the word-strings they represent, as describd in pages 106.1 -106.5, below.

TR Condition 1: Denotation Structure

At any given occurrence, under a given way of understanding it, any meaningful word-string or word-string part of the language realizes an expression [38.2] that denotes either a thing of some kind, a relation among things, or a modifier function on things and relations which imposes a variation of some sort on those things and relations, and is said to realize, respectively, a thing-denoting expression, a relation-denoting expression, or a modifier expression at that occurrence.[38.3]

TR Condition 2. Structure of Thing-Denoting Expressions

Every thing-denoting expression denotes exactly one of four types of things:

(i) a definite thing (e.g., such as denoted by expressions realized in the word-strings "John" and "the book" in most of their occurrences)

(ii) an indefinite thing bounded only from below (e.g., such as denoted by expressions realized in the word-strings "some man" and "at least five men" in most of their occurrences)

(iii) an indefinite thing bounded only from above (e.g., such as

Note 38.2. By "expression" here, I mean "underlying abstract expression" (AE) in the sense of Note 11 on pages 18, 19.

Note 38.3 "Things" can be abstract or concrete, individual entities or aggregations of whatever sort, physically continuous or discontinuous, composed of contemporaneous or non-contemporaneous parts; they can be sets of entities, events, concepts, etc.; in short, a "thing" is anything that is conceptualized as an entity. "Relations" are either characteristics or states of one thing, in which case they are called "1-place relations," or are interconnections among two, three or more "things," in which case they are called "2-place relations," "3-place relations," and so on.

denoted by expressions realized in the word-strings "at most one man," and "not all men" in most of their occurrences)

(iv) an indefinite thing bounded both from above and from below (e.g., such as denoted by expression realized in the word-strings "exactly two men," and "between five and seven men" in most of their occurrences)

TR Condition 3. Structure of Relation-Denoting Expressions

Every relation-denoting expression contains a base relation which, intuitively, carries the basic meaning of the relation, and one or more special role-indicating morphemes, called case morphemes (which need not be explicitly realized in word-strings) together with an ordering of them, called a relative case ordering. When the case-morphemes are applied to the base relation according to the relative case ordering, the base relation is converted, successively, into an expression denoting a 1-place relation, a 2-place relation, and so on.

TR Condition 4: Assertion Structure

Every sentence[39] of the language realizes an expression, called a sentence expression, which can be partitioned into m major, that is, immediate, thing-denoting expresisons, for some positive integer m, and exactly one major relation-denoting expression which denotes the m-place relation that inter-connects the m denotations of those m thing-denoting

Note 39. We continue, of course, to understand the notion of a sentence of a natural language, as opposed to a sentence of a representation language, as a precritical notion.

expressions. The m major thing-denoting expressions have, (besides their order of occurrence in the sentence-expression,) two orderings: One is that given by the order in which the major thing-denoting expressions of the sentence expression are considered with respect to the major relation-denoting expression of the sentence, called their relative-place ordering, and which is such that the i^{th} major thing-denoting expression in the relative-place ordering corresponds to the i^{th} case morpheme of the major relation-denoting expression of the sentence, and signifies that the "thing" denoted by the i^{th} major thing-denoting expression in this ordering has the "role" indicated by the i^{th} case morpheme of the major relation-denoting expression. The second ordering of the major thing-denoting expressions in a sentence, and which may or may not be identical with their relative-place ordering, is the order in which the major thing-denoting expressions are considered relative to their governing determiners, and is called their relative scope ordering, since it corresponds to the relative scope of their governing determiners.

At the word-string level TR-Condition 4 means that, at any given occurrence, every sentence of the language is decomposable into an m-place relation together with the m things that it relates and, if the sentence is "read assertionally," that is, if it is to be understood as a declarative assertion rather than as a modifier, then that sentence asserts that those m things "stand in" that m-place relation, i.e., that that m-place relation holds among those m things, taken relative to both their relative place and relative scope orderings.

Let us consider some examples from English[40]. Each of the sample sentences below, when read assertionally, can be understood as asserting that certain things (i.e., those denoted by the expressions realized by the exhibited noun phrases), stand in certain relations, (i.e., those denoted by the expressions realized by the exhibited verb phrases) as indicated parenthetically:

(1)	John	loves	Mary
	(thing)	(2-place relation)	(thing)
(2)	Mary	is loved by	John
	(thing)	(2-place relation)	(thing)
(3)	John's loving Mary	annoyed	Agnes
	(thing: i.e., an event)	(2-place relation)	(thing)
(4)	John loved Mary	and	Agnes was angry
	(thing: i.e., an event)	(2-place relation)	(thing: i.e., an event)
(5)	Some man	loves	some woman
	(thing)	(2-place relation)	(thing)

Note 40. In English active-voice sentences, the agentive and direct object case morphemes are implicitly rather than explicitly realized, thus in sentences (1)-(7), and (11)-(13), no explicit case morphemes occur, and in (8)-(10) only the indirect case morpheme is explicitly realized.

(6) every man loves some woman

(thing) (2-place relation) (thing)

(7) All the students but John love more than one girl

(thing) (2-place relation) (thing)

(8) John gave the book to Mary

(thing) (thing) (thing)

(3-place relation)

(9) Some men gave some books to some women

(thing) (thing) (thing)

3-place relation

(10) Many men gave many books to many women

(thing) (thing) (thing)

3-place relation

(11) John is a man

(thing) (2-place relation) (thing)

(12) The men are tall

(thing) (2-place relation) (thing)

(13) John walks

(thing) (1-place relation)

The relative-place ordering of the major thing-denoting expressions realized in an English sentence by noun phrases is usually uniformly "marked" and therefore tends not to give rise to multiple readings by the order of occurrence of those noun phrases; on the other hand, relative scope ordering is not so uniformly marked in English (nor in any other language, so far as I know) and can give rise to multiple readings. In sentences (1), (2), (3), (4), (5), (8), (11), (12), and (13), the different readings that arise through different relative scope orderings are intuitively equivalent; in sentences (6), (7), and (10), they are not.

From a cognitive psychological point of view, these four TR conditions can be regarded as hypotheses concerning the way that language users of TR languages intuitively conceptualize the syntactic organization and semantic interpretation of word-strings.

More particularly, the TR-conditions can be considered as hypotheses concerning the way that language users of TR-languages understand word-strings of those languages, that is, as hypotheses concerning the nature of particular ways of understanding those word-strings, as practiced by language users. As hypotheses of this kind, TR-conditions (1)-(4) could be formulated as (i)-(iv) respectively:

(i) The language user intuitively assigns to every meaningful word-string part a denotation which is either a <u>thing</u> of some kind, a <u>relation</u> among things, or a <u>modifier function</u> which, when applied to things, relations, or modifier functions, converts them into other things, relations, and modifier functions.

(ii) The language user intuitively assigns to every thing-denotation a status of (a) definite thing, (b) indefinite thing bounded only from below, (c) indefinite thing bounded only from above, (d) indefinite thing bounded from above and below.

(iii) The language-user intuitively partitions every relation-denotation into (a) a base relation that carries the quality of the relation and (b) one or more case-roles which carry the orientation of the domains of the base-relation (i.e., its arguments) relative to each other.

(iv) The language-user intuitively regards every sentence of the language when used assertionally, as asserting that m thing-denotations (of sub-strings of that sentence) stand in a certain relation-denotation (of a substring of that sentence), where the relation of "standing-in" is taken relative to two simultaneously understood orderings of the m thing-denotations: namely, the relative place ordering and the relative scope ordering of their respective denoting thing-expressions.

The intuitive notion of a TR-language will be formalized in Section 2.3 in the precise concept of a reading framework for TR languages, to follow in the next section. The remainder of this chapter is devoted to developing the more general concept of reading framework and illustrating its applicability to the special case of English, which, as remarked, will be argued to be a TR-language.[41]

Note 41. There is also a global condition which, while not part of the defining conditions of TR languages, appears to be

Note 41 - Cont'd.

fulfilled by English and may hold for other TR-languages as well. I call this the completeness condition, and will refer to TR languages fulfilling it as complete TR-languages.

The Completeness Condition: Every sentence of the language is intuitively equivalent to a sentence (of the same language) that realizes a sentence expression all of whose major thing-denoting expressions denote definite things or indefinite things bounded only from below.

The Determination of TR-Languages

In general, the determination that any given natural language L were actually a TR language is approachable in one of two ways. First, assuming that the theory of readings as formalized in this chapter is an accurate formalization of the TR conditions given above, then if that theory applied to L accounted for intuitively perceived entailment in L in the sense that it allowed for the relatively simple[41.1] specification of normal readings for all possible understandable word-strings of L, i.e., readings that correctly predicted the entailments that native speakers of L would assent to, then this could be taken as evidence that that the language in question were indeed a TR-language, for the TR-model imposed on it would have the right sort of consequences. Second, if native speakers of that language who were familiar with the meanings of the TR-conditions found them to provide a correct informal description of their intuitive conceptions about how arbitrary understandable word-strings of their language carried meaning, then that, in my judgment, would also constitute evidence that that language were a TR-language. From the point of view of implementation these two ways are independent. On the other hand, from the point of view of what one would expect from an adequate theory of readings, if the first way yielded that the language were a TR-language while the second did not, then the theory could not be considered an accurate formalization of the TR-conditions insofar

---

Note 41.1. See Note 38.1.

as it was too weak, i.e., permissive, hence yielding that some non-TR-languages were actually TR-languages. If the second way yielded that the language were a TR-language while the first did not, then the theory would have to be judged too strong, i.e., restrictive, hence failed to yield that some given TR-language were indeed such.

From the vantage point of an investigator who is, say, a native English speaker and who is not a native speaker of the language in question, one's intuitions that are brought to bear derive wholly from English or English-like translations of the word-strings of the language in question, the aptness of which begs the whole issue in question. That is, it might be urged that the fact that the English or English-like translations of word-strings of the language under investigation permit a thing-relation construction simply points up an inadequacy in those translations.[42]

Note 42. Many of the world's languages may not be what we have called TR-languages. In particular, the assertion condition TR4 stated above may fail to be satisfied by some natural language. A reasonable candidate for such a language would be the so-called "topic prominent" languages like Mandarin, as characterized by Li and Thompson[4], which they distinguish from "subject prominent" languages like English. One characteristic of such languages is that they split off the initial phrase of a sentence as a "topic phrase" (e.g. "Nèike shù" in (1M) below) which somehow stands separate from the rest of the sentence ("yèzi dà, in (1M) below) and significantly, is not related to the other phrases of the sentence by the dominant relation of the sentence as part of its "case frame." Let us examine one example which Li and Thompson mention from Mandarin:

(1M) Nèike shù yèzi dà

(1) That tree leaves big

The meaning of which can be expressed by:

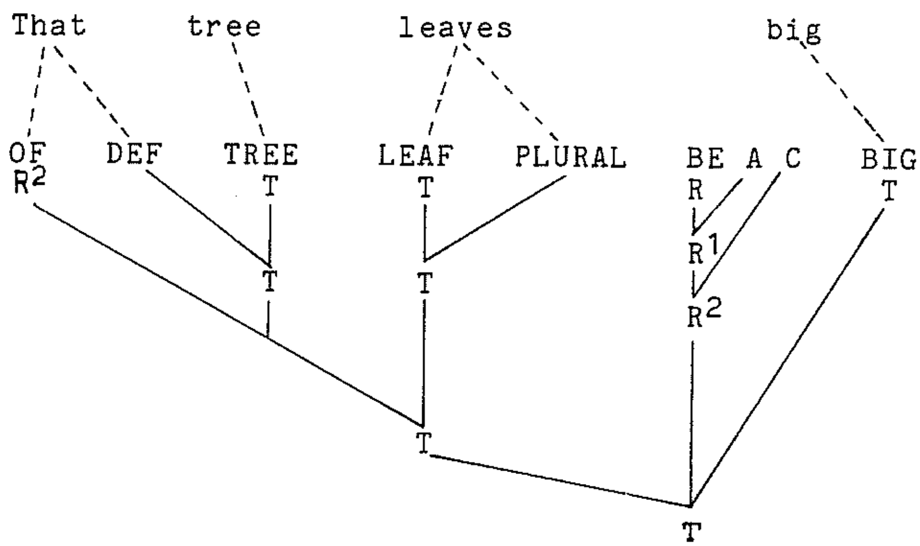
(1°) That tree (topic), the leaves are big.

By Li and Thompson's characterization of topic prominent languages, it would appear that the assertion condition would not be fulfilled by such languages, at least in the weak sense that there would exist some sentences of the language that could not be partitioned as the assertion condition describes. They would have to make the point that the intuitive semantic meaning of at least some sentences in Mandarin, such as 1M, below, could not be accommodated by the intuitive semantic condition described by the assertion condition. At an intuitive level, it is not at all clear that a model topic-prominent sentence like the sentence 1M would not fulfill the assertion condition. For consider the following syntactic representation (1') of (1), which utilizes a construction introduced later in this chapter in Section 2.3.1.3.1.2, called the <u>differentiated relative</u>: (The reader might wish to complete reading this example after reading Section 2.3.1.3.1.2).

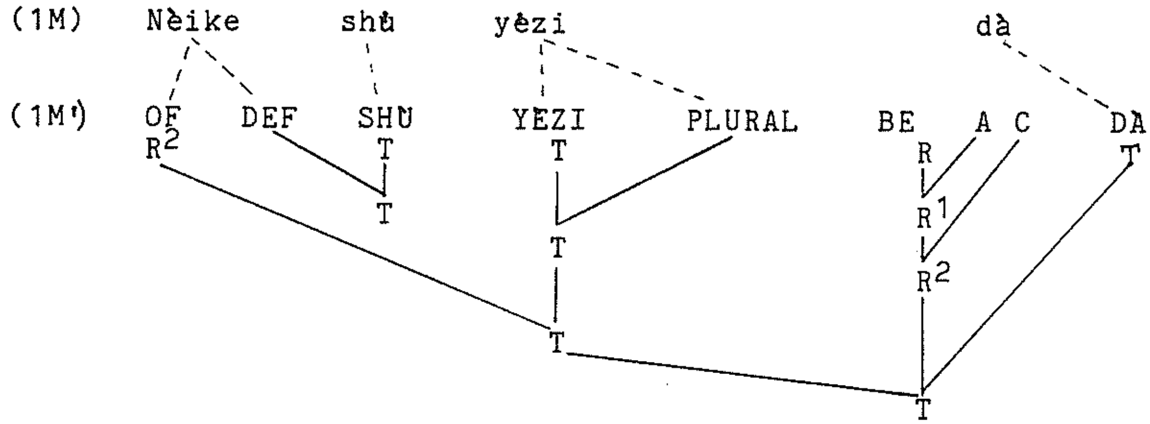
(1M) Nèike shù yèzi dà

(1)

(1')



The question as to whether (1) satisfies the assertion condition is not readily resolvable at an intuitive level. According to my semantic intuition, it does satisfy it and is expressed in the syntactic representation (1'). However, one could as easily claim that, even allowing that the English sentence (1) is an intuitively equivalent translation of the Mandarin sentence (1M), that (1) itself does not really satisfy the assertion condition, and that (1') is not really a correct syntactic representation of (1), (hence of (1M),) in the sense that any reading of (1) whose syntactic component is (1') and whose semantic component s is a refinement of the minimal semantic theory s_o satisfying L.S.A. (1)-L.S.A. (31) would fail to be normal (when taken together with other English sentences). Therefore, in order to establish that Mandarin, say, satisfied the assertion condition, it would be required, at the very least, to establish that (1') was a correct syntactic representation of (1) in the above sense. Assuming still, then, that sentences like (1M) of Mandarin are adequately translatable into English sentences like (1), and that, therefore, (1M) could be accorded a syntactic representation (1M') in $SYN^{TR}_{Mandarin}$ which was wholly analogous to the syntactic representation (1') in $SYN^{TR}_{English}$ of (1):



This would mean that there is a non-trivial set K of sentences of Mandarin and a reading assignment A on the sentences of K such that the syntactic component of A (1M) was (1M') and the semantic component of A (1M) was s and which is such that A induces an entailment relation among the sentences of K that is consistent with the intuitions of Mandarin speakers regarding what follows from what.

If the assertion condition is not fulfilled by some language, e.g., by Mandarin, then it represents a marginal case for the applications which would actually be proved or disproved by determining whether normal readings could be found for all its sentences within our framework, such as (1M') above.

The primary difficulty with intuitive semantic characterizations, like those employed by Li and Thompson, is that they are made independently of the problem of developing a formalization relative to which those intuitive judgments would be borne out by accounting for entailment relations using syntactic representations that were modeled along a "topic prominent" characterization. I do not of course know whether one could actually account for entailment in Mandarin using syntactic representations like (1M'); but it is an issue that can be properly formulated only with respect to a formal framework of some sort.

Let us briefly inquire into the requirements that any formalization might impose on a topic prominent language like Mandarin. In formalizing (1M), whatever syntactic representation one's theory would assign to (1M), that syntactic representation when interpreted by one's semantic theory, would accord some sort of set-theoretic relationship between the syntactic representation of the word-string "Nèike shù" and that of "yèzi," that is, between those of "that tree" and "leaves." In (1M'), by our choice of syntactic representation and by our semantic theory, we have related these phrases by the set-theoretic relationship interpreting the differentiated-relative-construction as specified by the logical semantic axiom 14.21 stated in Section 2.3.2.2 below of this chapter. There are other ways we could also have interpreted the relationship between these phrases within our TR reading framework which would, like (1M'), have comprised readings of (1M), that would appear, at least initially, to be normal.

If one were to find that no syntactic representations of (1M) in $SYN^{TR}_{Mandarin}$ were adequate relative to accounting for intuitive entailment, then Mandarin could not be regarded as satisfying the assertion condition, hence Mandarin could not be shown to be a TR-language. If this last were indeed borne out, then, in order to develop a satisfactory account of intuitively-felt entailment for Mandarin and, beyond that, a satisfactory formal characterization of "topic-prominence", one would have to develop a characterization of topic-prominent languages which provided another way of imposing a set-theoretic relationship between the phrase carrying the "topic" of the sentence and the rest of the sentence, relative to which one could develop a satisfactory account of intuitively-felt entailment in Mandarin.

A further examination of examples from topic prominent languages would go beyond the scope of this study. My conjecture at the present time is that topic-prominent languages differ from subject-prominent languages only in the nature of their reading rules, rather than in their underlying formal syntactic and semantic structure, i.e., rather than in the structure of their normal readings. Thus, the underlying intuitive semantic structure of sentences of languages like Mandarin is not inconsistent with the TR-conditions and, in particular, Mandarin appears to satisfy the assertion condition mentioned earlier. Relative to our reading framework, this means that the topic-prominent sentences of languages like Mandarin appear to have the same range of normal readings as do the subject-prominent sentences of languages like English in the sense that such readings comprise the underlying formal structure of those sentences. Thus the essential difference between topic-prominent and non-topic-prominent languages would be found not in their underlying syntactic and semantic structure but in the <u>reading rules</u> of those languages that assign such structures to sentences of those languages. That is, the difference is in the way that word-strings <u>signal</u> their underlying structure, i.e., readings. For example, (1M) would signal a differentiated relative differently than a sentence of English or, more generally, possibly differently than would be signalled in non-topic-prominent languages. The precise situation would have to await deeper examination.

2.2 General Reading Frameworks for Natural Languages

In this section we define the notion of a General Reading Framework for arbitrary natural languages. This notion is intended to identify the broader features of a reading framework without regard to whether the natural languages to which it applies are TR-languages. In the next section this notion is particularized to include special features appropriate to the syntactic and semantic characterization of TR-languages.

The notion of a general reading framework precisely interrelates word-strings of an arbitrary natural language to readings of those word-strings, and readings of word-strings to the entailment relations that they induce.[43]

General reading frameworks can be particularized in other ways as well, that is, to include special features appropriate to the characterization of languages other than TR-languages, as soon as reasonable alternative types of languages become identified.[44]

Let L be a natural language, understood as a set of finite strings of words together with a distinguished subset S of "sentences"[45] of L.

Note 43. As remarked earlier in Sections 1.1 and 1.3 of Chapter 1, readings of word-strings are specified in this study only to that extent necessary to account for intuitively-felt entailment.

Note 44. There may be none, i.e., it may be the case that all natural languages are TR-languages.

Note 45. Ultimately, the notion of a natural language sentence is a semantic notion, indeed one that is defined as a word-string having a certain kind of normal reading, called a "sentential" reading of that word-string. (See Chapter 3.) Accordingly, the

A *reading framework for L* is an ordered triple $\langle SYN_L, INT_L, M_L \rangle$ such that:[46]

(1) SYN_L is a set of objects collectively called a *syntactic representation language for L* which has a distinguished subset S';

(2) INT_L is a set of ordered triples $s = \langle F_s, V_s, R_s \rangle$ called *semantic theories for* SYN_L such that:

 (a) F_s is a set of ordered pairs $\langle D, f \rangle$, called *interpretations*, where D is a non-empty set and f is a function whose domain is SYN_L and whose range is included in D;

 (b) V_s is a function, whose domain is $S' \times F_s$ and whose range is included in {truth, falsehood, nil}; and

 (c) R_s is a set of binary relations on F_s;

(3) M_L is a set of functions r called *reading functions* such that the domain of r is the set of pairs <w#,C>, where w# is an occurrence (token) of a word-string or word-string part w of L and C is a context-of-utterance[47] of w# and which is such that,

---

notion of "sentence" used here is pre-critical, serving only to identify, for the sake of the exposition, those word-strings that are to be accorded sentential readings as one of their normal readings.

Note 46. The sets SYN_L, INT_L and M_L are intended, respectively, to correspond to the syntactic, semantic and pragmatic dimensions in the linguistic analysis of natural language word-strings.

Note 47. Context-of-utterance is used throughout this chapter in the sense of *interpreted* context-of-utterance, as characterized in Section 1.7.2 of Chapter 1. Our usage of this notion allows the possibility of multiplicity of contexts-of-utterance of a single word-string token w#.

to every pair <w#,C> in its domain, r associates a partially ordered set r(<w#,C>) of pairs <x,y> where $x \in SYN_L$ and $y \in INT_L$, called the *reading hierarchy of w# with respect to C*.

Intuitively: SYN_L is a set of syntactic representations of finite strings of words of L, each such string having multiple possible syntactic representations in SYN_L; S' is the set of "sentential"[48] readings of those word-strings; INT_L is a set of syntactic representations of word-strings of L that comprise semantic theories, each specifying a way of interpreting every syntactic representation in SYN_L; and M_L is the set of all functions that assign reading hierarchies to occurrences of word-strings and word-string-parts of L with respect to given contexts-of-utterance. The partial ordering of the readings in the set r (<w#,C>) is intended to represent the relative degrees of normality of the readings of w with respect to the context C.

Different reading frameworks for a given natural language L can differ regarding any of the three components: SYN_L--the set of syntactic representations of finite strings of words of L, INT_L--the set of semantic theories which interpret them, or M_L--the set of functions which associate reading hierarchies with occurrences of word-strings of L and contexts-of-utterance.

Generally speaking, we wish to develop the simplest possible reading framework for any given natural language that is yet sufficient to account for the intuitively perceived entailment relationships among the sentences of that language.

Note 48. See Section 3.1 .

We will shortly forward the outline of a reading framework for TR-languages that appears "simplest possible" and which is sufficient to account for the intuitively perceived entailment relationships among the sentences of TR-languages.

A _logic for L_ is an ordered pair $\langle SYN_L, INT_L \rangle$ derived from a reading framework for L by suppressing the third component M_L of that framework and is called _the logic associated with that reading framework_.

2.2.1 Readings Derived from Reading Frameworks

Given a reading framework $\langle SYN_L, INT_L, M_L \rangle$ for a natural language L and a finite string e of words of L, we define a $\langle SYN_L, INT_L, M_L \rangle$-*reading of e* as an ordered pair $\langle e', s \rangle$ where $e' \in SYN_L$ and $s \in INT_L$: e' is a syntactic representation of e which identifies the interpretable parts of e; $s \in INT_L$ is a semantic theory which identifies the set-theoretic structure of those interpretable parts. We will also refer to $\langle e', s \rangle$ as a *reading derived from that reading framework.* In cases where the reading framework is already understood, we will suppress reference to it and refer to $\langle e', s \rangle$ simply as a *reading of e*. If the word-string e is also understood, we will suppress reference to e and refer to $\langle e', s \rangle$ simply as a *reading*.

Parallel to our informal remarks in Section 1.2, we introduce the following notion: If $S^o \subseteq S'$ is a subset of sentences of L, and if $\langle SYN_L, INT_L, M_L \rangle$ is a reading framework for L, then we define a $\langle SYN_L, INT_L, M_L \rangle$-*reading assignment on* S^o as a function A whose domain is S^o and which assigns to each sentence $x \in S^o$ a $\langle SYN_L, INT_L, M_L \rangle$-reading of x, and which is such that, for all sentences $x, y \in S^o$, if $A(x) = \langle x', s_1 \rangle$ and $A(y) = \langle y', s_2 \rangle$, then $s_1 = s_2$. This last condition means that the same semantic theory is used to interpret all syntactic representations (in the range of A) of sentences of S^o.

In order to simplify formulations in subsequent discussions we will designate the first and second terms of a reading r respectively as 1(r) and 2(r), and will designate the set of first terms of the range of a reading assignment A in application to a set S^o of sentences of L as $1(A(S^o))$ and will designate the semantic theory that is common to all readings in $A(S^o)$ by $2(A(S^o))$.

2.2.2 Entailments Induced by Readings

Readings have been defined in such a way that a reading assignment on a set of sentences can be associated with a single entailment relation on that set of sentences, called the entailment relation induced on that set of sentences by that reading assignment. We have heretofore indicated the content of this notion in an indirect and informal way. We now define this notion precisely.

Let $\langle SYN_L, INT_L, M_L \rangle$ be a reading framework for L; then the entailment relation induced on a set K of sentences of L by a $\langle SYN_L, INT_L, M_L \rangle$-reading assignment A on K, in symbols $A^E(K)$, as follows: $A^E(K) = \{\langle J,k\rangle \mid J \neq \phi, J \subseteq K, k \in K$ and, letting $2A(K)=s=\langle F_s, V_s, R_s\rangle$ the following holds: for all $\langle D,f\rangle \in F_s$, if $V_s(\langle 1A(j), \langle D,f\rangle\rangle) =$ truth for every $j \in J$, then $V_s(\langle 1A(k), \langle D,f\rangle\rangle) =$ truth.$\}$

We can now define the notion of entails, for two closely related usages:

(i) a subset J of sentences of L entails a given sentence k of L under the $\langle SYN_L, INT_L, M_L \rangle$-reading assignment A to $J \cup \{k\}$ if and only if $\langle J,k\rangle \in A^E(J \cup \{k\})$.

(ii) The sentences $j_1, \ldots, j_n$ of L entail the sentence k of L under the $\langle SYN_L, INT_L, M_L \rangle$-reading assignment A to $\{j_1, \ldots, j_n, k\}$ if and only if $\{j_1, \ldots, j_n\}$ entails k under A.

2.2.3 Sensitivity of Readings

Different possible readings <e',s> of a given natural language word-string e can differ in the choice of the syntactic representation e' of e and in the choice of the semantic theory s that interprets that syntactic representation. In this section we discuss a certain dimension of variability in readings we refer to as <u>sensitivity</u>.

Roughly speaking, a given reading assignment A_1 on a set K of sentences of L is <u>more sensitive than</u> a reading assignment A_2 on K if and only if A_1 induces a "larger" entailment relation on K than A_2 does. That is, the more sensitive a reading assignment on a given set of sentences is, the more extensive is the set of entailments that it yields.

We can obtain increasingly more sensitive reading assignments on a given set K of sentences by using finer syntactic representations of the sentences of K and/or by using semantic theories which impose finer set-theoretic structures on the interpretable parts of those representations, hence allowing for the extraction of more entailments. Accordingly we define:

Let A_1, A_2 be reading assignments on a set K of sentences of L. Then A_1 <u>is more sensitive than</u> A_2 on K if and only if $A_1^{E}(K) \supsetneq A_2^{E}(K)$. Under the same conditions, A_1 is also called a <u>refinement</u> of A_2.

2.3 Reading frameworks for TR-Languages

In this section we begin to specialize the notion of a general reading framework for an arbitrary natural language L that was introduced in Section 2.2 to a form that more closely reflects the defining conditions for I R - languages.[49] We call reading frameworks of this special form reading frameworks of this special form <u>reading frameworks for TR languages</u>. Each such framework can be considered as a particular way of formalizing the (informal) defining conditions for TR languages.

The specialization to TR-languages is obtained by imposing additional constraints on each of SYN_L^{TR}, INT_L^{TR}, and M_L^{TR}, to obtain, respectively, SYN_L^{TR}, INT_L^{TR}, and M_L^{TR} where, given the TR-language L, SYN_L^{TR} is a specific syntactic representation language, INT_L^{TR} is a specific family of semantic theories for interpreting the expressions of SYN_L^{TR} ,different semantic theories in INT_L^{TR} being obtained by fixing the interpretations of all logical representational morphemes of SYN_L^{TR} and, allowing the lexical representational morphemes of SYN_L^{TR} to vary, and M_L^{TR} is a specific set of reading functions whose domain is the set of pairs of occurrences of word-strings or word-string parts w of L and contexts-of-utterance and whose range is a partially ordered subset of SYN_L^{TR} x INT_L^{TR}.[50]

Note 49. In particular: to a form that permits the specification of homologous readings, as defined later in this section.

Note 50. In accord with our earlier remarks in Sections 1.1 and 1.3 of Chapter 1, and in Note 43, the characterization of SYN_L^{TR} and INI_L^{TR} to follow are, like those of SYN_L^{TR} and INT_L^{TR}, also to be considered as "partial" in the sense that they describe only the

Recall further that a _logic_ for an arbitrary natural language L is obtained from a reading framework $\langle SYN_L, INT_L, M_L \rangle$ for L as the initial pair $\langle SYN_L, INT_L \rangle$ of the triple comprising that reading framework. By specializing $\langle SYN_L, INTL, M_L \rangle$ to $\langle SYN_L^{TR}, INT_L^{TR}, M_L^{TR} \rangle$, we thereby obtain a specialization to what we will call a _TR-logic_ $\langle SYN_L^{TR}, INT_L^{TR} \rangle$ _for L_.

Further Aspects of Motivation: Homologous Readings

Now that TR-languages have been characterized, at least in outline, we are in a position to amplify further on the motivation underlying the kinds of TR-Reading Frameworks we are attempting to develop: namely, such as can generate readings of word-strings of arbitrary TR-languages that are both _normal_ as well as what we will call (structurally) _homologous_ relative to the word-strings of which they are readings. It is perhaps perspicuous to describe the requisite sorts of readings that TR-Reading Frameworks are intended to generate as readings that satisfy (i) a _functional condition_, which concerns the concept of a "normal" reading, which has been persistently articulated

---

entailment relevant parts of the syntactic and semantic components of a reading. Accordingly, as one undertakes the formalization of aspects of readings other than those that are entailment-relevant, such as, possibly, focal aspects of readings, the structure of SYN_L^{TR} would have to be correspondingly enriched to accommodate whatever sorts of additional notation was required to syntactically represent those aspects, and the structure of the semantic theories of INT_L^{TR} would have to be correspondingly

enriched to interpret the additional syntactic notation.

throughout Chapter 1, and which we summarize explicitly below under (i1) and (i2) for completeness, and (ii) a structural condition, which concerns the concept of a "homologous" reading, which has not been explicitly articulated in any of the preceding exposition, but which has in some sense already been implicit in our examples.

Let us first summarize our general orientation to readings, as recounted in Chapter 1: A reading of a word-string is a structure intended to afford a complete formal explication of a "way of understanding" that word-string including, in part, aspects of understanding that are relevant to the intuitively perceived entailment relationships into which that word-string enters with respect to other word-strings. A reading of a word-string is comprised, roughly, of a syntactic representation of that word-string, together with a semantic theory that interprets that representation. That part of a reading which formalizes only the entailment-relevant aspects of understanding I call the entailment-relevant-part of a reading and, when thus narrowed, the syntactic representation component of a reading of a given word-string provides a description of that part of the syntactic structure of that word-string that is relevant to entailment, while the semantic theory component provides a description of that part of the underlying semantic structure of that word-string that provides a recursive set-theoretic interpretation of that entailment-relevant syntactic structure.

(i) The functional condition. This is comprised of two subconditions (which have already been variously touched on in Chapter 1):

(i.1) First functional subcondition: The entailment-relevant part of a given reading of a given word-string should wholly determine, in a precise and explicit sense, the entailment relations into which that word-string enters with respect to given readings of other word-strings; in particular, if a reading is assigned to each sentence k in a set K of natural language sentences such that all readings so assigned have the same semantic theory, then this assignment of readings should wholly determine -- that is, induce -- a unique entailment relation on the set K (where an entailment relation on K is a relation R whose domain is the set of subsets of K and whose range is K, and which is such that (a) if $k \in K^1 \subseteq K$, then $\langle K^1, k\rangle \in R$, and (b) if $\langle K^1, k_1\rangle \in R, \ldots, \langle K^1, k_n\rangle \in R$, and $\langle \{k_1, \ldots, k_n\}, k_{n+1}\rangle \in R$, then $\langle K^1, k_{n+1}\rangle \in R$.)

(i.2) Second functional subcondition: We say that a given reading assignment A on a set K of natural language sentences accounts for a given entailment relation R on K just in case the entailment relation induced on K by that reading assignment is precisely the entailment relation R, and we say that the reading assignment A on a set K of natural language sentences is normal with respect to a given context-of utterance in which the sentences of K are produced just in case the entailment relation that A induces on K is consistent with the intuitions of language users regarding entailment with respect to that context-of-utterance. As we have noted throughout Chapter 1, there are usually multiple possible normal reading assignments on K, each reflecting a different way of understanding the sentences of K with respect to the context-of-utterance. We also say that a reading assignment on K is normal (without reference to context-of-utterance) just in case it is normal with respect to some possible context-of-utterance in which the sentences of K are produced. Furthermore, we say that a reading of a _single_ sentence e is normal with respect to a given context-of-utterance in which e is produced just in case there is a reading assignment A on some set K of sentences containing e that is normal with respect to that context of utterance, and that a reading of a _subsentential_ word-string, such as a phrase or clause, is normal with respect to a given context-of-utterance in which it is produced just in case it is a sub-reading of a reading of a sentence containing it, which reading is itself normal with respect to that context-of-utterance. We can then formulate _the second functional subcondition_ for readings as follows: for any context-of-utterance in which the sentences of K are produced, and for any entailment relation R on K that is consistent with the intuitions of language-users regarding entailment with respect to that context-of-utterance, there is a reading assignment A on K that accounts for R.

(ii) _The Structural Condition_. Generally, this condition requires that the structure of readings of word-strings should be such that a normal reading of a given word-string would reflect, through a direct and fairly apparent pattern of correspondences between the parts of the word-string and the parts of its syntactic representation within that reading, the intuitive grammatical structure underlying that way of understanding that word-string that that reading purports to formalize. A reading of a word-string that satisfies this relationship is said to be _homologous_.

The requirement that, in a homologous reading, the correspondence should be "direct and fairly apparent" is a _syntactic_ requirement; the requirement that the reading, through such a correspondence, should reflect "the intuitive grammatical structure underlying that way of understanding that word-string that that reading purports to formalize" is a <u>semantic</u> requirement.

The semantic requirement of a homologous reading of a word-string, which is to be formulated in terms of intuitive grammatical structure, can be formulated only relative to an independent initial characterization of the intuitive grammatical structure of word-strings. There are numerous possible characterizations of intuitive grammatical structure of the word-strings of a language -- each would give rise to a different sort of notion of reading under the requirement that the readings of the language be homologous. We characterize the sort of natural languages we purport to treat as <u>TR-languages</u> (see Section 2.1) with the intent that the notion of reading they give rise to, called <u>TR-readings</u>, is optimally suited to formalize ways of understanding the word-strings of those languages in the sense that they provide suitable homologous normal readings of the word-strings of such languages.

We will attempt to indicate more exactly the relationship that homologous readings of word-strings of TR-languages are to bear to those word-strings, by characterizing the notion of a homologous reading for word-strings of arbitrary natural languages ((a), below) and then indicate how it particularizes to TR-languages ((b), below) in the sense in which we invoke it, as follows:

(a) Let e be a word-string of a natural language L, let $\langle e', s \rangle$ be a reading of e, let C be the context-of-utterance in which e is produced and let U(e) be a way of understanding e with respect to the context C, (which may or may not be normal with respect to C). Let J

be a binary relation whose domain is the set of all entailment-relevant semantically meaningful parts of e under U(e) and whose range is the set of interpretable parts of e': Then ⟨e' ,s⟩ is <u>homologous to e under U(e), C, and J</u> if and only if (i) ⟨e' ,s⟩ <u>syntactically reflects</u> e under U(e), C, and J in the sense that J preserves the syntactic order of parts by which is meant that if a,b are parts of e, a',b' are parts of e' such that J(a,a') and J(b,b') and a precedes b in e, then a' precedes b' in e', under some reasonable sense of "precedes", and (ii) ⟨e',s⟩ <u>semantically reflects</u> e under U(e), C, and J in the sense that J preserves the entailment-relevant semantic relationship among parts by which is meant that if $a, a_1, \ldots, a_m$ are parts of e, and if a', $a_1', \ldots, a_m'$ are parts of e' such that, for each $1 \leq i \leq m$, if $J(a_i, a_i')$, then every intuitive entailment-relevant semantic property of a or semantic relationship among $a_1, \ldots, a_m$ that holds under U(e) is reflected in an analogous semantic property of a' or relationship among $a_1', \ldots, a_m'$.
(b) Under the hypotheses of (a), together with the additional hypothesis that L is a TR-language, the correspondence relation J defined above is such that (i) ⟨e',s⟩ <u>syntactically reflects e under U(e), C, and J</u> in the sense that if a,b are parts of e, a',b' are parts of e' such that J(a,a'), J(b,b'), and a precedes b in e in the sense that the left-most symbol of a is to the left of the left-most symbol of b, then a' precedes b' in the sense that the left-most symbol of a' is to the left of the left-most symbol of b'; and (ii) ⟨e',s⟩ <u>semantically reflects e under U(e), C, and J</u> in the sense that if a,b are parts of e, and if a',b' are parts of e', such that J(a,a') and J(b,b') then

Note 50.1. This notion of "precedes" for subexpressions of SYN_L^{TR}-expressions, while adequate for the present purpose, is not a total relation on the subexpressions of a given SYN_L^{TR}-expression, owing to the fact that SYN_L^{TR}-expressions are not "linear". For other purposes, it needs to be extended to a total relation, such as that defined on page 186 as the occurrence precedence relation on the subexpressions of a given SYN_L^{TR}-expression.

(1) if a intuitively denotes a thing under U(e), an m-place relation under U(e), or an m-place modifier under U(e), then a' is a thing-expression, an m-place relation-expression or an m-place modifier in e'.

(2) if a intuitively denotes a definite thing, an indefinite thing bounded only from below, an indefinite thing bounded only from above, or a doubly-bounded thing, then J(a) denotes (under s) a definite set, an indefinite set bounded only from below, an indefinite set bounded only from above, or a doubly-bounded set.[50.2]

(3) if a intuitively modifies b under U(e), then a' is a modifier on b'.

(4) if e is a sentence under U(e) and a is the i^{th} major thing-denoting word-string in the relative scope-ordering and the j^{th} thing-denoting expression in the relative-place ordering of the major thing-denoting word-strings of e under U(e), then the relative-scope index on a' is i, and the relative-place index on a' is j.[50.3]

(5) if e is a sentence that is an assertion under U(e) then e' is a thing-expression.

The syntax of SYN_L^{TR} and the semantic theories s that satisfy the logical semantic axioms of Chapter 2 are intended to permit one to specify homologous readings of word-strings that account for intuitively perceived entailment relations among them; that is, they are

---

Note 50.2. See pages 202, 203 for definitions of these sets.

Note 50.3. See pages 171, 172, 173 for these notions.

intended to permit one to specify for any given word-string e of a thing-relation language, any ordinary sort of context-of-utterance C in which e is produced, and any normal way U(e) of understanding e with respect to C, a normal reading ⟨e',s⟩ of e and a correspondence relation J whose domain is the set of all entailment-relevant semantically meaningful parts of e under U(e) and whose range is the set of interpretable parts of e', such that ⟨e',s⟩ both syntactically and semantically reflects e under U(e), C, and J, in the sense of (i) and (ii) above.

2.3.1 *The Syntactic Representation Language SYN_L^{TR}*

2.3.1.1 *Informal Description of SYN_L^{TR}*:

Let L be a TR-language, and let $\langle SYN_L^{TR}, INT_L^{TR} \rangle$ be a TR-logic for L.

The building-blocks of the syntactic representation language SYN_L^{TR} are certain "atomic" expressions called *representational morphemes*. These are of two kinds: (1) *logical representational morphemes*, which are the same for all TR-languages, and (2) *lexical representational morphemes* which are specific to a given TR-language L and which, generally, vary from one TR-language to another.[51]

Note 51. This does not necessarily mean that for all TR languages, each logical representational morpheme of SYN_L^{TR} would occur in some logically normal reading of some word-string of that language, but only that the same fund of such morphemes is available for syntactic representations of word-strings of arbitrary TR-languages.

One might consider also the possibility of treating lexical representational morphemes in a way analogous to the treatment of logical representational morphemes by developing a fixed set of lexical representational morphemes that was "universal" in the sense that it would represent natural language morphemes of arbitrary TR-languages. Thus for example, the word "man" in English and "homme" in French (when considered, say, as nouns) realize at least one pair of natural language lexical morphemes whose respective meanings might be regardable as identical and hence representable by a single common lexical representational morpheme belonging to the universal set of lexical representational morphemes.

The reasonableness of constructing such a universal set of lexical representational morphemes requires that certain further issues be examined: for whereas the set-theoretic meanings of logical representational morphemes can be specified either in isolation from each other or with a minimal degree of interconnectedness, the set-theoretic meanings of lexical representational morphemes are multiply interconnected within a lexical "network." (See Chapter 4 regarding the notion of semantic network.) Thus, what we have above referred to as a "single" or "common" meaning of two natural language representational morphemes would need to be formally specified within a more comprehensive network of inter-related meanings of lexical representational morphemes.

Thus, for example, a particular natural language morpheme which is realized by the English word "love" might be distinguished from another natural language morpheme, say, one which is realized by the English word "adore," by the way that their respective corresponding lexical representational morphemes were networked with lexical representational morphemes corresponding to particular lexical natural language morphemes realized by the English words, "know," "respect," "hate," etc. Moreover, a particular lexical morpheme in one language would be equivalent (or near equivalent) to a particular lexical morpheme in another language just in case their corresponding lexical representational morphemes entered into isomorphic or near isomorphic lexical networks, wherein a 1-1 (or nearly so) correspondence between them would be identified which preserved the network relationships. It would appear then, that the assumption that a pair of lexical morphemes of two languages had the same meaning is perhaps a very strong one, for, by the above reasoning, it involves the existence of isomorphic or near isomorphic lexical networks across the two languages, involving inevitably a great many other lexical morphemes of those languages.

There is no intrinsic difficulty in treating lexical representational morphemes as we had treated logical representational morphemes, namely, by using the same set of morphemes for all languages. We prefer not to do this in order to allow for a systematic notation that would be useful in discussing translations from one language to another, wherein it is clearer to use distinct lexical root-forms (i.e., morphemes in the narrow sense) than to employ subscripted variants of a universal lexical morpheme. Thus, in particular, we prefer to distinguish MAN and HOMME, rather than distinguish between them by affixing subscripts to a "master lexical morpheme" which underlies both in order to distinguish them.

If we were to use universal lexical morphemes in this sense there would be no essential distinction between SYN_L^{TR} and $SYN_{L'}^{TR}$, for distinct TR-languages L and L'. The difference would occur only between INT_L^{TR}, and $INT_{L'}^{TR}$; and between M_L^{TR} and $M_{L'}^{TR}$ where there would be differences in the semantic networks of normal readings of English word-strings involving "man," and French word-strings involving "homme."

We do not pursue any of these questions or possibilities in the present study; instead, for the purposes of this study, the logical representational morphemes are treated as "universal" and the lexical representational morphemes are treated as language-specific.

The set of expressions of $SYN^{T\,R}_{L}$ consists of the logical and lexical representational morphemes of $SYN^{T\,R}_{L}$ together with the syntactic representations (of word-strings of L) that can be built up out of the representational morphemes. Every syntactic representation of a word-string e of L can be regarded as obtained in two successive steps: (1) first, by a <u>morphological decomposition</u> of e into representational morphemes of $SYN^{T\,R}_{L}$, which replaces strings of words in e by strings of representational morphemes-in-the-narrow-sense of $SYN^{T\,R}_{L}$, followed by (2) a <u>phrasing</u> of that morphological decomposition of e, which imposes an upward branching labeled tree onto that decomposition, whose uppermost "leaves" are the component representational morphemes of that morphological decomposition. The resultant phrasing is an expression of $SYN^{T\,R}_{L}$. If e is the word-string of L from which the phrasing is developed, the resultant phrasing is called a <u>syntactic representation</u> of e in $SYN^{T\,R}_{L}$.

A <u>morphological decomposition</u> of e in $SYN^{T\,R}_{L}$ ranges between "coarse" and "fine"; roughly speaking, the finer the decomposition of e, the simpler need be the semantic rules required to interpret the resulting analysis e' of e relative to inducing the expected entailments from e. There is thus an inverse relationship between the complexity of the analysis of e and the complexity of the semantic rules required to interpret that analysis; that is, the more complex the analysis of e we develop, the simpler the semantic rules required to interpret it, and vice versa.

Generally, one of the ways in which we can construct refinements of given readings of given word-strings is by using finer morphological decompositions of those word-strings.

As remarked earlier, the set of <u>logical</u> representational morphemes of SYN_L^{TR} is the same for all TR-languages L. Those logical morphemes it has been useful to identify for the purposes of this study are listed and described in Section 2.3.1.1.1 Two kinds of special logical representational morphemes of SYN_L^{TR} that directly reflect the fact that L is assumed to be a TR-language are the following:

(i) <u>Thing-labels</u>. All symbols comprised of the letter "T" with or without subscripts or superscripts are logical representational morphemes of SYN_L^{TR}. These function as labels, labeling expressions of SYN_L^{TR} that intuitively denote "things".

(ii) <u>Relation-labels</u>. All symbols comprised of the letter "R" with or without subscripts or superscripts are logical representational morphemes of SYN_L^{TR}. These also function as labels, labeling expressions of SYN_L^{TR} that are to be interpreted as relations that interconnect "things." Superscripts on "R" indicate the place number on the relation so labeled; for example, R^1 indicates a 1-place relation, R^2 indicates a 2-place relation, and so on. An unsuperscripted R indicates simply that the expression so labeled is to be interpreted as a relation.

For every TR-language L, every expression of SYN_L^{TR} is to be interpreted either (a) as a "thing," (in which case it is an upward branching labeled tree with bottom-most node labeled "T," with or without subscripts or superscripts), or (b) as a "relation",(in which case it is an upward branching labeled tree with bottom-most node labeled "R," with or without subscripts - superscripts), or (c) as a "modifier" ,(in which case it is either unlabeled or labeled by *.)

We illustrate some of the above concepts by returning to the syntactic representation of the example discussed in Section 1.5, namely that of the English sentence:

(1) John loves Mary

The <u>logical representational morphemes</u>[52] of $SYN^{TR}_{English}$ entering into the syntactic representation of (1) are:

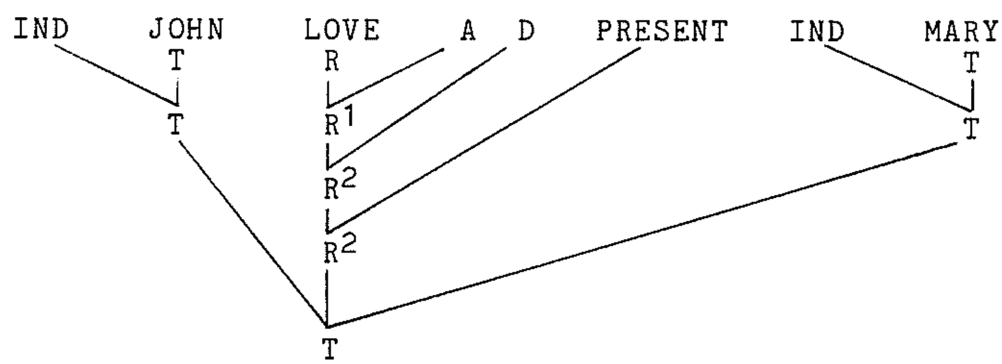
T, R, R^1, R^2, PRESENT, IND

The <u>lexical representational morphemes</u>[52] of $SYN^{TR}_{English}$ entering into that syntactic representation are

A, D, JOHN, LOVE, MARY

The <u>syntactic representation</u> of (1), then, which is the syntactic component of the dominant normal reading of (1) is the following expression of $SYN^{TR}_{English}$:

(2)



Note 52. These are representational morphemes in the "narrow sense," that is, morphemes that are both syntactically and semantically minimal.

and is obtained by first obtaining a morphological decomposition of (1), namely the sequence of "leaves":

(3) IND JOHN LOVE A D PRESENT IND MARY

followed by the imposition of the indicated phrasing on this morphological decomposition.

The precise meaning of the syntactic representation (2) of (1) can be spelled out only by a semantic theory that interprets it. That is, even with the fixed syntactic representation (2), there are many possible readings of (1), each obtainable by varying the semantic theory that interprets (2). Different semantic theories are obtained by imposing different sorts of set-theoretic conditions on the interpretations of the lexical representational morphemes of (2), namely:

JOHN, LOVE, MARY.

As noted earlier, the set-theoretic meanings of the logical representational morphemes of (1), namely:

T, R, PRESENT, IND

are the same for all semantic theories in $INT^{TR}_{English}$.

As a second example, consider the English sentence

(4) The tall boys love Mary

In order to discuss this example we need to mention two further logical morphemes of SYN^{TR}_{L} for all TR-languages L, namely:

DEF,

which is interpreted[53] as the definite-article morpheme; and

PLURAL,

which is interpreted[53] as the plurality morpheme.

The logical representational morphemes, then, of $SYN^{TR}_{English}$ entering into the syntactic representation of (4) are

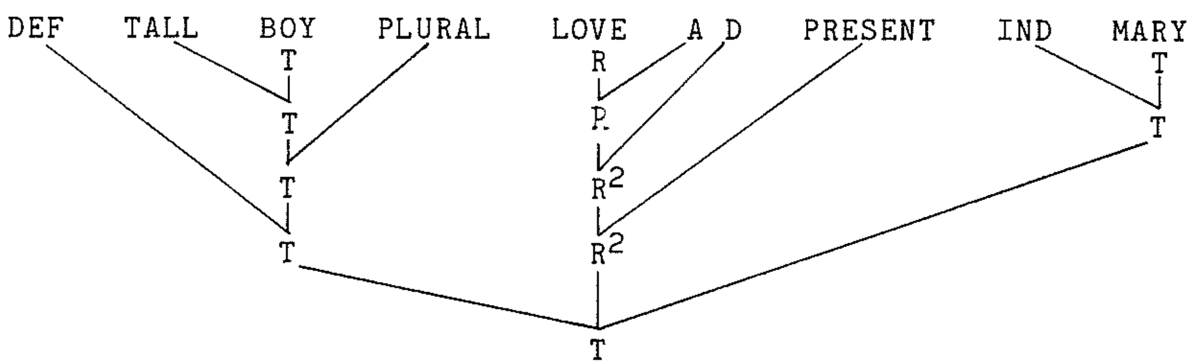
T, R, R^1, R^2, PRESENT, DEF, PLURAL

The lexical representational morphemes of $SYN^{TR}_{English}$ entering into that syntactic representation are

TALL, BOY, LOVE, MARY, A, D

The <u>syntactic representation</u> of (4), then, which is the syntactic component of the dominant normal reading of (4), is the following expression of $SYN^{TR}_{English}$:

(5)



and is obtained by first obtaining a morphological decomposition of (4), namely, the sequence of "leaves":

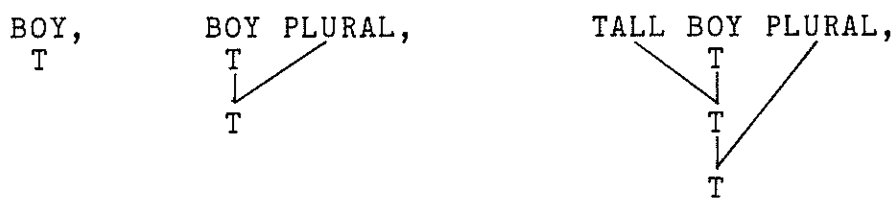
(6) DEF TALL BOY PLURAL LOVE A D PRESENT IND MARY

followed by the imposition of the indicated phrasing on this morphological decomposition.

Note 53. The precise meaning of "interpreted" will be made clear in Section 2.3.2, where we specify the way that the semantic theories of INT^{TR}_{L} assign meanings to the logical representational morphemes.

(5) then asserts, very roughly, that the "thing" comprised of some particular set of tall boys and the individual thing (that is) Mary are interconnected by the 2-place "relation" of loving-at-present, relative to which the former is the agent and the latter the direct object.

Note that among other sub-expressions of (5) that are to be interpreted as "things," we have:



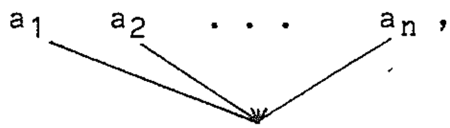
as well as the entire expression (6).

In general, for an arbitrary TR-language L, SYN_L^{TR} is a language whose expressions are upward branching trees[54] (we regard a single representational morpheme as a "degenerate"--i.e., branchless-expression). More exactly, letting n be a positive integer, <u>an</u> <u>expression</u> of SYN_L^{TR} is either (a) a labeling representational morpheme T, or T_n^j, called a <u>thing-label</u>, (b) a

---

Note 54. Equivalently, we could have expressed such phrasings in <u>subscript</u> <u>form</u>, in which the same information carried by the labeled tree is carried by subscripts on each morpheme in the morphological decomposition of e, namely that sequence of labels which label the nodes, taken in top-to-down order, dominating a substring of morphemes in which that morpheme occurs. That is, each morpheme is subscripted by the sequence of those labels which occur at nodes which are connected to it by a branch line, taking the nodes in a top-to-down order. The subscript form is more compact but less perspicuous than the tree form; accordingly, we will continue to employ only the tree-form phrasings of morphological decompositions of natural language word strings.

labeling representational morpheme R or R^n, called a _relation-label_, (c) a non-labeling representational morpheme, (d) an upward branching tree with an unlabeled root node:



for some expressions $a_1,\ldots,a_n$ of $\mathrm{SYN}_{L}^{T\,R}$

(e)[55] An upward branching tree with a labeled root node, labeled either by a thing-label

(e.1.1) $\frac{a}{T}$ or

(e.1.2) $\frac{a}{T_n}$, $\frac{a}{T^j}$, or $\frac{a}{T^j_n}$

or by a relation-label:

(e.2.1) $\frac{a}{R}$ or

(e.2.2) $\frac{a}{R^n}$

Expressions of types (a), (e.1.1) and (e.1.2) are called _thing-expressions of_ $\mathrm{SYN}_{L}^{T\,R}$; expressions of types (b) (e.2.1), and (e.2.2) are called _relation-expressions of_ $\mathrm{SYN}_{L}^{T\,R}$; expressions of type (e.2.1) are called _base relations_; expressions of type (e.2.2) are called _relation expressions of_ $\mathrm{SYN}_{L}^{T\,R}$ _of degree n_ or,

Note 55. As an example of (e) we have the lexical morpheme BOY (a degenerate tree) labeled by the label "T": $\frac{\text{BOY}}{T}$

equivalently, *n-place relation-expressions of* $\underline{\mathrm{SYN}_{L}^{T}{}^{R}}$; expressions of types (c) and (d) are called *modifiers of* $\underline{\mathrm{SYN}_{L}^{T}{}^{R}}$.

Thus a morpheme of $\mathrm{SYN}_{L}^{T}{}^{R}$ is either a modifier, a thing-label, or a relation-label. A modifier, thing-expression, or relation-expression of $\mathrm{SYN}_{L}^{T}{}^{R}$ that is not a morpheme is called, respectively, a *compound modifier*, a *compound thing-expression*, or a *compound relation-expression of* $\underline{\mathrm{SYN}_{L}^{T}{}^{R}}$.

We distinguish, then, only three grammatical categories: thing-expressions, relation-expressions, and modifiers. Thing-expressions of $\mathrm{SYN}_{L}^{T}{}^{R}$ are any expressions of $\mathrm{SYN}_{L}^{T}{}^{R}$ that contain a thing-label at their lowest point; relation-expressions of $\mathrm{SYN}_{L}^{T}{}^{R}$ are any expressions of $\mathrm{SYN}_{L}^{T}{}^{R}$ that contain a relation-label at their lowest point; modifiers of $\mathrm{SYN}_{L}^{T}{}^{R}$ are expressions of $\mathrm{SYN}_{L}^{T}{}^{R}$ that contain *no* label at their lowest point.

Thing-expressions of $\mathrm{SYN}_{L}^{T}{}^{R}$ are interpreted as sets of subsets of the domain of discourse (called "things"); for n a positive integer, n-place relation-expressions of $\mathrm{SYN}_{L}^{T}{}^{R}$ are interpreted as sets of n-tuples of elements of the domain of discourse, i.e, as set-theoretic relations; base-relation-expressions of $\mathrm{SYN}_{L}^{T}{}^{R}$ are interpreted as arbitrary elements of D; modifiers of $\mathrm{SYN}_{L}^{T}{}^{R}$ are interpreted as functions whose domain is a set of m-tuples for some positive integer m, and whose entries are things and relations and whose range is included in the set of things and relations.

It is useful, however, to distinguish further subclasses of representational morphemes, and we shall do so in the following sections.

2.3.1.1.1 Partial Listing of Logical Morphemes of SYN_L^{TR}

All expressions of SYN_L^{TR} are ultimately built out of logical and lexical representational morphemes of SYN_L^{TR}. As remarked earlier, the logical representational morphemes of SYN_L^{TR} are common to all syntactic representation languages SYN_L^{TR} as L ranges over arbitrary TR-languages, whereas lexical representational morphemes are specific to specific languages.

Whether a given morpheme or, more generally, a given expression e of SYN_L^{TR} is logical or lexical depends on whether it receives a special structure in whatever semantic theory is minimal in the family of semantic theories which one is willing to consider in interpreting the expressions of SYN_L^{TR}. In particular, the semantic theory, defined by the logical semantic axioms S.L.A 1 -S.L.A.31 given in Chapter 2, is a partial specification of the minimal semantic theory.

Essentially, we elect to regard an expression of SYN_L^{TR} as logical when we impose a set-theoretic structure upon it beyond that it would have simply on the basis of its being a modifier, a thing-expression, or a relation-expression.

For ease in inter-relating (occurrences of) word-strings of L with their syntactic representations in SYN_L^{TR}, it will be useful to introduce the following notion: a word-string e of a natural language L is called an L-analogue of an expression e' of SYN_L^{TR} if and only if there is at least one normal reading r of e with respect to some context-of-utterance C such that r = <e',s>, for some semantic theory s.

Among the 9 classes[57] of representational morphemes, expressions of classes (1) through (7) are logical expressions, while those of classes (8) and (9) are lexical expressions.

(1) *T and R labels*. These are the special logical representational morphemes in the narrow sense[58] of $SYN_{L}^{T\,R}$ discussed in the preceding section, which may have superscripts or subscripts. Roughly, "T", with or without subscripts, stands for *thing*, and "R", with or without superscripts, for *relation*. The T labels have analogues in English in the word "thing," as in "any*thing*," "every*thing*," "some*thing*," "no*thing*," etc., but in most cases, the morphemes of English that would be analyzed as a T label would be only implicitly marked. The R label has an analogue in English in the verb "to do" as in "what he did was to make her cry."

(2) *Determiners*. These are logical representational morphemes of $SYN_{L}^{T\,R}$ (some in the narrow sense and some in the wider sense) that formalize a generalization of the linguistic notion of determiner (which includes the logical notion of

---

Note 57. This list contains only those representational morphemes I have thus far found it useful to distinguish, and should not be considered complete. Moreover, the list is probably weighted in the direction of logical representational morphemes that are analogues of English logical morphemes that are explicitly realized by morphs of English. The compilation of a more comprehensive list that could more legitimately be regarded as a "universal" set of logical representational morphemes, must await an in-depth examination of explicitly realized logical morphemes of natural languages other than English or related languages.

Note 58. For the distinction between representational morphemes in the narrow and wider senses, see pages 25,26.

quantifier). Most determiners of $\mathrm{SYN}^{TR}_{English}$ have analogues in English, though many of these do not have single word analogues, or even word-strings that could be considered as single morphemes in English. I do not know if English is impoverished in this regard, but I would suspect that it is fairly well endowed, as compared to other natural languages. Some determiners of $\mathrm{SYN}^{TR}_{English}$ have no analogues in English, e.g., the massing determines MASS, and the individuating determiner IND. Also included among determiners are determiners called *complementors*. These are special modifiers that have not been distinguished as such in either the linguistic or logical literature, but which have an important complementary relationship with determiners. Analogues of complementors in English include "not," "all but," and "non." Complementors have the semantic effect of inverting the set theoretic structure of the denotations of expressions to which they are applied.

(3) *Affixive Morphemes*. These are logical representational morphemes in the narrow sense that enter into the construction of thing and relation expressions that re-orient the meaning of the expression to which they are applied, by negating, repeating, undoing, etc. Analogues of affixive morphemes in English include "not," "un," "anti," "con," "pro," "re," "in," "dis," and "non."

(4) *Copula Morphemes*. These are representational morphemes in the wider sense that link thing-expressions together.

Analogues of copula morphemes in English include the forms of the verb "to be."

(5) Logical-Relation Morphemes. These are logical representational morphemes in the wider sense that have as analogues the traditional logical connectives of both truth functional and modal logic, as well as further sorts of relation expressions. Analogues of logical relation morphemes in English include "and," "or," "if, then," "strictly entails," "not," "besides," "therefore," and "however."

(6) Modal-Relation Morphemes. These are logical representational morphemes in the wider sense whose analogues of modal relation morphemes in English include "necessarily" and "possibly."

(7) Temporal-Relation Morphemes. These are logical representational morphemes in the wider sense which formalize many natural language temporal notions not heretofore attempted, to my knowledge. Analogues of temporal morphemes in English include "when," "whenever," "during," "after," "hereafter," "heretofore," and "before."

(8) Basic Case-Morphemes. These are lexical[59] representational morphemes in the narrow sense that enter into the construction of relation expressions and signify the basic case-roles (i.e., those of agent, direct object, indirect object, and complement that various thing-expressions bear to an underlying relation. Analogues of the basic case morphemes in English are fragmentary, e.g., "by" for

Note 59. See next page.

agentive case and "to" for indirect object case; that is, as is well known, basic case roles in English are not indicated explicitly but implicitly, that is, by such devices as order and, in oral speech, stress.

(9) <u>Adjunctive Case Morphemes</u>. These are lexical[59] representational morphemes that enter into the construction of

---

Note 59. The grounds for regarding basic and adjunctive case morphemes as lexical rather than logical are not decisive, but are rather loosely constituted by the following sorts of considerations: (1) intuitively, the basic and adjunctive case morphemes have distinct notional meanings which are intuitively interconnected with the meanings of other expressions which are less problematically lexical (for example, the basic case morpheme "A" is associated with - and identifies - that major thing-expression that has the role of agent of the action denoted by the major relation-expression of the sentence, and the intuitive meaning of agency is related to the notions of activity, passivity, etc., while the intuitive meaning of the adjunctive case morpheme "O N" is related to the notions of relative position, covering, etc.); (2) formally, the denotations of the basic and adjunctive case morphemes do not appear to require differentiation within the semantic theory of a reading, so that any differentiation among them would be rendered by lexical semantic axioms which interconnect case morphemes within a "semantic" network to those lexical expressions that represent those natural language word-strings that carry the pertinent intuitive meanings. Part of the difficulty in formulating a philosophically satisfying basis for regarding basic and adjunctive case morphemes as lexical rather than logical is that the distinction is possibly a matter of the degree to which the meaning of an expression is dependent on the meanings of other expressions: at the extremes, we have "pure" logical expressions like the determiners, whose meanings can be fully articulated without any reference to the specific denotations of other expressions, and "pure" lexical expressions, like "B O Y", whose meaning can be fully articulated only through reference to the meanings of other expressions, like "M A N", "W O M A N", "A G E", etc. The basic and adjunctive case morphemes appear to be more similar to the pure lexical expressions than to the pure logical ones, with adjunctive case morphemes enjoying the more pronounced similarity to pure lexical expressions. In any case, our decision to regard case morphemes as lexical rather than logical is a tactical and not a doctrinal one, where tactics are dictated by ease in formulating normal readings.

relation-expressions and signify further (that is, beyond those signified by the basic case morphemes, above) roles that thing-expressions bear relative to an underlying relation, which roles pertain to time, place, instrument, purpose, cause, motive, direction, source, goal, etc. Analogues of adjunctive case morphemes in English include a special class of prepositions, such as "at," "on," "by," "for," "to," "in-order-to," "with," "through," etc., conjunctions like "because," and other expressions.

We explicitly list some of the logical representational morphemes of SYN_L^{TR} that fall under these 9 classes.

Representational Morphemes of SYN_L^{TR}

(1) Labeling Morphemes

T or T_n^i	Descriptive name: Thing label English analogues: "thing," "something."
R or R^n	Descriptive name: Relation label English analogues: "did"

(2a) Monadic Determiner Morphemes

DEF	Descriptive name: definite determiner English analogues: "the"
INDEF	Descriptive name: indefinite determiner English analogues: "some," "a"
PLURAL	Descriptive name: plurality determiner English analogues: "s"
BLB	Descriptive name: basic lower bounding determiner English analogues: "at least," "some"
BUB	Descriptive name: basic upper bounding determiner English analogues: "at most"
UN	Descriptive name: universal determiner English analogues: "all," "every"
1, 2, 3, . . . 1/2, , . . .	Descriptive name: numeric determiners English analogues: "one," "two," "three," . . . "one-half," . . .
MID	Descriptive name: mid-point determiner English analogues: "half"
GMID	Descriptive name: greater-than-midpoint determiner English analogues: "most"
LMID	Descriptive name: less-than-midpoint determiner English analogues: -----
NULL	Descriptive name: null determiner English analogues: "no," "none"
SMLB	Descriptive name: strong multi-

plicity indicating lower bounding determiner
English analogues: "many"

WMLB	Descriptive name: weak multiplicity indicating lower bounding determiner English analogues: "at least a few"
MXPLB	Descriptive name: maximizing approximation indicating lower bounding determiner English analogues: "almost"
UB1, UB2, . . .	Descriptive name: numeric upper bounding determiner English analogues: "at most one," "at most two," . . .
LB1, LB2, . . .	Descriptive name: numeric lower bounding determiner English analogues: "at least one," "at least two,"...
G	Descriptive name: greater-than determiner English analogues: "greater than"
L	Descriptive name: less-than determiner English analogues: "less than"
EXCT	Descriptive name: univocality determiner English analogues: "exactly"
SMUB	Descriptive name: strong multiplicity indicating upper bounding determiner English analogues: "all but at most a few"
WMUB	Descriptive name: weak multiplicity indicating upper bounding determiner English analogues: "at most a few"
MXPUB	Descriptive name: maximizing approximation indicating upper bounding determiner English analogues: "all but at most a few"
IND	Descriptive name: individuator determiner English analogues: -----

MASS — Descriptive name: massing determiner
English analogues: -----

EXP — Descriptive name: expansion determiner
English analogues
"more than," "in addition to"

SHR — Descriptive name: shrinking determiner
English analogues:
"less than," "fewer than."

TC — Descriptive name: true complementor
English analogues: "not"

QC — Descriptive name: quasi complementor
English analogues: "all but"

DC — Descriptive name: diffuse complementor
English analogues: "non"

(2b) <u>Binary Determiner Morphemes</u>

CONJ — Descriptive name: conjunction operator
English analogues: "and"

DISJ — Descriptive name: disjunction operator
English analogues: "or"

BCOMP — Descriptive name: binary exclusion operator
English analogues:
"___and not___", "___but not___",
"___except___"

REP — Descriptive name: repetition affix
English analogues: "re ___"

COMP — Descriptive name: complement affix
English analogues: "un", "a", "in", "dis", "non"

(3) <u>Affixive Morphemes</u>

REV — Descriptive name: reversal affix
English analogues: "not," "un"

REP — Descriptive name: repetition affix
English analogues: "re"

OPP — Descriptive name: opposition affix
English analogues: "anti," "con"

SUPP — Descriptive name: supportive affix
English analogues: "pro"

EXS

Descriptive name: excess affix
English analogues: "too"

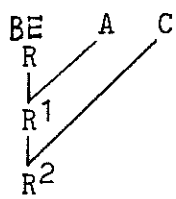
COMP

Descriptive name: complement affix
English analogues:
"un," "a," "in," "dis," "non"

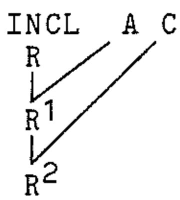
SUFF

Descriptive name: sufficiency affix
English analogues: -----

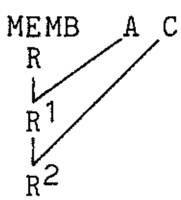
(4) <u>Copula Relation Morphemes</u>



Descriptive name: identity relation
English analogues: "is"

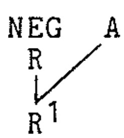


Descriptive name: inclusion relation
English analogues: "is"



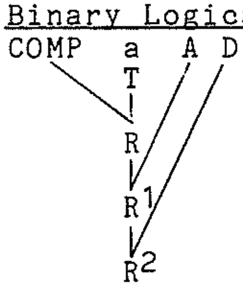
Descriptive name: membership relation
English analogues: "is"

(5a) <u>Unary Logical Relation-Morphemes</u>

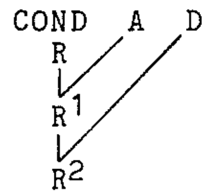


Descriptive name: negation relation
English analogues:
"not", "it is false that"

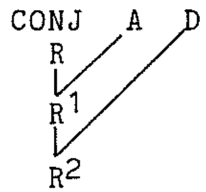
(5b) <u>Binary Logical Relation Morphemes</u>



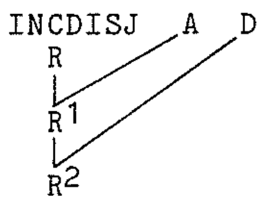
Descriptive name: comparative relation
English analogues:
"_er than", "more _ that"



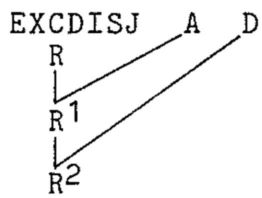
Descriptive name: conditional relation
English analogues:
"if,...then", "only if", "if"



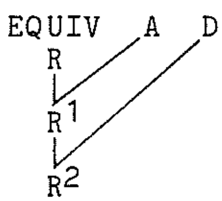
Descriptive name: conjunction relation
English analogues:
"and"



Descriptive name: inclusive disjunction relation
English analogues:
"or", "and/or"

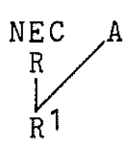


Descriptive name: exclusive disjunction relation
English analogues:
"or", "either-or"

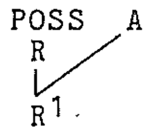


Descriptive name: equivalence relation
English analogues:
"just in case", "if and only if"

(6a) <u>Unary Modal-Relation Morphemes</u>

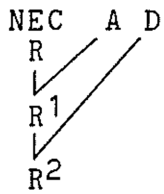


Descriptive name: monadic necessity relation
English analogues:
"necessarily", "it is necessary that"

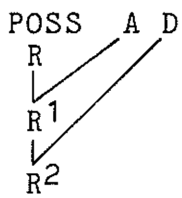


Descriptive name: monadic possibility relation
English analogues:
"possibly", it is possible that"

(6b) Binary Modal-Relation Morphemes

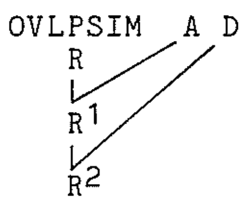


Descriptive name: binary necessity relation
English analogues:
"necessitates"

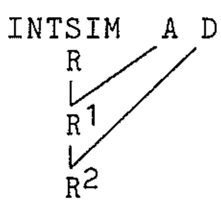


Descriptive name: binary possiblity relation
English analogues:
"makes it possible that"

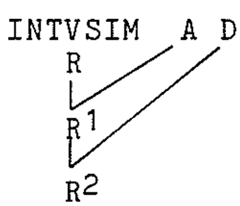
(7) Temporal Relation Morphemes



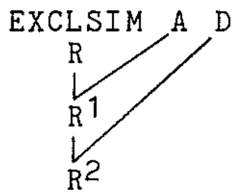
Descriptive name: overlap simultaneity relation
English analogues:
"when", "as"



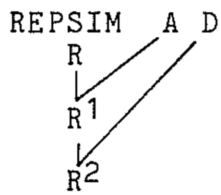
Descriptive name: instantaneous simultaneity relation
English analogues:
"as soon as"



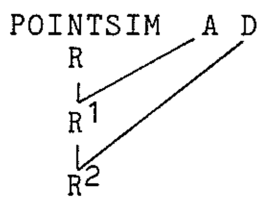
Descriptive name: interval simultaneity relation
English analogues:
"during", "while"



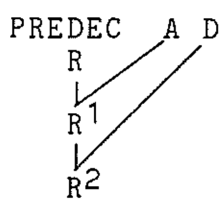
Descriptive name: exclusion simultaneity relation
English analogues:
"until"



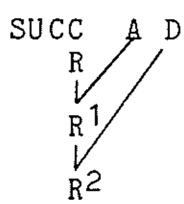
Descriptive name: repeated simultaneity relation
English analogues:
"whenever"



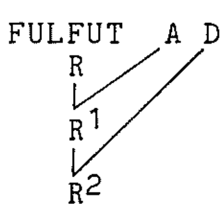
Descriptive name: point simultaneity relation
English analogues:
"at"



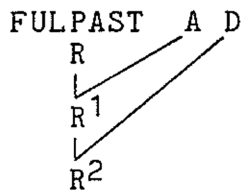
Descriptive name: predecessor relation
English analogues:
"before", "earlier than"



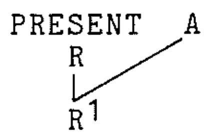
Descriptive name: succession relation
English analogues:
"after", "then"



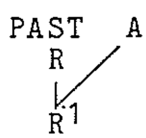
Descriptive name: full future relation
English analogues:
"henceforth", "hereafter"



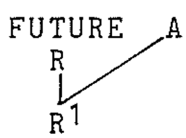
Descriptive name: full past relation
English analogues:
"heretofore", "up till now"



Descriptive name: present relation
English analogues: -----



Descriptive name: past relation
English analogues: -----



Descriptive name: future relation
English analogues: -----

(8) <u>Basic Case Morphemes</u>

A — Descriptive name: agentive case morpheme
English analogues: "by" (in passive)

D — Descriptive name: direct object case morpheme
English analogues: -----

I — Descriptive name: indirect object case morpheme
English analogues: "to," "for"

C

Descriptive name: complement case morpheme
English analogues: -----

(9) Adjunctive Case Morphemes

TEM

Descriptive name: temporal case morpheme
English analogues: "at"

LOC

Descriptive name: locative case morpheme
English analogues: "at"

INT

Descriptive name: interest case morpheme
English analogue "for"

PURP

Descriptive name: purpose case morpheme
English analogues: "for," "to," "in order to"

SUB

Descriptive name: substitution case morpheme
English analogues: "for"

ROLE

Descriptive name: role case morpheme
English analogues: "as"

CAUS

Descriptive name: causation case morpheme
English analogues: "cause," "because"

EXT

Descriptive name: extension case morpheme
English analogues: "over," "for"

GOAL

Descriptive name: goal case morpheme
English analogues: "for," "to"

DIR

Descriptive name: direction case morpheme
English analogues: "to," "towards"

DEST

Descriptive name: destination case morpheme
English analogues: "to"

2.3.1.1.2 Further Discussion of Some Special Classes of SYN_L^{TR} Expressions

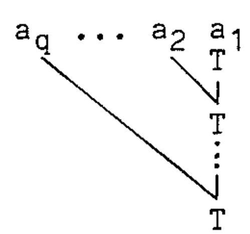
2.3.1.1.2.1 Determiners

The sentences of TR-languages L are represented in SYN_L^{TR} as composed of thing and relation expressions of SYN_L^{TR}. The role of determiners of SYN_L^{TR} in constructing thing-expressions is analogous to the role of case-expressions of SYN_L^{TR} in constructing relation-expressions of SYN_L^{TR}.

In this section we indicate the intuitive semantic role of determiners of SYN_L^{TR} in constructing thing-expressions of SYN_L^{TR}; in Section 2.3.2.5 we indicate the intuitive semantic role of cases of SYN_L^{TR} in constructing relation-expressions of SYN_L^{TR}.

Recall that a thing-expression of SYN_L^{TR} is either a thing-label or is an expression $\underset{x}{a}$ for some thing-label x, and for some modifier a of SYN_L^{TR}. Every thing-expression is composed of a series of modifiers $a_1,\ldots,a_q$, the first of which is applied to a thing-label, and each successive modifier is applied to the result of the immediately preceding application:

(1)



Certain of these and, in particular, a_q, are determiners in homologous readings of English noun phrases, where the determiners precede all the non-determiner modifiers, but this appears too strong a condition to impose on noun phrases of arbitrary TR-languages.

<u>Semantically, the role of each non-determiner modifier</u> among $a_1,\ldots,a_q$ is to identify the set of all subsets of some set of

interest: as successive application is made of such modifiers, since they are interpreted as subset operators, the semantic effect of their iterated application is to obtain increasingly restrictive subsets of the sets to which they are applied.

Semantically, the role of each of the determiners among $a_1, \ldots, a_q$ is to act on a set of subsets suitably restricted by the previous action of non-determiner modifiers and to impose a special structure on it.

In the previous section we encountered some determiners of SYN_L^{TR} that were single morphemes. Other determiners of SYN_L^{TR} are compound expressions of SYN_L^{TR}. What counts as a determiner of SYN_L^{TR} is defined in terms of the semantic theory of SYN_L^{TR}, to be described in Section 2.3.2 below. In the present section we will introduce some of the informal motivation underlying the semantic clauses of the semantic theory.

A determiner of English (as opposed to a determiner of $SYN_{English}^{TR}$) is an English word-string that is an analogue of a determiner of $SYN_{English}^{TR}$. Among the determiners of English we include the word-strings: "the," "every," "any," "no," "at least," "at least one," "most," "at most one," "five," "all but five," "half," "more than half," "less than half," "many," "few," "a few," "exactly five," "all but exactly five," "a," "some," "many," "most," and so on.

2.3.1.1.2.1.1. <u>Fourfold Division of Determiners</u>

In this sub-section we examine the intuitive entailment properties of the four kinds of thing-expressions mentioned under TR-Condition 3 suggesting the determiner structure of thing-relation languages.

For any TR-language L, the determiners of SYN_L^{TR} fall into four classes. In order to motivate the distinctions that this division represents, we consider, informally, an analogous fourfold division of noun phrases in English.

Consider the sentence forms:

1. X likes Carol
2. X likes Agnes
3. X likes Carol or Agnes
4. X likes Carol and Agnes

Let N be a noun phrase in English. Let 1(N), 2(N), 3(N), and 4(N) stand, respectively, for the result of replacing X by N in 1, 2, 3, and 4, above. Let R(N) be the entailment relation that is induced by the dominant normal reading assignment to the set {1(N), 2(N), 3(N), 4(N)}. Since the present discussion is informal, we can employ a rough characterization of the intended normal reading assignment as follows: (i) N denotes a thing, (ii) "Carol" and "Agnes" denote individual things, (iii) "likes" denotes a two-place relation whose agentive place is occupied by N and whose direct object place is occupied in 1 by "Carol"; in 2 by "Agnes"; in 3 by "Carol and Agnes"; and in 4 by "Carol and Agnes."

With these understandings, there are exactly four possibilities relative to the possible entries: $\langle\{1(N), 2(N)\}, 3(N)\rangle$ and $\langle\{1(N), 2(N)\}, 4(N)\rangle$: these are:

I. $\langle\{1(N), 2(N)\}, 3(N)\rangle \in R(N)$ and $\langle\{1(N), 2(N)\}, 4(N)\rangle \in R(N)$

II. $\langle\{1(N), 2(N)\}, 3(N)\rangle \in R(N)$ and $\langle\{1(N), 2(N)\}, 4(N)\rangle \notin R(N)$

III. $\langle\{1(N), 2(N)\}, 3(N)\rangle \notin R(N)$ and $\langle\{1(N), 2(N)\}, 4(N)\rangle \in R(N)$

IV. $\langle\{1(N), 2(N)\}, 3(N)\rangle \notin R(N)$ and $\langle\{1(N), 2(N)\}, 4(N)\rangle \notin R(N)$

These four possibilities give rise to a four-fold partition of the class of noun phrases of English, and correspond precisely to the four kinds of thing-expressions mentioned on pages 83,84 under TR-condition 2 dealing with determiner structure.

Definite Expressions (DE): These are thing-expressions satisfying I, and include: "John," "the man," "every man," "all men," "no men," "the men," ...

Lower Bounded (Indefinite) Expressions (LBE): These are thing-expressions satisfying II, and include: "some man," "a man," "at least one man," "many men," "almost all men," "men," "at least a few men," "all but at most a few men," "more than half the men," "all but at most one man," "more than one man," ...

Upper Bounded (Indefinite) Expressions (UBE): These are thing-expressions satisfying III, and include: "at most one man," "all but five men," "not all men," "no more than half the men," "at most a few men," ...

Doubly Bounded (Indefinite) Expressions (DBE) (These are both lower and upper bounded.) These are thing-expressions satisfying IV, and include: "five men," "exactly five men," between five and seven men," "half of the men," several men," ...

The sample memberships above could be easily continued. Moreover, the same classification would have been obtained had we used arbitrary distinct noun phrases in place of "Carol" and "Agnes," and an arbitrary transitive verb in place of "likes," indicating that the classification is in some sense fundamental.[60]

For example, replacing N in I, II, III, IV respectively by the DE "the man," by the LBE "some man," by the UBE "at most one man," and by the DBE "between five and seven men," the following will be true of the respective induced entailment relations R(N): "The man likes Carol" and "The man likes Agnes" together entail both "The man likes Carol and Agnes" and "The man likes Carol or Agnes"; "Some man likes Carol" and "Some man likes Agnes" together entail "Some man likes Carol or Agnes," but fail to entail "Some man likes Carol and Agnes"; "At most one man likes Carol" and "At most one man likes Agnes" together entail "At most one man likes Carol and Agnes" but fail to entail "At most one man likes Carol or Agnes"; "Between five and seven men like Carol" and "Between five and seven men like Agnes" together fail to entail either "Between five and seven men like Carol and Agnes" or "Between five and seven men like Carol or Agnes."

Note 60. On the other hand, if the variable "X" were to occur in the second argument place rather than the first, a certain restriction would have to be imposed on what occurs in the first argument place in order to obtain the same classification as above. We will remark on this restriction later when we develop the terminology to describe it. This restriction is important because it arises in certain other situations as well which we will encounter subsequently.

The four-fold division among thing-expressions gives rise to a corresponding four-fold division among determiners: Those determiners (like "the" in English) which, in application to a thing-expression, produce a definite expression (DE) are called definite determiners, and are abbreviated as DD; those determiners (like "at least" in English) which, in application to a thing-expression, produce an indefinite expression bounded only from below (LBE) are called lower bounding determiners, and are abbreviated as LBD; those determiners (like "at most" in English) which, in application to a thing-expression, produce an indefinite expression bounded only from above (UBE), are called upper bounding determiners, and are abbreviated as UBD; finally, those determiners (like "exactly" in English) which, in application to a thing-expression, produce an indefinite expression bounded from above and below (DBE) are called doubly bounding determiners, and are abbreviated as DBD.

This four-fold division of determiners of English is formalized by an analogous four-fold division of determiners in $SYN^{TR}_{English}$. Moreover, since determiners of $SYN^{TR}_{English}$ are logical expressions, hence fixed for all TR-languages, the same division holds for determiners of SYN^{TR}_{L}, for all TR-languages L. The division of determiners of SYN^{TR}_{L}, for arbitrary TR-languages L, will be semantically characterized within the logical semantic axioms to follow. Since the present section is intended to motivate those axioms, we include here an indication of the analogous division of determiners of $SYN^{TR}_{English}$, hence of SYN^{TR}_{L}, for all TR-languages L:

Definite Determiners (DD)of SYN_L^{TR}: IND, DEF, UN, NULL, ...

(Corresponding English analogues: _____, "the," "all," "no," ...)

Lower Bounding Determiners (LBD) of SYN_L^{TR}: INDEF,

SMLL , MXPLB UN_T , PLURAL , BLL WMLL_T , ...)

(Corresponding English analogues: "some," "many," "almost all," "s," "at least a few," "at least one,"...)

Upper Bounding Determiners (UBD) of SYN_L^{TR}:

BUL, $\text{BUL } 1_T$, $\text{QC } 5_T$, NEG UN_T BUL WMLL_T, ...

(Corresponding English analogues: "at most one," "all but five," "not all," "at most a few," ...)

Doubly Bounding Determiners (SD) of SYN_L^{TR}: 5 , $\text{EXCT } 5_T$, $\text{QC } 5_T$,...

(Corresponding English analogues: "five, "exactly five," "all but five," ...)

In Section 2.3.2, the intuitive meanings of these four kinds of thing-expressions will be formally cast within the set-theoretic structure of the denotations of thing-expressions of SYN_L^{TR} which is in turn "determined" by the four corresponding kinds of determiners.

2.3.1.1.2.1.2. <u>Compound Determiners of SYN_L^{TR}: The Differentiated Relative and the Ordinary Restrictive Relative Determiner</u>.

Various of the determiners of SYN_L^{TR} we have thus far considered have been morphemes in the narrow sense, such as IND, DEF, UN, NULL, SMLL, PLURAL, etc. Other determiners of SYN_L^{TR} that we have considered are compound expressions, whose denotations can be determined from the denotations of their parts. We call these <u>compound determiners</u>, and they include, for example, MXPLB UN, BLL WMLL, NEG UN, etc. In this section we
T T T

discuss two special compound determiners of a slightly more complex sort, which are useful in developing readings of restrictive relative-clause constructions in natural languages. The first such special compound determiner of SYN_L^{TR} is called the <u>differentiated relative determiner</u> of SYN_L^{TR}; the second is called the <u>ordinary relative determiner</u> of SYN_L^{TR}. In a subsequent section, after we have formulated a definition of truth for sentences of SYN_L^{TR}, we will discuss their semantic differences more precisely. In this section we attempt to motivate the distinction between these two compound determiners of SYN_L^{TR} by examining some intuitive semantic differences between their English analogues.

Consider the English noun phrase:

(1) many books written by several authors

This has at least four distinct readings which we might express by the following English paraphrases:

(2) an aggregate of many things, each of which is a book (collectively) written by several authors

(3) an aggregate of many books, each of which is collectively written by several authors

(4) an aggregate of many books such that, for some set consisting of several authors, each of whom has written some book in the aggregate, that aggregate is obtained by pooling together all of the books written collectively or singly by those several authors

(5) an aggregate of many books such that, for some set consisting of several authors each of whom has written a collection of many books in that aggregate, that aggregate is obtained by pooling together all the books in each collection written collectively or singly by those several authors

The difference between (2) and (3) and between (4) and (5) derives from a difference in the scope of application of the phrase "written by several authors" in (1); that is, (2) and (4) express the application of "written by many authors" in (1) to "books," whereas (3) and (5) express the application of "written by several authors" to "many books." We refer to the former as the <u>short scope application</u> of the phrase "written by several authors" and to the latter as the <u>long-scope application</u> of this phrase. Moreover, (2) and (3) express what we call <u>ordinary relative readings</u> of (1), whereas, (4) and (5) express what we call <u>differentiated relative readings</u> of (1). As will be described later in Section 3.5 of Chapter 3, the short-scope ordinary relative reading (2) of (1) expresses what we will call a <u>restrictive relative reading</u> of (1), and the long-scope ordinary relative reading (3) of (1) expresses what we will call a <u>non-restrictive relative reading</u> of (1). The differences in the short and long scope differentiated relative readings of (1) expressed, respectively, by (4) and (5), while important, have not been treated in the literature, and so do not correspond to any familiar distinction as does the distinction between the two ordinary relative readings (2) and (3) of (1). Accordingly, in the absence of established terminology, we will refer to the readings of (1) expressed by (4) and (5) simply as the long-scope differentiated relative reading of (1), and as the short-scope differentiated relative reading of (1), respectively.

In order to further delineate these differences we consider a case which, unlike (1), permits only differentiated relative readings as readings that are normal relative to entailment, that is, a case which has no normal ordinary relative readings:

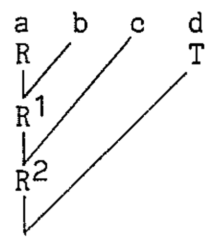
(8) Many children fathered by many composers are musically gifted

This assertion would be biologically bizarre if the word-string "fathered by many composers" were given an ordinary restrictive (i.e., short-scope) relative reading, or non-restrictive (i.e., long-scope) relative reading, but makes perfectly good sense if given either a short-scope or long-scope differentiated relative reading.

The syntactic representation in SYN_{L}^{TR} of a differentiated relative reading is a compound determiner which, in application to a given thing-expression, yields a compound thing-expression. It would be a simple matter technically to extend these considerations to m>2 thing-expressions and a connecting relation-expression, but these do not seem to be intuitively reasonable in any natural language known to me, so we will forego this apparently uninteresting generalization.

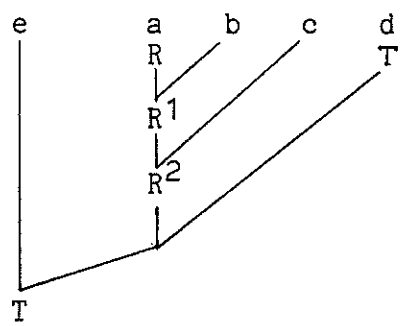
The differentiated relative is syntactically represented by the application to a thing-expression of a special modifier, called a relativized relation modifier (i), which has the form:

(i)



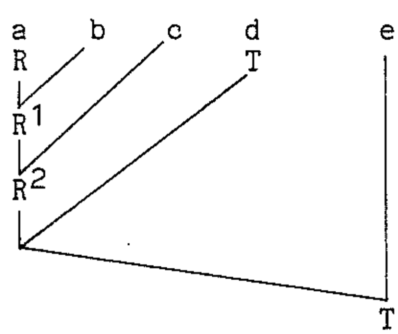
for some modifiers a, b, c, d of SYN_L^{TR}. When applied to a thing-expression e, the differentiated relative has the form:

(i.1)



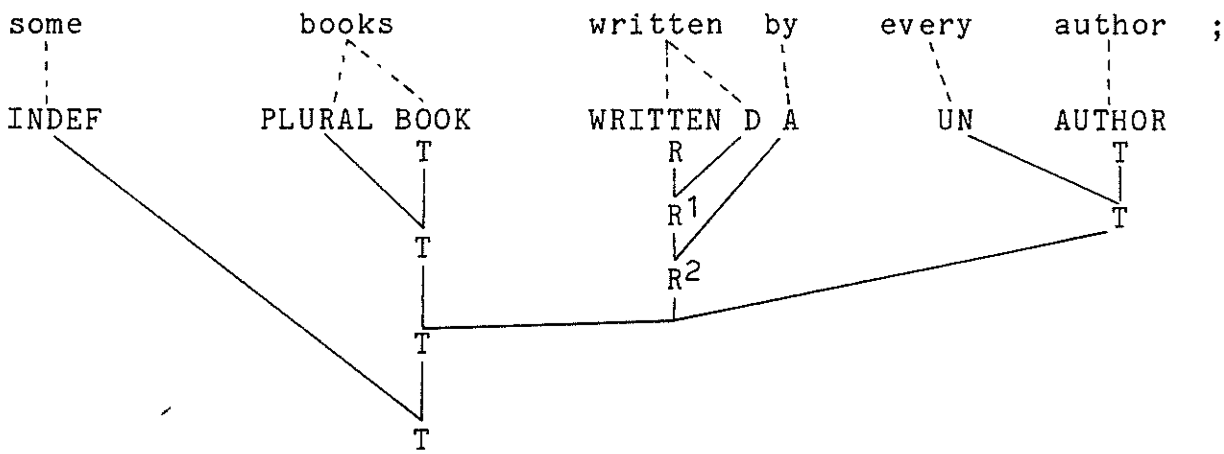
or

(i.2)

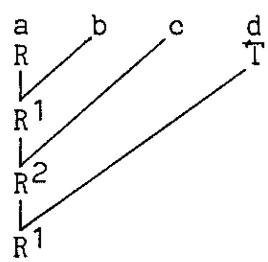


Semantically, (see page 206), the modifier (1), in application to the denotations of thing-expressions, is interpreted in such a way as to assure the correct entailments.[61]

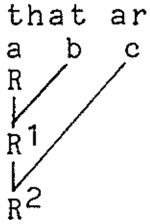
Let us examine some particular syntactic representations incorporating the differentiated relative construction. For example, a short scope differentiated relative reading[62] of the English word-string "some books written by every author" would be:



Note 61. In Section 2.3.1.1.3.3, we encounter another use of the relativized relative modifier (i) in forming relations



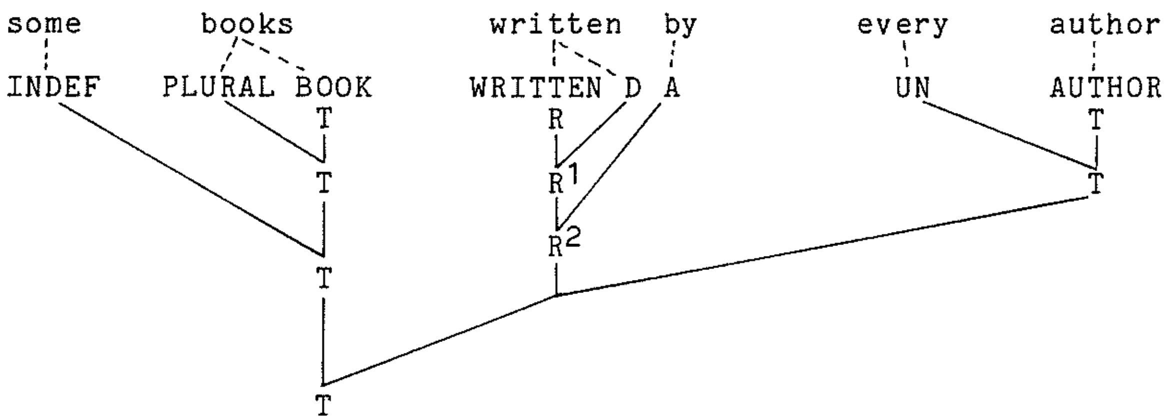
that are to be regarded as the relativization of the relation



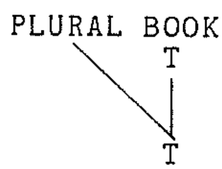
to the thing-expression $\underset{T}{d}$.

Note 62. For the sake of simplicity of expression, we often use the simpler expression "reading" to refer to the syntactic representation associated with that reading.

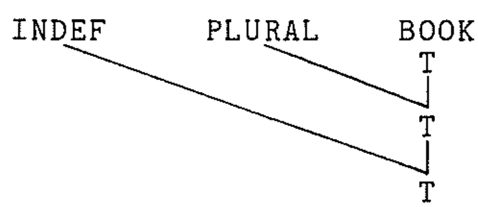
On the other hand, a long scope differentiated relative reading of this word-string would be:



The difference is that, in the short scope reading, the differentiated relative determiner is applied to the shorter thing-expression:



rather than to the longer thing-expression to which the differentiated relative is applied in the long scope analysis, namely:



Under the long scope differentiated relative reading of "some books written by every author," this word-string is interpreted as the set of those subsets B of books such that every book in B is written by some author and every author has some books written by him in B. The special case where in place of "every author" a word-string denoting an individual thing occurred, as would be the case if "every author" were replaced by "John," then the situation would be simpler and more intuitive. Specifically, the differentiated relative reading of "some book written by John" would have as its denotation the set of those subsets of books written by John. It can easily be seen that if John wrote no books then the denotation of "some books written by John" under a differentiated relative reading would be the empty set Ø. Similarly, if there is some author who wrote no books, then the denotation of "some books written by every author" under a long scope differentiated relative reading would also be the empty set. It will follow from our treatment (below) of the restrictive relative determiner that if an English word-string, like (12) or (14) below, has both differentiated and ordinary relative readings which are normal relative to entailment, then if it has an empty denotation under the differentiated reading, then it also has an empty denotation under the ordinary relative reading as well. Consider the following English word-strings:

(9) some books written by John

(10) some books that John wrote

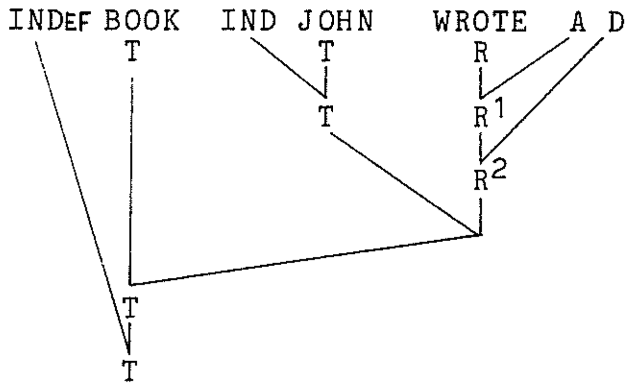
Now (9) and (10) are intuitively equivalent, whereas the following are not:

(11) some books written by every author

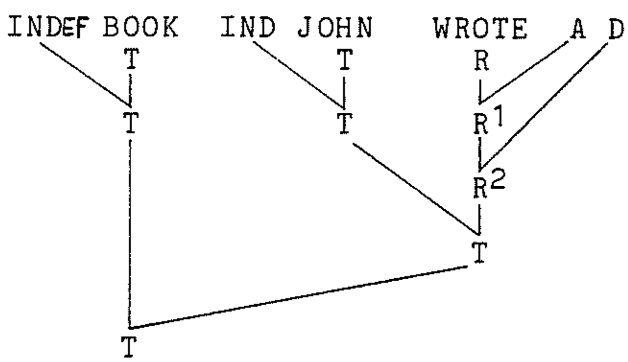
(12) some books that every author wrote

The intuitive equivalence of (9) and (10) can be explained within our treatment by the fact that both the ordinary and differentiated relative readings of (9) and (10) that are normal relative to entailment are interpreted similarly, i.e., have the same sets as their denotations, and the intuitive non-equivalence of (11) and (12) can be explained by the fact that this is not so for the latter: Specifically, under ordinary relative readings of each, (11) and (12) do have the same denotation whereas, under a differentiated relative reading of (11) they would not. For example, in an interpretation intended to capture the intuitive meaning of (11) and (12), (as, say, where there are books, each book is written by an author, and yet where no book is written collectively by every author) under ordinary relative readings of (11) and (12), (11) and (12) would each receive the empty set Ø as denotation, whereas under the differentiated relative reading, (11) would not receive the empty denotation.

The above examples of the differentiated relative illustrate the case where it occurred with a passive reading, that is, one where some case morpheme on the major relation precedes the case morpheme A. The differentiated relative can occur also in an active reading, i.e., one where the case morpheme A on the major relation is not preceded by any other case morpheme. The following examples illustrate the latter situation.[63]

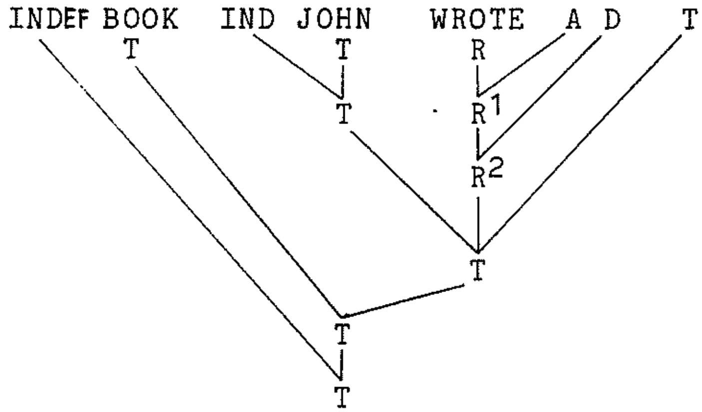


(short-scope differentiated relative)

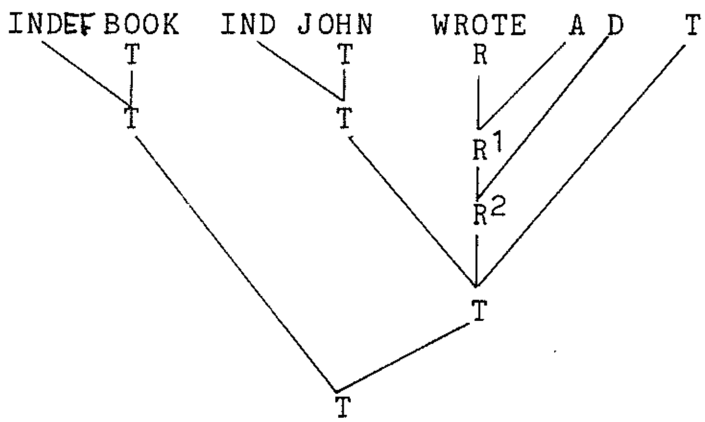


(long-scope differentiated relative)

Note 63. Syntactically, the differentiated relative can occur with any case morphemes in any order. The only restriction is that imposed by the normality of the obtained readings.



(short-scope
ordinary
relative;
i.e.,
ordinary
restrictive
relative)

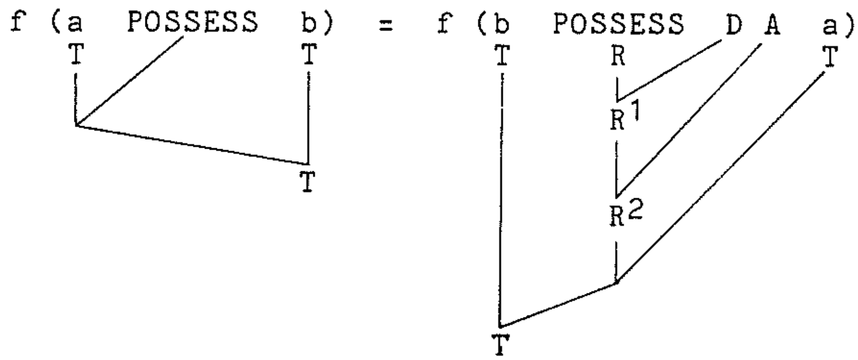


(long-scope
ordinary relative;
i.e., ordinary
non-restrictive
relative)

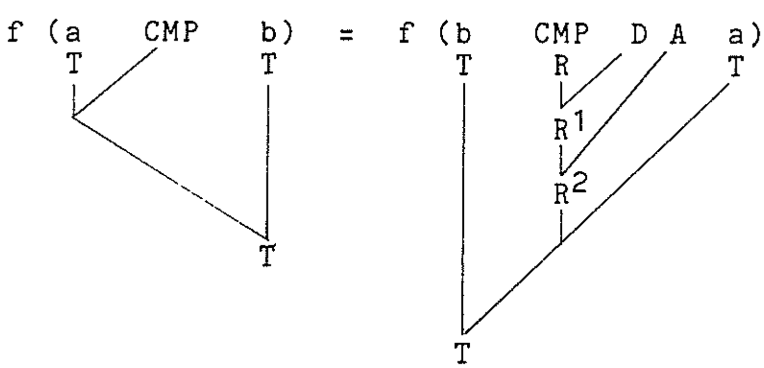
THE POSSESSIVE MODIFIER POSSESS

The possessive modifier POSSESS can be defined in terms of the differentiated relative. Analogous to the case with the differentiated relative, the POSSESS modifier has both a short or long scope form. The interpretation of the "possessive" modifier can be defined in terms of the differentiated relative as follows:

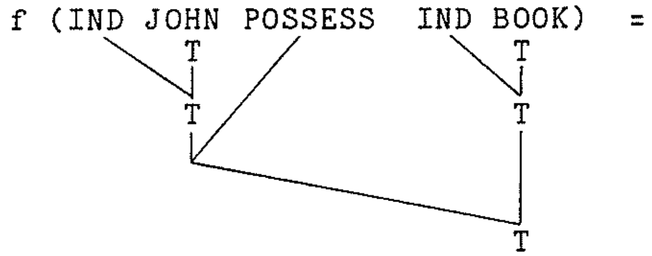
Let a, b, c be modifiers.[64] Then

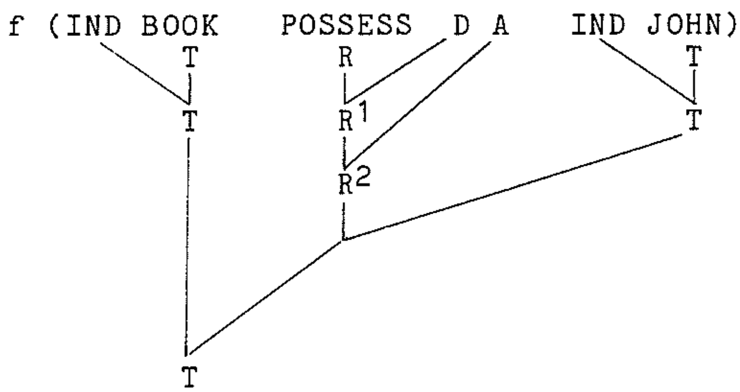


Note 64. One could conceive of natural languages in which, in analogy to the formation of the possessive, noun phrases had suffixes which indicate relationships that the denotation of that noun phrase bears to the denotation of another noun phrase other than possession, as for example the relationships of being written by, being comprised of, being filled with, etc. Say such suffixes were, respectively, "wr" for "written by", "cmp" for "comprised of", "fw" for "filled with" and were represented in SYN^{TR}_{L} by the representational morphemes WR, CMP, and FW, respectively. Then we could, for example, in analogy with the possessive construction above, have defined the following interpretation of the modifier CMP.



Some examples follow of special cases of the above definition, using the long-scope form of the differentiated relative:

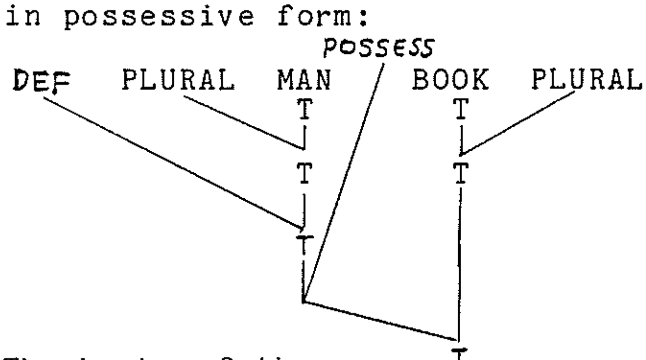




As another example, consider:

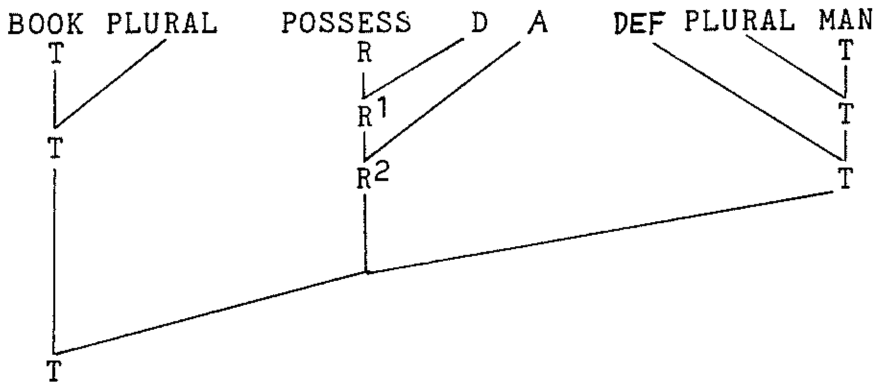
The men's books

in possessive form:



The books of the men

in differentiated relative form:



Thus the possessive modifier is regarded here as derivative, and as derived from a differentiated relative whose main relation is POSSESS, which in turn is just a special case of the differentiated relative which, generally, can have arbitrary main relations.

2.3.1.1.2.1.3 <u>Complementors</u>

Some of the determiners of English used in the examples of the previous section contain substrings of words called <u>complementors</u>, which were themselves determiners, such as "all but" and "not," and which intuitively reverse the sense of the other determiners to which they are applied. For example, the complementors "all but" and "not" reverse the sense of the determiners "a few," "many," "no," "at most two," etc. As is probably already obvious to the reader, "all but" and "not" reverse the sense of the above determiners in different ways. The precise semantic difference between them will be spelled-out in Section 2.3.2.3. In all, we distinguish three sorts of complementors in SYN_L^{TR} for TR-languages L:

True-complementors (TC) of SYN_L^{TR}

(English analogue: "all but")

Quasi-Complementors (QC) of SYN_L^{TR}

(English analogue: "not")

Diffuse complementors (RTC, ATC) of SYN_L^{TR}

(English analogue: "non", "not", "un")

The way in which complementors reverse the sense of determiners is reflected in the fact that determiners of English that are analogues of UBD determiners of $SYN_{English}^{TR}$ are transformed into determiners of English that are analogues of UBD determiners of $SYN_{English}^{TR}$, and conversely. The following examples illustrate the situation for English.

Determiner	Application of True-complementor to Determiner	Application of Quasi-complementor to Determiner
at least (LBD)	not at least (UBD) i.e., less than (UBD)	all but at least (UBD)
at most fewer than (UBD)	not at most fewer than (LBD) i.e., at least (LBD)	all but at most fewer than (LBD)
at most (UBD)	not at most (LBD) i.e., more than (LBD)	all but at most (LBD)
more than (LBD)	not more than, (UBD) i.e., at most (UBD)	all but more than (UBD)
many (LBD)	not many (UBD) i.e, a few (UBD)	all but many (UBD)
at most a few (UBD)	not at most a few (LBD)	all but at most a few (LBD)
at least a few (LBD)	not at least a few (UBD)	all but at least a few (UBD), i.e., at most many (UBD)
at most many (UBD)	not at most many (LBD) i.e.,more than many (LBD)	at least a few (LBD)
all (LBD)	not all (UBD) i.e., less than all (UBD)	all but all (UBD) i.e., no (UBD)
some (LBD)	not some (UBD) i.e., no (UBD)	not all (UBD)
______s (LBD)	not ______s (UBD)	all but ______s (UBD)
most (LBD)	not most (UBD)	all but most (UBD)
the (DD)	not the (LBD)	all but the (DD)
no (DD)	not no, (LBD) i.e., some (LBD)	all but no (DD) i.e., all (DD)
John (DD)	not John (LBD) i.e., something other than John (LBD)	all but John (DD)
a few (UBD)	not a few (LBD) i.e., many (LBD)	all but a few

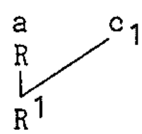
2.3.1.1.2.2 Case Morphemes

In this section we further discuss the semantic role of case modifiers of SYN_L^{TR} in constructing relation expressions of SYN_L^{TR}.

Every m-place relation-expression is composed of a series of modifiers a, $c_1,\ldots,c_m$, and the relation symbols R, $R^1,\ldots,R^m$ as follows: a is first applied to the relation symbol R to form the o-place relation-expression

$$\begin{matrix} a \\ R \end{matrix} \quad ;$$

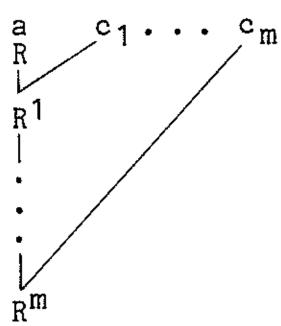
then c_1 and R^1 are applied to this o-place relation-expression to form:



;

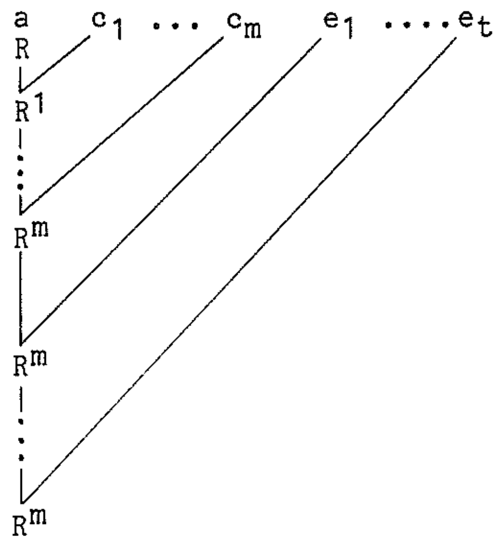
Then each successive modifier c_j,1 m, among $c_1,..,c_m$ is applied to the result of the immediately preceding application of c_{j-1} to form, ultimately, the m-place relation-expression:

(1)



Semantically, the role of the modifiers c_1 ,..., c_m is to build higher-place relations from lower-place relations "by accretion." Once these modifiers have been applied to build-up (1), then a sequence of non-place-increasing modifiers $e_1,..,e_t$ can be successively applied to form the fully modified m-place relation-expression:

(2)



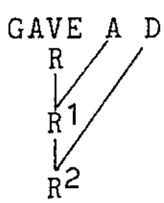
The modifiers $c_1,..,c_m$ of the relation (2) that increase the place-number of the relation are called the <u>case modifiers</u> of the relation-expression (2). Case modifiers that are morphemes are referred to as <u>case morphemes</u>. While all case modifiers illustrated in this study are morphemes, there is no theoretical basis for limiting case modifiers to morphemes of SYN_L^{TR}.

Letting L be a natural language, we define an occurrence of a word-string of L to be a <u>case-marker under a given reading</u> just in case it is represented by a case-modifier of SYN_L^{TR}. We further define a word-string of L to be a <u>case-marker of L</u> if it is an L-analogue of a case modifier of SYN_L^{TR}, that is, (by the definition of "L-analogue") just in case its occurrences are "usually" case-expressions of SYN_L^{TR} in normal readings of word-strings of L containing those occurrences.

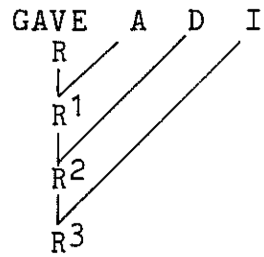
Basic Case Morphemes

For the situation where L is English, we have as case markers the usual case-indicating prepositions, e.g., "by," "for," "to," etc. which are explicit[65] case markers of English; among the implicit case markers of English we have those that indicate the usual cases, marked in English by order and other cues, and marked in SYN_L^{TR} by the basic morphemes: "A" for agent, "D" for direct object, "I" for indirect object, "C" for complement, "LOC" for location, and "TEMP" for time.

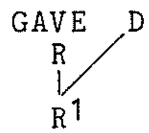
Composing relation-expressions of SYN_L^{TR}, where L = English

EXPRESSION OF $SYN_{ENGLISH}^{TR}$		INTERPRETATION
GAVE	:	a base morpheme
$GAVE_R$	:	a base relation (on that morpheme)
	:	a one-place relation expression, consisting of a base relation together with the agentive case morpheme; to be interpreted as a 1 place-relation whose only place is to be occupied by an agent of the action "gave."
	:	a two-place relation expression, consisting of a base relation together with agentive and direct object case morphemes; to be interpreted as a 2 place-relation whose places are to be occupied by an agent and by a direct object of the action "gave," in that order.

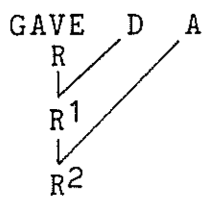
Note 65. See Section 1.5 for discussion of explicitness and implicitness.



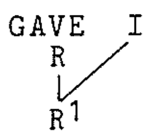
: a three-place relation-expression, consisting of a base relation, together with an agentive direct object, and indirect object case morphemes; to be interpreted as a 3 place-relation whose places are be occupied by an agent, by a direct object, and by an indirect object of the action "gave," in that order.



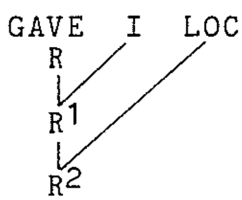
: a one-place relation-expression, consisting of a base relation together with a direct object case morpheme; to be interpreted as a 1-place relation whose only place is to be occupied by a direct object of the action "gave."



: a two-place relation-expression, consisting of a base relation together with direct object and agentive case morphemes; to be interpreted as a 2-place relation whose places are to be occupied by a direct object and agent of the action "gave," in that order.



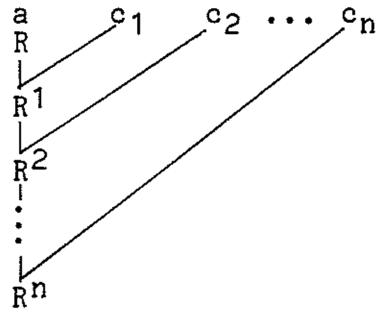
: a one-place relation-expression, consisting of a base relation together with an indirect object case morpheme; to be interpreted as a 1-place relation whose only place is to be occupied by an indirect object of the action "gave."



: a two-place relation-expression, consisting of a base relation together with an indirect object and locative case morphemes; to be interpreted as a 2-place relation whose places are to be occupied by an indirect object and location of the action "gave."

The relationships illustrated in these examples can be schematized in a general way as follows:

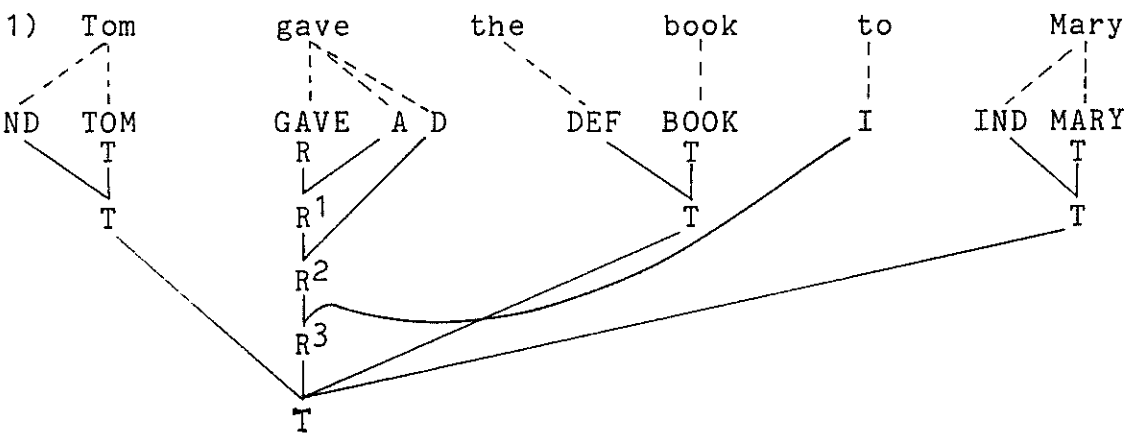
Let a, c_1, c_2,...,c_n be morphemes such that c_1, c_2,..., c_n are case markers;"a" will be used as a base morpheme for a base relation a:



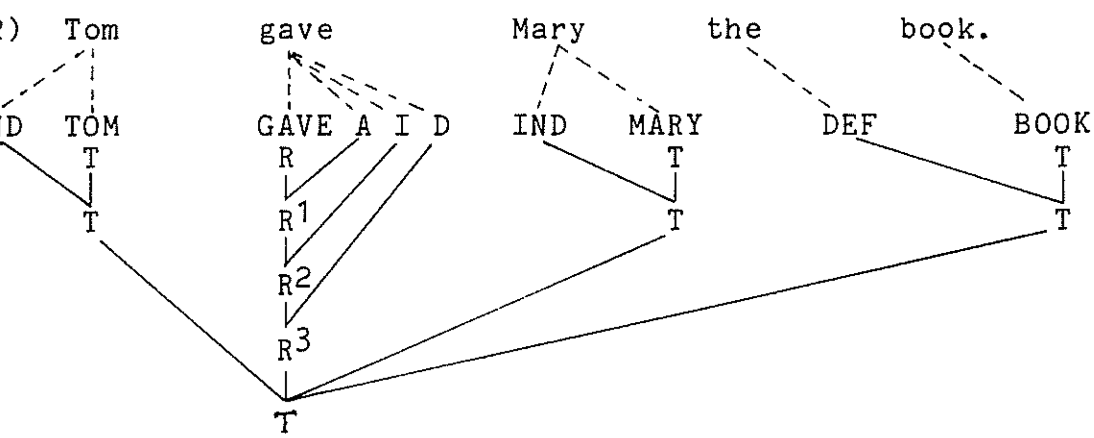
Syntactically, this is an n-place relation-expression consisting of a base relation a_R together with the case markers $c_1, c_2, \ldots, c_n$ interpreted as an n-place relation whose places are to be occupied by thing-expressions carrying the notional significance relative to the base relation a_R which intuitively corresponds to the case marker associated with that place.

The following examples illustrate the use of the basic case morphemes in composing relation-expressions of $SYN^{TR}_{English}$:

(1)



(2)



2.3.1.1.2.2.1 Adjunctive Case Morphemes

Consider the English sentence:

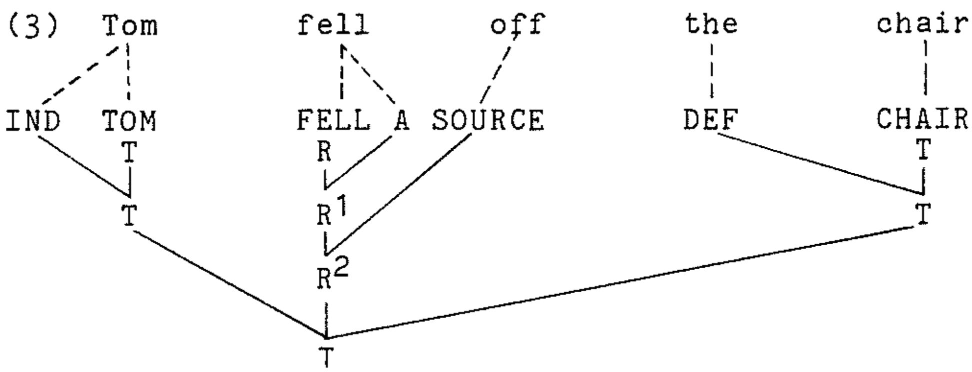
(1) John walked the dog to exercise

This is ambiguous in at least several ways: first, we can distinguish between "exercise" as an activity and as a place; second, as an intransitive or transitive activity; as a transitive activity, we can distinguish between the exercising of John and of the dog, that walking is to implement. When we consider "exercise" as a place, the dominant intuitive meaning of "to" in (1) is that of goal; when we consider "exercise" as an activity, (whether intransitive or transitive), the dominant intuitive meaning of "to" is that of purpose. Thus there are at least four distinct ways of understanding (1): under the first John's purpose in walking his dog is to exercise the dog; under the second, John's purpose is to exercise himself; under the third, John's purpose is simply to exercise, i.e., in the intransitive sense of "exercise"; under the fourth, the goal, i.e., the place to which John is walking the dog, is exercise.

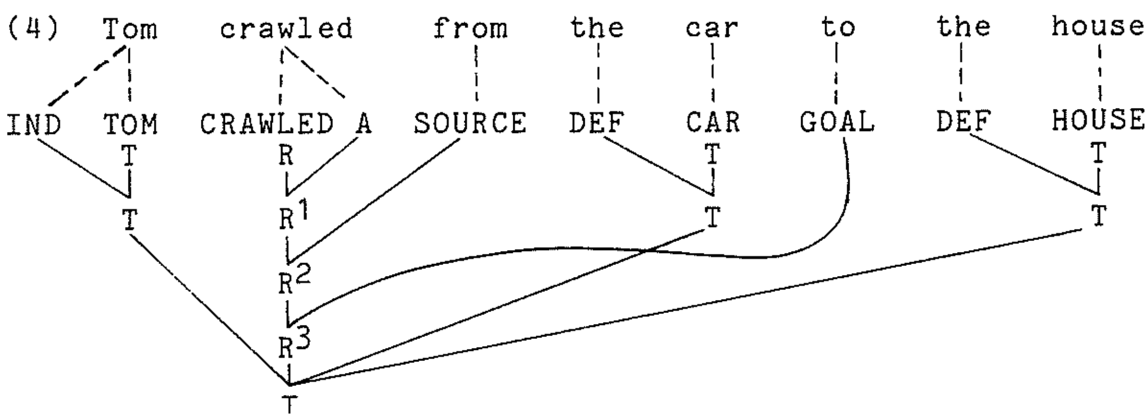
These considerations suggest the possible utility of introducing into SYN_L^{TR} case morphemes that have special notional meanings, such as the case morphemes PURP and GOAL, having purpose and goal as their respective notional meanings. These notional meanings can be expressed by inter-relating such case morphemes within suitable lexical semantic axioms, with other lexical morphemes of SYN_L^{TR}, to the effect, for example, that goals are places, that purposes are intentions of certain kinds, that only living things have purposes, etc. Such axioms would be language-specific, since the relation between purposes and intentions, say, that holds under the various English meanings of these words would not necessarily be duplicated within expressions of other languages. Accordingly, we treat adjunctive case morphemes as lexical rather than logical, and continue to distinguish lexical representational morphemes from logical ones as those which are language-specific from those which are not.

The following examples illustrate the use of adjunctive case morphemes[66] in composing relation-expressions of $SYN^{TR}_{English}$:

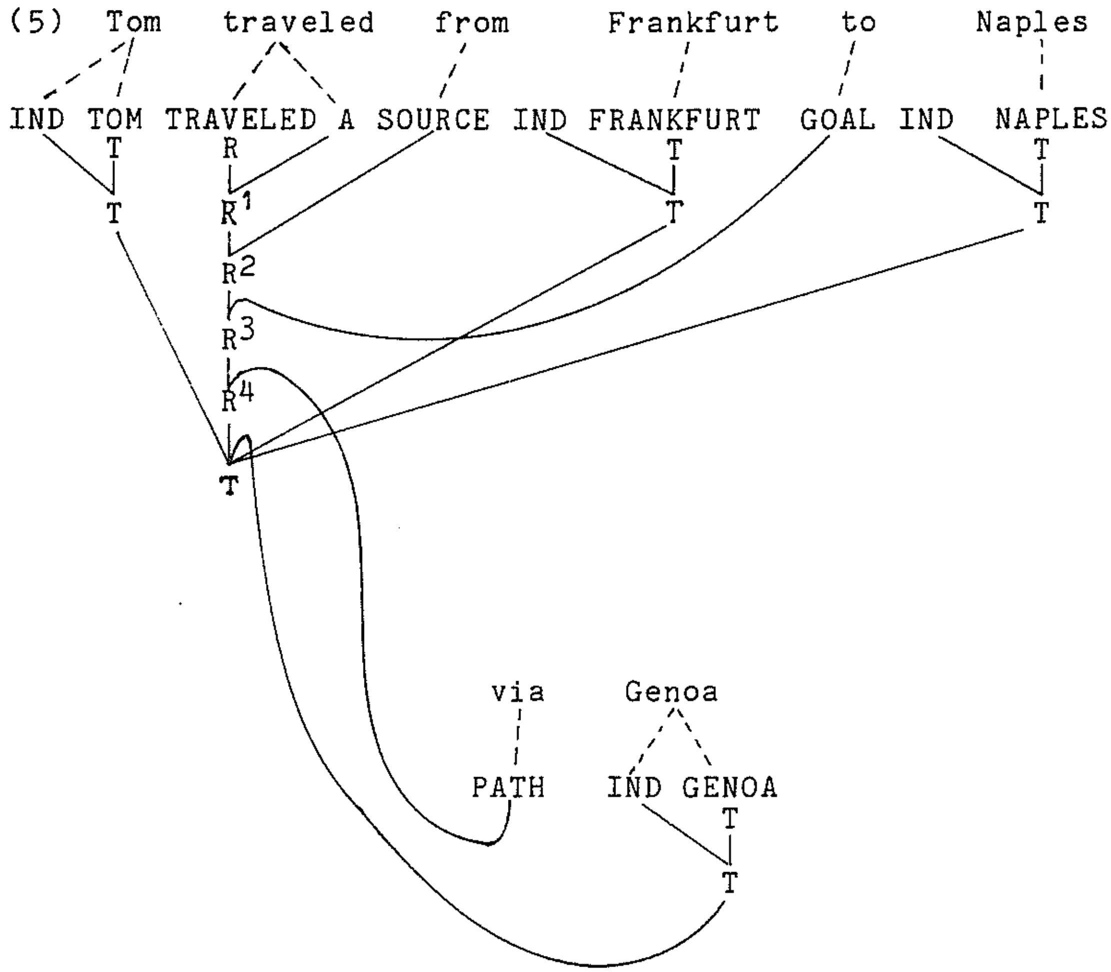
(3)



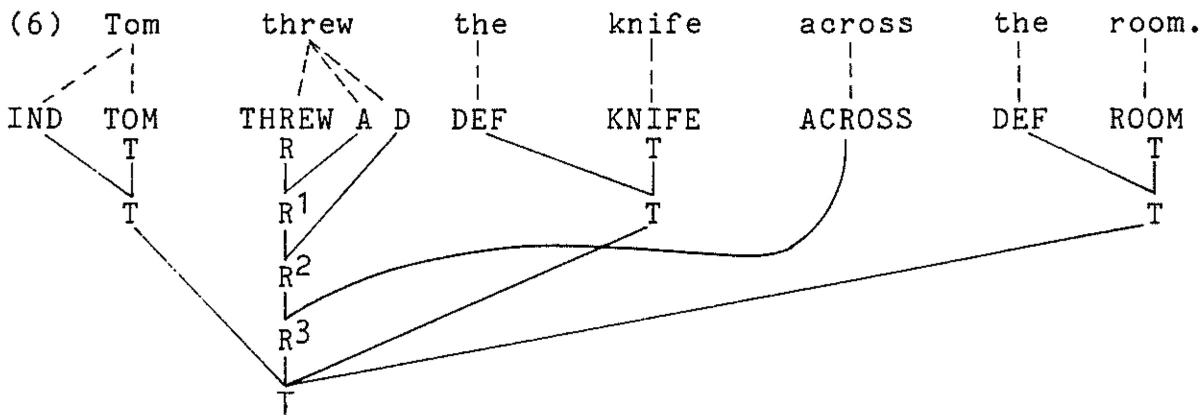
(4)



Note 66. The adjunctive case morphemes of SYN^{TR}_{L} are indicated here with English-like mnemonic lexical morphemes, such as PURP for "purpose," etc., but I do not wish by so doing to suggest that lexical representational morphemes of SYN^{TR}_{L} (or for that matter, logical ones) need to "resemble" the morphs of L in any way.

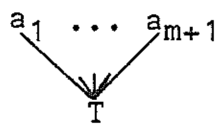

(5) Tom traveled from Frankfurt to Naples
IND TOM TRAVELED A SOURCE IND FRANKFURT GOAL IND NAPLES
T R T T
T R1 T T
R2
R3
R4
T
via Genoa
PATH IND GENOA
T
T

(6)


Tom threw the knife across the room.
IND TOM THREW A D DEF KNIFE ACROSS DEF ROOM
T R T T
T R1 T T
R2
R3
T

2.3.1.1.2.3 <u>Sentences</u>

Let L be a TR-language. As described in the preceding section, a sentence of SYN_L^{TR} has the form:

(1) 

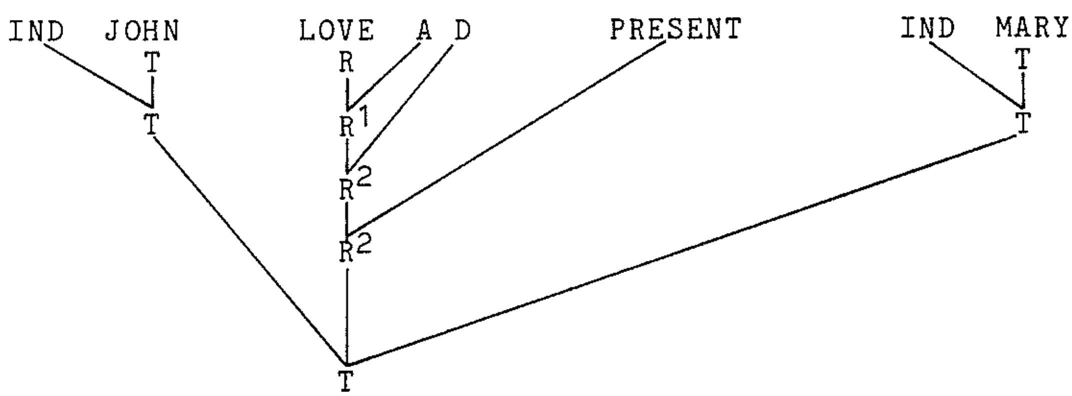
where exactly m among $a_1,\ldots,a_{m+1}$ are thing-expressions of SYN_L^{TR} and one among $a_1,\ldots,a_{m+1}$ is an m-place relation-expression of SYN_L^{TR}.

For simplicity, we will occasionally use the notation

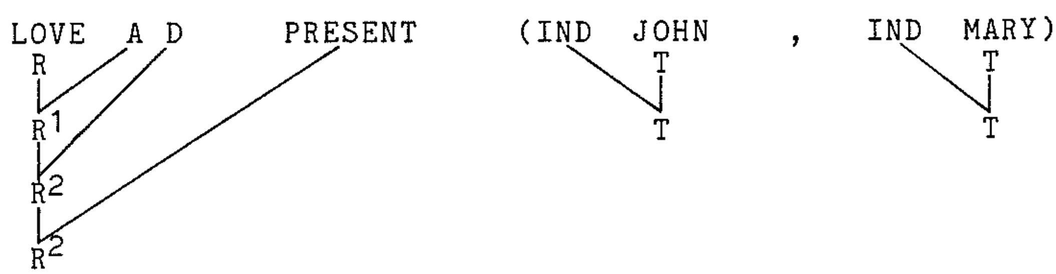
(2) $r^m(e_1 \ldots e_m)$

for a sentence of SYN_L^{TR}, where $e_1,\ldots, e_m$ are the m thing-expressions among $a_1,\ldots,a_{m+1}$, taken in order of occurrence, and r^m is the m-place relation expression among $a_1,\ldots, a_m$.

For example, a sentence of $SYN_{English}^{TR}$, which has the form:

(3) 

would be expressed in this simplified notation of (2) as:

(4) 

The Denotations of Sentences: Events and Event Particulars

The Basic Semantic Paradigm

In this section we define the denotations of sentences. For simplicity we here consider the special case where no referential links (see pages 177-180) occur among thing-expressions. When we abandon this constraint, an extension of the paradigm is required, and appears below as L.S.A. (8.1).

Now let <F, V, R> be a semantic theory belonging to INT_L^{TR}, and let <D,f> be an interpretation belonging to F. Let $G_V(b)$ be a condition [67.1] on the denotations of the proper sub-expressions of b associated with V and which is such that, for all sentences e of SYN_L^{TR},

$$V(<b,<D,f>>) = \begin{cases} \text{truth, if } f(b) \neq \phi \text{ and } G_V(b) \\ \text{falsehood, if } f(b) = \phi \text{ and } G_V(b) \\ \text{nil, if not } G_V(b) \end{cases}$$

The Basic Paradigm. The following basic paradigm motivates the definitions of event and event particulars to follow. Let e = $r^m(a_1,\ldots,a_m)_{p,q}$ be a sentence of SYN_L^{TR} where each of $a_1,\ldots,a_m$ is a thing-express n and p,q are permutations on the set of integers $\{1,\ldots,m\}$ such that, for each i, $1 \leqslant i \leqslant m$, p maps i into

Note 67.1 The precise character of the condition G_V will, generally speaking, vary as the semantic theory <F,V,R> varies, and will govern the sorts of existential presuppositional conditions one might wish to impose in the denotations of subexpressions of sentences under the interpretations <D,f>, such as, for example, the condition that no lower-bounded thing-expression occurring in the sentence have the denotation $\{\emptyset\}$ under <D,f>, that no definite thing-expression occurring in the same sentence whose leftmost determiner is different from "UN" has the denotation $\{0\}$ under <D,f>, or the like.

the scope index $p(i)$ of a_i in e, and q maps i into the place index $q(i)$ of e_i in e. Then e is <u>true</u> in <D,f> just in case $G_V(e)$ and there is some set $x_1 \in f(a_{[p^{-1}(1)]})$ such that for all $y_1 \in x_1$ there is some $x_2 \in f(a_{p^{-1}(2)})$ such that for all $y_2 \in x_2$ ... ... there is some set $x_m \in f(e_{p^{-1}(m)})$ such that for all $y_m \in x_m$, $\langle y_{q^{-1}(1)}, y_{q^{-1}(2)}, \ldots, y_{q^{-1}(m)} \rangle \in f(r^m)$.

Let us see how this paradigm specializes to the example sentence (4) (i.e., (3)) on page 162. Since the scope and place indexes of the two thing-expressions of (4) coincide with the order of occurrence of those thing-expressions in (4), the permutations p,q on $\{1,2\}$ are just the identity permutations on $\{1,2\}$, so that $p^{-1}(1) = q^{-1}(1) = 1$ and $p^{-1}(2) = q^{-1}(2) = 2$, hence that $a_{p^{-1}(1)} = a_{q^{-1}(1)} = a_1$ = IND JOHN (T, T)

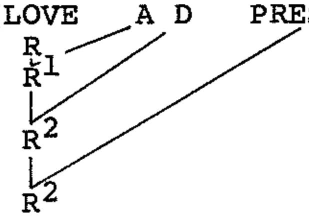
and

$a_{p^{-1}(2)} = a_{q^{-1}(2)} = a_2$ = IND MARY (T, T)

The sentence (4) given on page 162 (hence (3)) is <u>true</u> in <D,f> just in case $G_V((3))$ and there is some set $x_1 \in$ f(IND JOHN (T, T)) such that for all $y_1 \in x_1$,
there is some set $x_2 \in$ f(IND MARY (T, T))

such that for all $y_2 \in x_2$,

$\langle y_1, y_2 \rangle \in f$(LOVE A D PRESENT).



As remarked earlier in Section 2.1, there are two important orderings among the thing-expressions in a SYN^{TR}-sentence e: first, their relative-place ordering which orders the thing-expressions relative to the major relation of e which interconnects them; second, their relative-scope ordering which orders the thing-expressions relative to the scope of their determiners. (We can find a partial analogue in the structure of a quantified atomic formula of predicate logic: the first sort of order specializes to the order of occurrence within argument places in that formula; the second sort of order is that of the order of the quantifiers on those argument places.)

The basic paradigm stated above affords a basis for the definition of a suitable denotation f[e] for a SYN_L^{TR}-sentence e under an interpretation (D,F), called an <u>event particular</u>, and a sort of "generalized" interpretation s[e] of e under the full semantic theory $s = (F_s, V_s, R_s)$, called an <u>event</u> which is defined as the set of all event particulars yielded by the individual interpretations $(D,f) \in F_s$.

These definitions proceed as follows:

Let $B_1,\ldots,B_m \subseteq PD$. A <u>chain function</u> on the sequence $(B_1,\ldots,B_m)$ is defined as a function g which assigns, for every $1 \leq i \leq m-1$ and for every $y \in \bigcup B_i$, a set $g(i,y)$[67.2] $\in B_{i+1}$.

Let $B_1,\ldots,B_m \subseteq PD$, let q be a permutation of $\{1,\ldots,m\}$, and let g be a chain function on the sequence $(B_1,\ldots,B_m)$. Then the trace of g through $(B_1,\ldots,B_m)$ with respect to q is the set:

$$\{(z_{q^{-1}(1)},\ldots,z_{q^{-1}(m)}) \in D^m \mid \text{for some } x_1 \in B_1,\ z_1 \in x_1, \text{ and } z_2 \in g\ (1,z_1) \text{ and } z_3 \in g\ (2,z_2) \text{ and } \ldots \text{ and } z_m \in g\,(m-1,\ z_{m-1})\}$$

Let $r^m(a_1,\ldots,a_m)p,q$ be a sentence of L' and let (D,f) be an interpretation. Then:

(i) the positive relational structure of $r^m(a_1,\ldots,a_m)p,q$ under (D,f) is the set $P_{(D,f)}[r^m(a_1,\ldots,a_m)p,q]$:

$$f(r^m) \cap [\bigcup f(a_{q-1(1)}) \times \ldots \times \bigcup f(a_{q-1(m)})]$$

(ii) the denotation $f[r^m(a_1,\ldots,a_m)p,q]$ of $r^m(a_1,\ldots,a_m)p,q$ under (D,f) is the set:

$\{\{(f[r^m], P_{(D,f)}\ [r^m(a_1,\ldots,a_m)p,q])\}\}$, if the positive relational structure of $r^m(a_1,\ldots,a_m)p,q$ under (D,f) is consistent with the determiner structure of $r^m(a_1,\ldots,a_m)p,q$ under (D,f) in the sense that there are sets $B_1 \subseteq f(a_{p-1(1)}),\ldots,\ B_m \subseteq f(a_{p-1(m)})$ and there is a chain function g on $(B_1,\ldots,B_m)$ such that the trace of g through $(B_1,\ldots,B_m)$ with respect to q is identical with $P_{(D,f)}[r^m(a_1,\ldots,a_m)p,q]$; otherwise, $f[r^m(a_1,\ldots,a_m)p,q] = \phi$

Given the notion of consistency defined in (ii) above, we can express the defintion (ii) above more succinctly as follows:

Note 67.2. We introduce the parameter i in g(i,y) to accommodate the case where y occurs in both B_i and B_j for $i \neq j$.

$$f[r^m(a_1,\ldots,a_m)p,q] = \begin{cases} \{(f[r^m], P_{(D,f)}[r^m(a_1,\ldots,a_m)p,q])\} & \text{if the positive relational profile of } r^m(a_1,\ldots,a_m)p,q \text{ under } (D,f) \text{ is consistent with the determiner structure of } r^m(a_1,\ldots,a_n)p,q \text{ under } (D,f)\text{; and} \\ \phi & \text{otherwise.} \end{cases}$$

By the above definition, if $f[r^m(a_1,\ldots,a_m)p,q \neq \phi$, then $f[r^m(a_1,\ldots,a_m)p,q] = \{(f[r^m], P_{(D,f)}[r^m(a_1,\ldots,a_m)p,q])\}$. In the case that $f[r^m(a_1,\ldots,a_m)p,q] \neq \phi$, we call the pair $(f[r^m], P_{(D,f)}[r^m(a_1,\ldots,a_m)p,q]$ <u>the event particular corresponding to $r^m(a_1,\ldots,a_m)p,q$ relative to (D,f)</u>. Also, we define the <u>event corresponding to $r^m(a_1,\ldots,a_m)p,q$ relative to the semantic theory $s = (F_s,V_s,R_s)$</u> as the set of all event particulars corresponding to $r^m(a_1,\ldots,a_m)p,q$ relative to (D,f), as (D,f) ranges over F_s.

Example

$$f[\underset{\substack{R\\R^1\\R^2\\T}}{\mathrm{INCDISJ}}\ A\ \ D\ \ a_1\ a_2] \underset{\substack{\uparrow\\ \text{by def.}\\ \text{of } f}}{=} \{\{P_{D,f)}\ [\underset{\substack{R\\R^1\\R^2\\T}}{\mathrm{INCDISJ}}\ A\ \ D\ \ a_1\ a_2]\}\}$$

$$\underset{\substack{\uparrow\\ \text{by def.}\\ \text{of } P_{(D,f)}}}{=} \{\{f[\underset{\substack{R\\R^1\\R^2}}{\mathrm{INCDISJ}}\ A\ \ D] \cap [\bigcup f(a_1) \quad x \quad \bigcup f(a_2)]\}\}$$

$$\underset{\substack{\uparrow\\ \text{by def.}\\ \text{of } f<\underset{\substack{R\\R^1\\R^2}}{\mathrm{INCDISJ}}\ A\ \ D]}}{=} \{P_{(D,f)}[b_1] \quad x \quad P_{(D,f)}[b_2] \mid f(b_1) \neq \phi \text{ or } f(b_2) \neq \phi\} \cap [\bigcup f(a_1) \ x \ \bigcup f(a_2)]$$

$$\underset{\substack{\uparrow\\ \text{by set theory}}}{=} \begin{cases} \{P_{(D,f)}[a_1) \ x \ P_{(D,f)}[a_2]\}, & \text{if either } f(a_1) \neq \phi \text{ or } f(a_2) \neq \phi \\ \phi, & \text{otherwise} \end{cases}$$

$$\text{i.e.} \underset{\substack{\uparrow\\ \text{by set}\\ \text{theory } 67.3}}{=} \begin{cases} \bigcup f(a_1) \ x \ \bigcup f(a_2), & \text{if either } f(a_1) \neq \phi \text{ or } f(a_2) \neq \phi \\ \phi & \text{, otherwise} \end{cases}$$

Note 67.3.
Since $\bigcup f(a_1)$ = the set of els of els of $f(a_1) = \{P_{(D,f)}[a_1]\}$, we have: $f(a_1) = \{\{P_{(D,f)}[a_1]\}\}$ if $P_{(D,f)}[a_1]$ is consistent with the determiner structure of a_1, and $f(a_1) = \phi$ otherwise. So, $\{P_{D,f)}[a_1] \ x \ P_{(D,f)}[a_2]\}$ $= \bigcup f(a_1) \ x \ \bigcup f(a_2)$.

By L.S.A.(18.1) (page 240, below), we have:

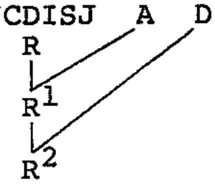
$f(\mathrm{INCDISJ}_{R,R^1,R^2}\ A\ D) = \{\langle x,y\rangle \in D^2 : x \neq \phi \text{ or } y \neq \phi\}$; it follows then that

$a_T\ \mathrm{INCDISJ}_{R,R^1,R^2,T}\ A\ D\ b_T$ is true in $\langle D,f\rangle$ just in case

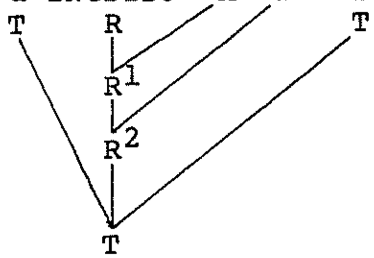
$f(a_T) \neq \{\{\phi\}\}$ or $f(b_T) \neq \{\{\phi\}\}$, that is, just in case $f(a_T)$ contains at least one event particular (i.e., some event particular is an element of $\bigcup f(a_T)$) or $f(b_T)$ contains at least one event particular (i.e., some event particular is an element of $\bigcup f(b_T)$).

By L.S.A.(18.1) (page 240, below), we have:

$f(\text{INCDISJ} \; A \; D) = \{(x,y) \mid D^2 : x \neq \phi \text{ or } y \neq \phi\}$; it follows then that



a INCDISJ A D b is true in (D,f) just in case



$f(a_T) \neq \{\{\phi\}\}$ or $f(b_T) \neq \{\{\phi\}\}$, that is, just in case $f(a_T)$ contains at least one event particular (i.e., some event particular is an element of $\bigcup f(a_T)$) or $f(b_T)$ contains at least one event particular (i.e., some event particular is an element of $\bigcup f(b_T)$).

2.3.1.1.2.4 Further Notational Devices in SYN_L^{TR}

There are four further notational devices within SYN_L^{TR} whose purpose is to preserve contiguity relations[67] between natural language word-strings and their syntactic representations in SYN_L^{TR}. The first device is to use subscripts and superscripts on simple thing labels to indicate, respectively, a particular relative scope ordering and a particular relative-place ordering on the major thing-expressions of a sentence. The second device consists in the use of multiple labels, wherein a collection of simple thing and relation labels are used to differentiate the different ways in which a given occurrence of an expression can function grammatically within a containing expression. The third device involves the use of dummy thing and relation labels. The fourth involves the linking of one or more expressions by a branch to signify that they are to have the same denotation.

We will next describe each of these four notational devices in turn.

Note 67. Preservation of contiguity relations simplifies the specification of reading and generation rules, and makes it possible to specify homologous readings to word-strings, in the sense of Note 49.

Subscripts and Superscripts on Thing Labels to Indicate Relative Scope and Relative-Place Ordering

There are two dominant normal readings of the sentence

(1) Every man loves some woman

which we can identify (as we had done earlier in Section 1.8) by exhibiting its canonical alternates:

(2) Every man is such that he loves some woman

and

(3) Some woman is such that every man loves her

It is intuitively clear that (2) expresses a more dominant normal reading of (1) than does (3). The basis for this difference will be partly evidenced in this section.

The intuitive difference in meaning between (2) and (3) derives, of course, from a difference in the relative scope of the quantifiers "every" and "some": in (2) "some" lies within the scope of "every"; in (3), "every" lies within the scope of "some."

This difference is familiarly expressible within a predicate logic representation as follows:

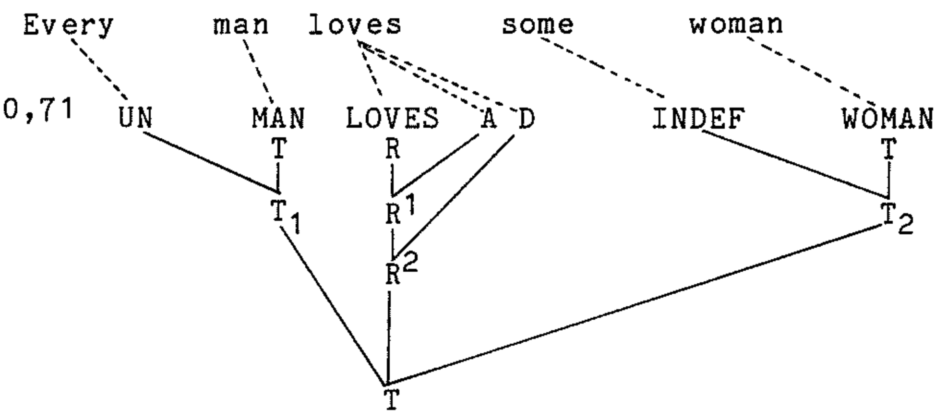
(2') (x)(man x $\rightarrow$ (Ey)(woman y & loves xy))

(3') (Ey)(woman y & (x)(man x $\rightarrow$ loves xy))

We express the difference in which the relative scope of quantifiers within $SYN^{TR}_{English}$ by imposing numerical subscripts on the thing-labels which indicate the relative scope of their respective quantifiers: for example, a subscript of "1" on a simple thing-label indicates that the quantifier[68,69] on that expression is taken first, i.e., all other quantifiers lie within its scope; a subscript of "2" indicates that it is to be taken second, i.e., all remaining quantifiers lie within its scope, and so on. We call the subscripts "1," "2," etc. on the thing-labels the scope _indexes_ of the thing-expressions that they label. Accordingly, we express the intended readings (2) and (3) of (1) within $SYN^{TR}_{English}$ as (2'') and (3'' respectively:

(1)

(2'')[70,71]



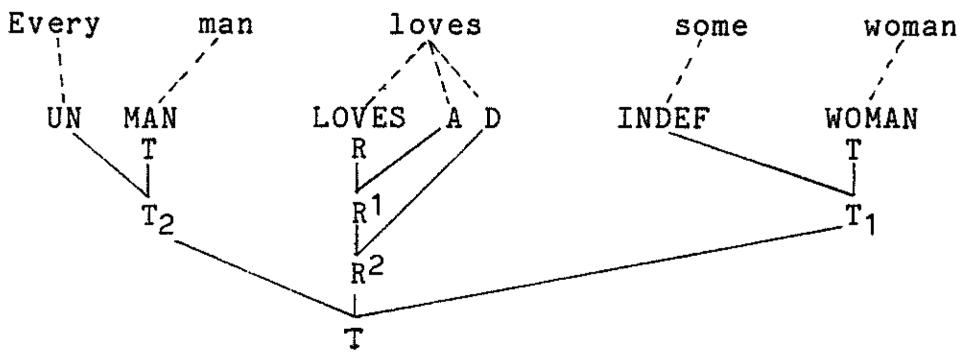
Note 68. In SYN^{TR}_{L}, we treat arbitrary determiners, not only the quantifiers "every" (or "all") and "some," so that the subscripting notation being described here applies indifferently to all determiners of L.

Note 69. Since the effect of the notation of the predicate calculus is to separate a quantifier from the predicate (i.e., the remaining part of the noun phrase) on which it operates, we speak here of the "ordering of the quantifiers" rather than of "the ordering of the (full) noun phrases"; when working within SYN^{TR}_{L} this separation between a determiner and the remaining part of the noun phrase does not occur, so that the ordering of the determiners need not be distinguished from the ordering of the full noun phrases to which they apply. Indeed, the fact that the language SYN^{TR}_{L} preserves the contiguity properties of L makes it possible to develop a fairly simple set of reading rules by which syntactic representations of natural language word-strings can be coherently related to those word-strings.

Note 70. We suppress the tense morpheme PRESENT here for simplicity.

(1)

(3")[70,71]



Up to this point we had suppressed the indexes on thing-expressions, since in earlier examples the relative scope orderings and relative-place orderings of the thing-expressions in a sentence coincided with the occurrence ordering of those thing-expressions in that sentence. For the sake of notational simplicity, when either of these two orderings coincides with the occurrence ordering, we will continue to suppress the relevant indexes. In addition, we will suppress other notational elements such as tense modifiers, in examples when those elements are redundant or not relevant to the topic at hand.[70]

Relative-Place Orderings

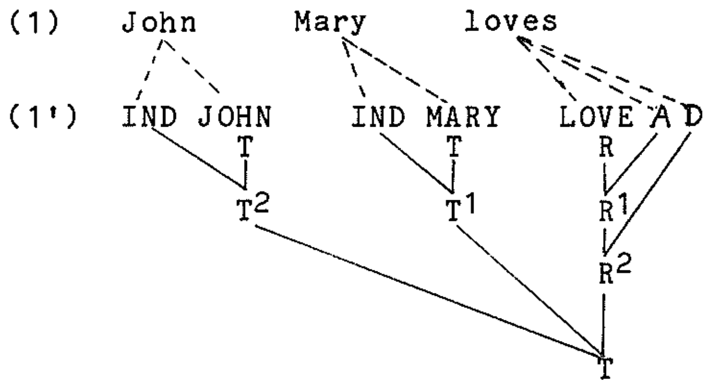
There are no natural examples in English, so far as I can determine, where the relative-place ordering is different from the occurrence ordering. To illustrate the use of superscripts to indicate the relative place order, let us consider a

---

Note 71. As remarked above, the scope index of the thing-expression UN MAN is 1 in (2") and is 2 in (3"), and the scope index of INDEF WOMAN is 2 in (2") and is 1 in (3").

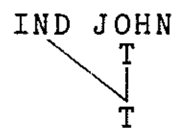
hypothetical language L that formed the passive by situating both the intended subject and direct object on the same side, say the left, of the main verb phrase, whereby the intended subject was the noun phrase closest to the verb phrase.

Using English-like lexical representational morphemes within L' we would represent then the sentence (1) by (1'):



The superscripts indicate that the thing-expression IND MARY is

taken first and the thing-expression IND JOHN is taken second



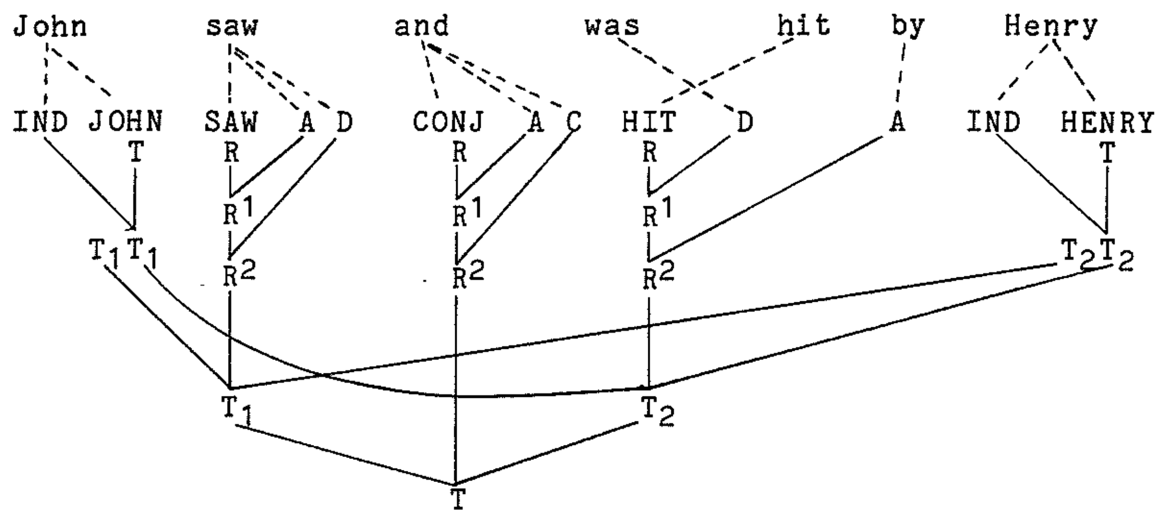
with respect to the relation LOVE A D

Multiple Labels

Multiple labels are used to indicate the different ways that a given expression can function grammatically within a containing expression. As indicated, there are three general grammatical functions, namely an expression can function as a thing, a relation, or as a modifier. Let us consider some examples:

(4)

(5)



We use the multiple labels T_1T_1 and T_2T_2 immediately below IND JOHN and IND HENRY, respectively, obtaining thereby

IND JOHN (T) and IND HENRY (T)

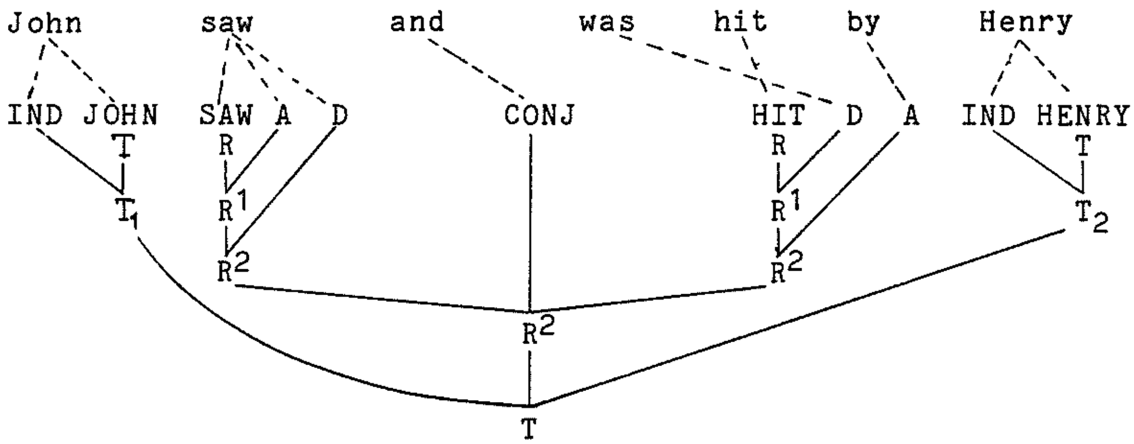
IND JOHN (T, T_1T_1) and IND HENRY (T, T_2T_2)

to indicate that IND JOHN (T, T_1) (IND HENRY (T, T_2)) functions grammatically in (4') (hence derivatively, in (4)) both as the agent (direct object) of SAW (R) and as the direct object (agent) of HIT (R). This permits us to avoid introducing second occurrences of IND JOHN (T, T_1) and IND HENRY (T, T^2) in the syntactic representation component of a normal reading of (4).

Note that multiple labels are not required in the following syntactic representation of (4).

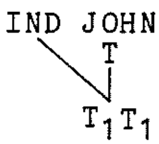
(4)

(6)

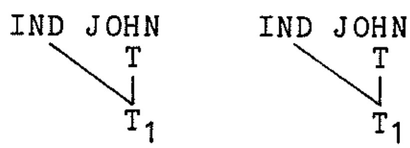


We note that

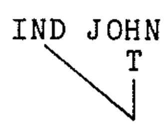
(i)



is not an expression[72] of $SYN^{TR}_{English}$ but, rather, is two expressions (which here happen to be identical), namely,



which have the same modifier part



Note 72. We could, alternatively, have allowed configurations like (i) as expressions of $SYN^{TR}_{English}$, but this would have required us then to allow for an infinite number of types of expressions, one for each finite sequence of labels T_n and R^m. This in itself would not be noxious; however, the fact that an expression with a multiple label would not denote a thing, relation, or modifier function would, in my judgment, destroy what I regard as part of the fundamental intuitive character of thing-relation languages, namely, that each expression is intuitively interpreted as a thing, a relation, or modifier function.

See Section 3.5 of Chapter 3 for examples of multiple labels on a modifier which are not identical.

The difference between the readings (5) and (6) of (4) is that the reading (5) of (4) is that equivalent to the dominant normal reading of

(4') John saw Henry and John was hit by Henry

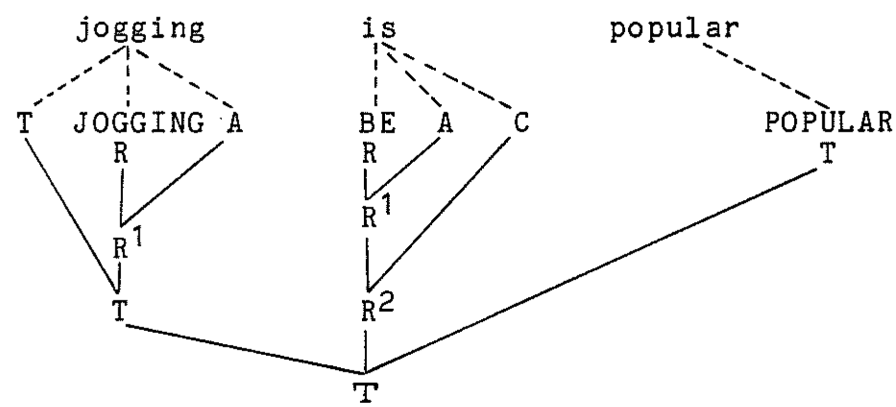
whereas the reading (6) of (4) is not.

Dummy Thing and Relation Labels

Dummy thing and relation labels are used to designate a degenerate thing or relation-expression when no suitable lexical base exists for it. As examples of each we have:

(7)

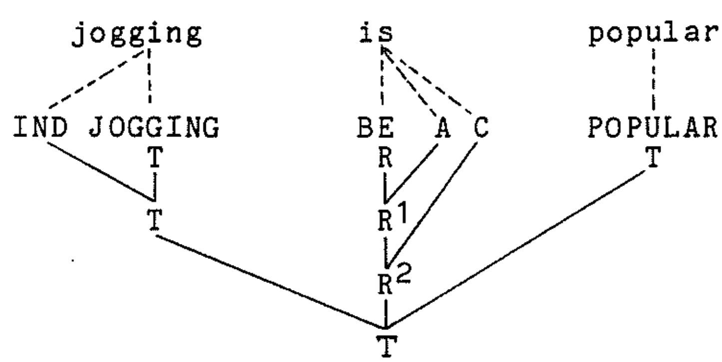
(7')



which intuitively means that the event of jogging is popular as opposed to, say, the activity of jogging being popular, which would be syntactically represented by

(8)

(8')



The use of dummy relation labels will be further illustrated in Section 3.1 of Chapter 3, when we discuss non-restrictive relative constructions.

Linking

There are two sorts of ways that pronouns can be related to their referent nouns, which we distinguish as their being _co-referentially distributed_ with their referent noun phrases and as being _co-referentially non-distributed_ with their referent noun phrases. The dominant normal reading of (1) below is one in which "his" is co-referentially distributed with "every man"

(1) Every man loves his mother

by which is meant, roughly, that every man loves his own mother but not necessarily other men's mothers; that is, "his mother" and "every man's mother" need not denote the same individual(s); that reading of (1) in which "his" is co-referentially non-distributed with "every man," every man loves every man's mother, that is, that reading in which "his mother" and "every man's mother" denote the same individual(s), and is clearly non-normal. Unlike (1), where the co-referentially nondistributed reading is inequivalent to the co-referentially distributed one, the situation is different in the following sentence

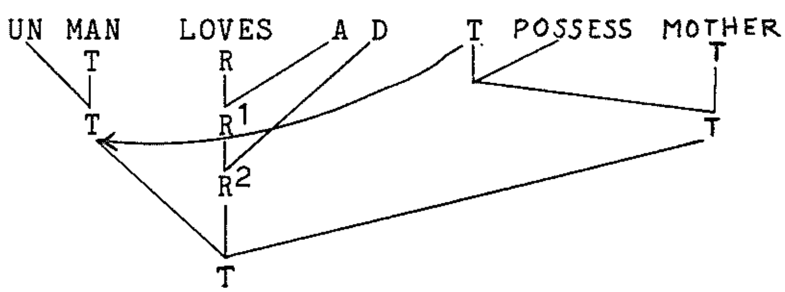
(2) John loves his mother

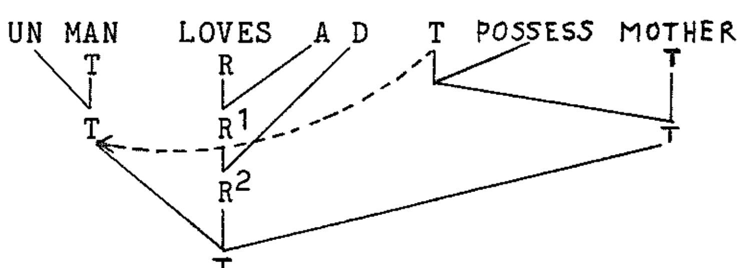
where the co-referentially nondistributed reading is equivalent to the co-referentially distributed reading so that, given that "John" is the referent noun for "his" (which in other normal readings of (2) need not be the case), "his mother" and "John's mother" denote the same individual. Semantically, this follows from the fact that "John" is represented within $SYN_{English}^{TR}$ by an expression that is interpreted as a singleton in a normal reading.

We wish to distinguish syntactically between co-referentially distributed and nondistributed relations among expressions of SYN_L^{TR}, so that we can interpret them differently within a suitable semantic theory.

Syntactically, within SYN_L^{TR}, we indicate that a thing expression e is to be (interpreted as) co-referentially non-distributed with its referrent thing-expression b by joining them by a solid arrow from e to b, and we indicate that e is to be (interpreted as) co-referentially distributed with its referrent thing-expression b by joining them by a dotted arrow from e to b.

For example, we would distinguish syntactically between the co-referentially nondistributed and co-referentially dis-tributed readings of (1) as (1') and (1''), respectively:

(1')


(1'')


Applications of these linking devices, in particular, the co-referentially nondistributed case, to restrictive and non-restrictive relative clauses, anaphora, and ellipsis, are given later in Chapter 3.

We note that pronouns that are co-referentially non-distributed with their referent noun phrases are _eliminable_ in the sense that they can be replaced by their referent noun phrases, and that pronouns that are co-referentially distributed with their referent noun phrases are _ineliminable_ in the sense that they cannot be replaced by their referent noun phrases.[73]

For ease in exposition, I will often use the word "eliminable" to refer to a pronoun that is co-referentially non-distributed with its referent noun phrase, and will use the word "ineliminable" to refer to a pronoun that is co-referentially distributed with its referent noun phrase.

Note 73. For example, the pronoun "he" in

(1) John loves Mary or he loves Martha

is eliminable, because "he" can be replaced by its referent noun phrase "John", to yield the equivalent

(1.1) John loves Mary or John loves Martha

On the other hand, the pronoun "he" in (2) cannot be replaced by its referent noun phrase "every man"

(2) Every man loves Mary or he loves Martha.

for such replacement would yield

(2.1) Every man loves Mary or every man loves Martha
which is not equivalent to (2). Such cases are common across many constructions; consider the following:

(3) John loves his mother

which is clearly equivalent to

(3.1) John loves John's mother

whereas

(4) Every man loves his mother

is clearly _not_ equivalent to

(4.1) Every man loves every man's mother.

Let us consider also cases where co-referentiality crosses sentence boundaries: the referent of an eliminable pronoun can be in the preceding sentences, as in

(5) John loves Mary. He also loves Agnes.

or in the following sentence, as in

(6) He loves Mary. John also loves Agnes.

There may even be an intervening sentence, as in:

(7) John loves Mary. Mary knows Agnes. He also loves Agnes

or in

(8) He loves Mary. Mary knows Agnes. John also loves Agnes.

The referent of an ineliminable pronoun, while not usually in a different sentence, can be, as in (9) where the referent is in the preceding sentence:

(9) Every man loves Mary. He also loves Agnes.

The referent of an ineliminable pronoun can also be in a following sentence, but only very awkwardly, as in:

(10) He loves Mary. Every man also loves Agnes.

Also, it is very awkward to have an intervening sentence unless that sentence also contains an ineliminable pronoun with the same referent, as does (11); (12), on the other hand, contains an intervening sentence without such an ineliminable referrent:

(11) Every man loves Mary. He also loves Agnes. He also loves his mother.

Compare

(12) Every man loves Mary. Mary knows Agnes. He also loves Agnes.

2.3.1.2 Formal Description of SYN_L^{TR}

Let L be a TR-language. Let LOGMORPH be a set of logical representational morphemes, which contains at least those listed in Section 2.3.1.1.2. Recall that such morphemes are common to all $SYN_{L'}^{TR}$, as L' ranges over TR-languages. Let LEX(L) be the set of lexical representational morphemes of L. Then the syntactic representation language SYN_L^{TR} is defined as the smallest set K such that, if m is a positive integer, and if each of i,j is a positive integer or the blank expression, then:

(1) LOGMORPH U LEX(L) $\subseteq$ K

(2)[74] If $a_1, \ldots, a_m \in K$, then $\underset{\bigvee}{a_1 \ \ldots \ a_m} \in K$

(3)[75] If $b \in$ LOGMORPH U LEX(L) or if $b = \underset{\bigvee}{b_1 \ldots b_m}$, for some $b_1, \ldots, b_m \in K$, then

$\underset{T_i^j}{b} \in K$ and $\underset{R^j}{b} \in K$ 75.1

Note 74. The notation $\underset{\bigvee}{a_1 \ldots a_m}$ means that the descending lines are attached to the lowest points of $a_1, \ldots, a_m$. If m = 1, then we suppress the single descending line from a_1.

Note 75. The notation $\underset{\vee}{X \ Y}$ means that X and Y are joined by a branch as indicated, and that the leftmost symbol of Y is to the right of the rightmost symbol of X. This leaves open the possibility that there may be other symbols or full intervening expressions between (in the ordinary sense of "between") X and Y.

Note 75.1 We will continue to refer to special classes of SYN_L^{TR}-expressions by the terms introduced for them in the preceding Section 2.3.1.1.

The intuitive meaning of the clauses (1), (2), and (3) is respectively as follows:

Clause (1) means that all logical and lexical representational morphemes in the narrow sense are in SYN_L^{TR}. The remaining clauses (2) and (3) specify how larger expressions of SYN_L^{TR} are built out of these morphemes.

Clause (2) means that any expression or sequence of expressions of SYN_L^{TR} can be used to form a modifier of SYN_L^{TR}

Clause (3) means that any modifier can be labeled by a thing or relation label to form, respectively, a thing or relation expression.

2.3.2 Semantic Axioms for SYN_L^{TR} Defining INT_L^{TR}

INT_L^{TR} is a set of semantic theories, each semantic theory being a triple $\langle F,V,R\rangle$ which satisfies, among others, the following conditions:

(a) F is a set of pairs $\langle D,f\rangle$, called interpretations, where D is a non-empty set, and f is a function whose domain is SYN_L^{TR} and whose range is included in D.

(b) V is a function, called a valuation function, whose domain is the set $S^1 \times F$ (where S^1 is the set of sentences[75.2] of SYN_L^{TR}), and whose range is included in {truth, falsehood, nil.}

(c) R is a set of binary relations on F.[75.3]

We now specify further constraints on INT_L^{TR} by imposing conditions called logical semantic axioms, (abbreviated as L.S.A.) on the structure of all $\langle D,f\rangle \in F$. We will describe a number of these conditions throughout the remainder of this chapter. We first impose general conditions (1)-(9); later we impose more specific conditions governing special representational morphemes and special constructions.

2.3.2.1 Basic Logical Semantic Axioms for SYN_L^{TR}.

For all $\langle D,f\rangle \in F$, $\langle D,f\rangle$ satisfies the following conditions:

L.S.A.1 $f(T_m^n) = PD - \{\phi\}$, where each of m,n is a positive integer or the blank expression

L.S.A.2 $f(R^m) = D^m$, where m is a positive integer or the blank expression[75.4]

L.S.A.3 Letting $r = PPD \cup \{x \subseteq D^m:$ for some positive integer $m\}$,
$f(*) = r^x : x = r^n$, for some positive integer n [75.5]
$= {}_r\{r^n : n \text{ is a positive integer}\}$

L.S.A.4 If a is a thing-expression of SYN_L^{TR}, then $f(a) \subseteq f(T)$

L.S.A.5 If a is an m-place relation-expression of SYN_L^{TR}, then $f(a) \subseteq f(R^m)$

Note 75.2. See remarks on page 163, especially Note 67.1.

Note 75.3. See remarks on page 241, especially Note 84.

Note 75.4. If n is a positive integer, and if y is a set then y^n is the n-Cartesian power of y.

Note 75.5. If x and y are sets, then y^x is the set of all functions whose domain is x and whose range is included in y; thus f(*) is the set of all functions whose domain is r^n, for some positive integer n, and whose range is included in r.

L.S.A.6 If a is a modifier, then $f(a) \in f(*)$

L.S.A.7 Let each of i,j be either the blank expression or some positive integer, and let m,n,k be positive integers. Let $x = T_i^j$ or $x = R^j$, or $x = *$, and let $b, b_1, \ldots, b_n$ be expressions. Then:

(i) $f\binom{b}{x} = f(b)[f(x)]$, if b is a modifier; if $b \in LEX(L)$, then $f(b)[f(T)] = P(\bigcup [f(b)[f(T)]]) - \{\phi\}$;

(ii) $f(\underbrace{b_1 \quad {}^{b_2}_{x_2}}_{x}) = f(\underbrace{{}^{b_2}_{x_2} \quad b_1}_{x}) \subseteq f({}^{b_2}_{x_2})$, if b_1 is a modifier;

(iii) $f(\underbrace{{}^{b_1}_{x} \ldots {}^{b_n}_{x}}_{x}) = f({}^{b_1}_{x}) \cap \ldots \cap f({}^{b_n}_{x})$;

(iv) $f(\underbrace{b_1 \ldots b_n}_{x}) = f(b_k)[\langle f(b_1), \ldots, f(b_{k-1}),$

$f(b_{k+1}), \ldots, f(b_n)\rangle]$,

if $n > 1$, b_k is a modifier

and, for all $1 \leq j \neq k \leq n$, $b_j = {}^{a}_{x}$,

for some modifier a.

L.S.A.7 defines the conditions under which the denotations of thing- and relation-expressions can be evaluated as the application of the denotation of a modifier to the denotation of a thing or relation label ((i) above), or to the denotations of one or more thing- or relation-expressions. The specific denotation values obtained from such applications are identified in the remaining semantic axioms.

We have earlier stated the basic semantic paradigm for interpreting sentences. The following axiom generalizes that paradigm.

We next state L.S.A. 8, which interprets sentences. We formulate L.S.A. at three levels of generality: L.S.A. 8A deals with the case where there are no referential links issuing from the sentence being interpreted, L.S.A. 8B generalizes 8A to the case where there may also be non-distributed referential links issuing from the sentence being interpreted, and L.S.A. 8C generalizes 8B to the case where there may also be distributed (as well as non-distributed) referential links issuing from the sentence being interpreted. If there are no referential links issuing L.S.A. 8A. from $r^m(a_1,\ldots,a_m)p,q$, then $f[r^m(a_1,\ldots,a_m)p,q] = \{\{(f[r^m], P_{(D,f)}[r^m(a_1,\ldots,a_m)p,q])\}\}$, if $P_{(D,f)}[r^m(a_1,\ldots,a_m)p,q]$ is consistent with the determiner structure of $r^m(a_1,\ldots,a_m)p,q$ under (D,f), that is, if there are non-empty subsets $B_1 \subseteq f(a_{p^{-1}(1)}),\ldots,B_m \subseteq f(a_{p^{-1}(m)})$ and there is a chain function g on $(B_1,\ldots,B_m)$ such that the trace of g through $(B_1,\ldots,B_m)$ with respect to q is identical with $P_{(D,f)}[r^m(a_1,\ldots,a_m)p,q]$; otherwise $f[r^m(a_1,\ldots,a_m)p,q] = \phi$.

Before stating L.S.A. 8B, which covers the case where there may be non-distributed referential links issuing from $r^m(a_1,\ldots,a_m p,q$, we need to first introduce the following auxiliary definition:

Let S be a set of sentences, and let (D,f) be an interpretation. Then $^{[S]}f$ is that interpretation function which is like f on all expressions of L' except that, for every referring expression b occurring in any sentence of S (i.e., for every expression of S from which a referential link issues) whose referrent expression is an expression b^o (i.e., the expression at which that referential link terminates), ${}^{[S]}f[b] = f[b^o]$. (We note that if there are no referring expressions in S, then ${}^{[S]}f = f$.)

L.S.A. 8.B. If there are no distributed referential expressions issuing from $r^m(a_1,\ldots,a_m)p,q$ and if S is a set of sentences containing $r^m(a_1,\ldots,a_m)p,q$ as well as containing all referrent expressions at which any non-distributed referential link issuing from any expression of $r^m(a_1,\ldots,a_m)p,q$ terminates. Then $f[r^m(a_1,\ldots,a_m)p,q] = {}^{[S]}f[r^m(a_1,\ldots,a_m)p,q] = \{\langle f[r^m], P_{(D,{}^{[S]}f)}[r^m(a_1,\ldots,a_m)p,q]\rangle\}$, if $P_{(D,{}^{[S]}f)}[r^m(a_1,\ldots,a_m)p,q]$ is consistent with the determiner structure of $r^m(a_1,\ldots,a_m p,q$ under $(D,{}^{[S]}f)$, (that is, if there are non-empty subsets $B_1 \subseteq {}^{[S]}f(a_{p-1(1)})$, ..., $B_m \subseteq {}^{[S]}f(a_{p-1(m)})$,) and there is a chain function g on $(B_1,\ldots,B_m)$ such that the trace of g through $(B_1,\ldots,B_m)$ with respect to q is identical with $P_{[D,{}^{[S]}f)}[r^m(a_1\ldots,a_m)p,q]$; otherwise, ${}^{[S]}f[r^m(a_1,\ldots,a_m)p,q] = \phi$.

Before stating L.S.A. 8C which covers the case where there may be both non-distributed and distributed referential links issuing from $r^m(a_1,\ldots,a_m)p,q$, we need to introduce a series of definitions.

First of all, we note the following: the extension to cover distributed referential links requires that we consider not simply single sentences or a set of sentences, but some ordering among the sentences. We choose a sequential ordering rather than, say, a partial ordering, since it simplifies some of the formulations. Accordingly, we let S be a sequence of sentences

Note. The reader might wish to skip the discussion of L.S.A. 8C and return to it at a later point.

whose component sentences contain all subexpressions to which subexpressions of a given sentence are referentially linked. Item (iii) in the following definition makes essential reference to the ordering of S.

Definition.

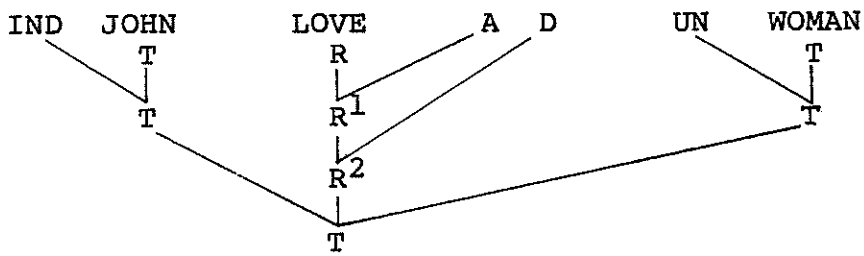
(i) Let A be an expression occurrence of some expression of SYN_L^{TR}. The occurrence precedence relation for A is that relation R on the subexpression occurrences within A such that, for all subexpression occurrences a,b within A, a R b holds if and only if every symbol occurrence within b and including b is to the right of or below some symbol occurrence within a and including a, unless there is a sentence B occurring within A or identical to A such that there are major thing-expressions a', b' of B occurring within B such that b' has lower scope index than a' and a occurs as a (not necessarily proper) subexpression of a' and b occurs as a not necessarily proper subexpression of b', in which case a R b fails to hold and b R a holds instead.

It can be verified that the occurrence precedence relation R for A orders the subexpression occurrences within A, in the sense that for all subexpression occurrences a, b, c within A: (i) a R b or b R a, (ii) not a R a; (iii) if a R b, then not b R a; and (iv) if a R b and b R c then a R c.

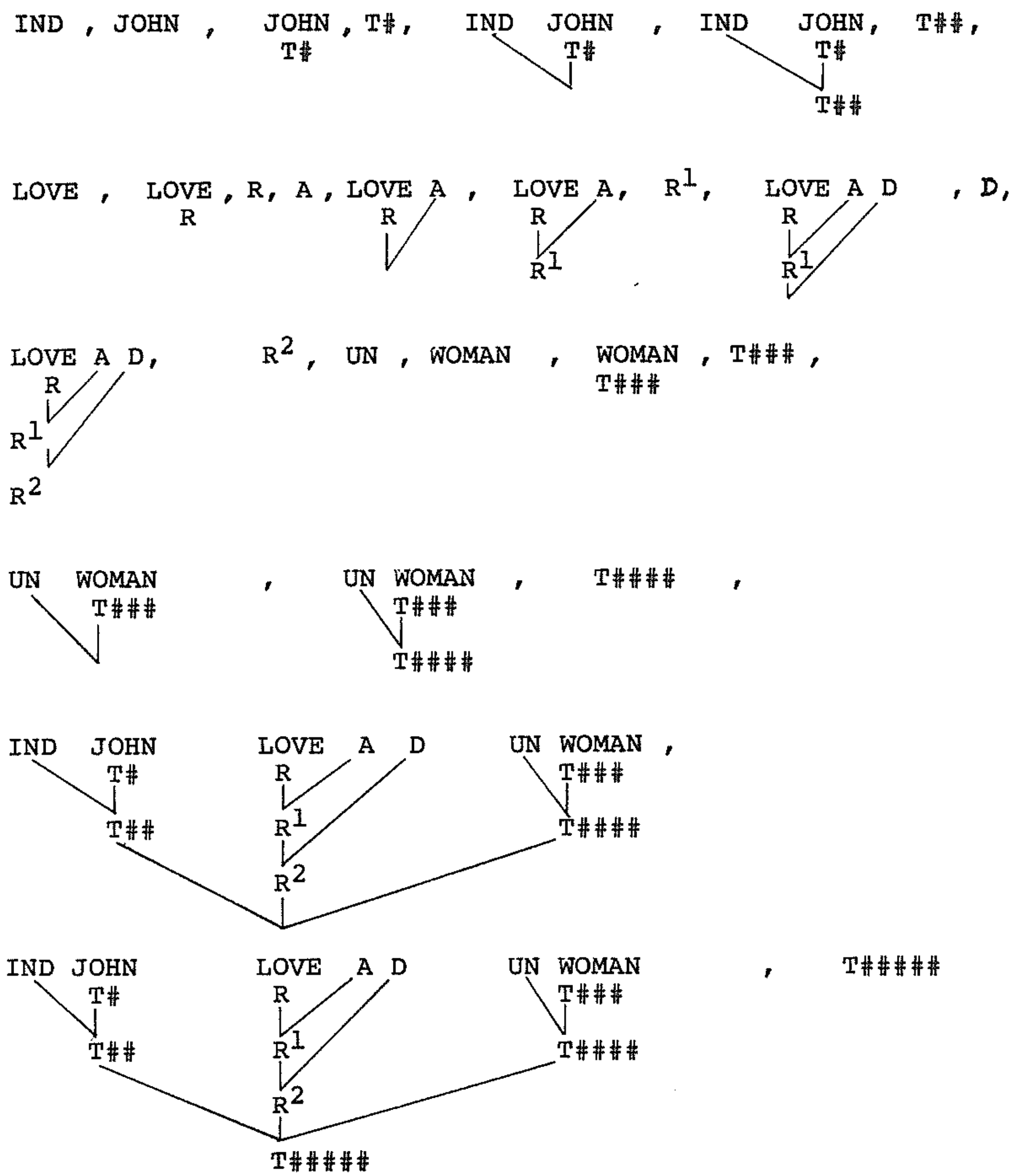
(ii) The occurrence number of a subexpression occurrence a within a containing expression occurrence A is the place (a non-negative integer) that a occupies relative to the occurrence precedence relation on A. That is, if a occupies the j^{th}-

place relative to the occurrence precedence relation on A, then j is the occurrence number of a in A.

For example, if A is the expression:



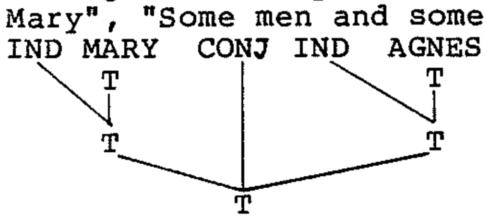
and if we let T#, T##, T###, T#### designate the 1st, 2nd, 3rd, and 4th occurrences of T in A relative to the occurrence precedence relation for A, and designate each subexpression B in A which has only one occurrence in A simply by B, then the occurrence number of any subexpression B of A is the place in the following ordering of the sub-expressions of A in which B occurs (for example, the occurrence number of IND in A is 1, the occurrence number of JOHN in A is 2, the occurrence number of T# in A is 3, and so on):


IND , JOHN , JOHN , T#, IND JOHN , IND JOHN, T##,
T#
T##
LOVE , LOVE , R, A , LOVE A , LOVE A, R1, LOVE A D , D,
R
R1
LOVE A D, R2 , UN , WOMAN , WOMAN , T### ,
T###
R2
UN WOMAN , UN WOMAN , T#### ,
T####
IND JOHN LOVE A D UN WOMAN ,
IND JOHN LOVE A D UN WOMAN , T#####
T#####

(iii) Let S be a sequence of SYN_L^{TR} sentences $S_1 \ldots S_t$, such that S is closed under referential links in the sense that all referential links issuing from any subexpression of a sentence among $S_1,\ldots,S_t$ link up to a referenced expression that is a subexpression of one of $S_1,\ldots,S_t$, and all referential links linking up to a referenced expression that is a subexpression of one of $S_1,\ldots,S_t$ issues from a subexpression of $S_1,\ldots,S_t$. Then we let g_S (h_S) be the co-referentially distributed (co-referentially nondistributed) linking function on S, that is, the function that links each referencing expression x occurring in S with its referenced expression g(x) (h(x)) in S.[75.1]

The following semantic axiom governs the interpretation of a sentence S_i occurring in a sequence of sentences $S_1 \ldots S_t$; as a

Note 75.1. Thus for every referencing expression there is exactly one referenced expression, so that the referencing relation is a function. We can, nonetheless, fully adequately treat English sentences like "John loves Mary and Agnes, and they love him too", "John will pursue Agnes only if Mary rejects him, but he scarcely knows them", "Some men and some women love their mothers and their mothers love them too," etc. We handle such cases by using syntactic representations of the involved word-strings which represent phrases such as "Mary and Agnes", "Agnes Mary", "Some men and some women" as explicit compounds, e.g.,



Such representations allow us to treat referential relationships as functions, that is, to assign exactly one referenced expression to each referring expression, wherein the compound syntactic representation of two or more word-string parts becomes that single referenced expression. We could have proceeded differently, but at the cost of increased complexity.

special case, of course, where i=t=1, this axiom governs the interpretation of a single sentence that is not part of such a sequence.

Let S be a sequence of sentences $S_1 \ldots S_t$, where $S_1 = r_1(a_{11} \ldots a_{1n_1}), \ldots, S_t = r_t(a_{t1} \ldots a_{tn_t})$ and which is closed under referential links. For each $1 \leq i \leq t$, let p_{S_i}, q_{S_i} be permutations on the set $\{1, \ldots, n_i\}$ of argument places of S_i, which are such that, for each $1 \leq j \leq n_i$, $p_{S_i}(j)$ is the scope index and $q_{S_i}(j)$ is the place index of the thing-expression a_{ij} in S_i; and let $p_{S_i}^{-1}$, $q_{S_i}^{-1}$ be the inverses of p_{S_i}, q_{S_i}, respectively. It is convenient to introduce two abbreviations (a), (b) below:

(a) Let s be the number of occurrences of not necessarily distinct referencing expressions in S, let r_1 be the number of referenced thing-expressions in S that are linked by solid lines, let r_2 be the number of referenced thing expressions in S that are linked by dotted lines, and let $\alpha_1, \ldots, \alpha_s$ be the not necessarily distinct referencing expression in S. Let $\gamma_1, \ldots, \gamma_s$ be any s distinct expressions of SYN_L^{TR} not occurring in S.[76] Let $w_1, \ldots, w_{r1}$ be subsets of D, and let $u_1, \ldots, u_{r_2}$ be

Let s_o be a positive integer such that $1 \leq s_o \leq s$.

Note 76. The precise identity of $\gamma_1, \ldots, \gamma_s$ is irrelevant; for the sake of definiteness, we could have ordered the expressions of SYN_L^{TR} and let $\gamma_1, \ldots, \gamma_s$ function as distinct surrogates for the not necessarily distinct referencing expressions $\alpha_1, \ldots, \alpha_s$ in S. Thus, two occurrences, α_i and α_j, say, of a given expression may reference different expressions in S and, by the expedient of using the distinct expressions γ_i and γ_j in their place we are able to take semantic account of their possibly distinct referenced expressions.

elements of D. Let β_S be a function that assigns to every expression occurrence in S its occurrence number in S. Let f be an interpretation function. Then we define

$$f_S\langle w_v ; u_v\rangle_{v<s_0}$$

to be that interpretation function such that $f_S\langle w_v ; u_v\rangle_{v<s} = f$ on all expressions other than $\gamma_1, \ldots, \gamma_s$ and, for all $1 \leq i \leq s$, we have:

$$f_S\langle w_v ; u_v\rangle_{v<s_0}(\gamma_i) \begin{cases} = \{ w_{\beta_S(y)} \}, & \text{if } \alpha_i \text{ is a thing-expression and } y = h(\alpha_i) \text{ and } \beta_S(y) < s_0 ; \\ = \{\{ u_{\beta_S(y)} \}\}, & \text{if } \alpha_i \text{ is a thing-expression and } y = g(\alpha_i) \text{ and } \beta_S(y) < s_0 ; \\ = f_S\langle w ; u_v\rangle_{v<s_0}(y), & \text{if } \alpha_i \text{ is a thing-expression and } y = h(\alpha_i) \text{ or } y = g(\alpha_i) \text{ and } \beta_S(y) \geq s_0 ; \\ = f_S\langle w_v ; u_v\rangle_{v<s_0}(h(\alpha_i)) & \text{if } \alpha_i \text{ is a relation-expression or a modifier expression.} \end{cases}$$

(b). Let β_S^o be that subfunction of β_S obtained by restricting β_S to the occurrences of major thing-expressions of S and referenced thing-expressions of S. Thus $\beta_S^o(e) = i$ if and only if e is the i^{th} expression relative to the occurrence order β_S that is either a major thing-expression of some sentence $S_1, \ldots, S_t$ of S or else is a referenced thing-expression of S.

Let $a_{i p_{S_i}^{-1}(j)k}$ be the k^{th} subexpression occurrence within $a_{i\, p_{S_i}^{-1}(j)}$ relative to the ordering β°_{S}, where $1 \leq i \leq t$ and $1 \leq k \leq m_{ij}$, where m_{ij} is the number of thing-expressions in $a_{i\, p_{S_i}^{-1}(j)}$. Let μ_{ijk} be the result of replacing $\alpha_1, \ldots, \alpha_{\beta^{\circ}_{S}(a_{i p_{S_i}^{-1}(j)k})}$ in $\alpha_{i p_{S_i}^{-1}(j)k}$ by $\gamma_1, \ldots, \gamma_{\beta^{\circ}_{S}(a_{i p_{S_i}^{-1}(j)k})}$ respectively.

We define:

J (i,j) [W] is the expression (A):

(A): there is an $x_{\beta^{\circ}_{S}(a_{i\, p_{S_i}^{-1}(j)1})} \in$

$f^{\langle x_v ; y_v \rangle_{v < \beta^{\circ}_{S}(a_{i\, p_{S_i}^{-1}(j)1})}}(\mu_{ij1})$ such that

for all $y_{\beta^{\circ}_{S}(a_{i p_{S_i}^{-1}(j)1})} \in x_{\beta^{\circ}_{S}(a_{i p_{S_i}^{-1}(j)1})}$

there is an $x_{\beta^{\circ}_{S}(a_{i\, p_{S_i}^{-1}(j)2})} \in$

$f^{\langle x_v ; y_v \rangle_{v < \beta^{\circ}_{S}(a_{i\, p_{S_i}^{-1}(j)2})}}(\mu_{ij2})$ such that

for all $y_{\beta^{\circ}_{S}(a_{i p_{S_i}^{-1}(j)2})} \in x_{\beta^{\circ}_{S}(a_{i p_{S_i}^{-1}(j)2})}$

there is ... ... there is

$x_{\beta^{\circ}_{S}(a_{i p_{S_i}^{-1}(j)m_{ij}})} \in B_i \subseteq f^{\langle x_v ; y_v \rangle_{v < \beta^{\circ}_{S}(a_{i p_{S_i}^{-1}(j)m_{ij}})}}(\mu_{ijm_{ij}})$

such that W holds.

We also define:

Let $J_S[W]$ be the expression: $J(1,1)\ [J(1,2)[\ldots$
$J(1,n_1)\ [J(2,1)\ [J(2,2)\ [\ \ldots\ J\ (2,n_2)\ [\ldots,\ldots$
$J(t,1)\ [J(t,2)\ [\ldots\ J(t,n_t)\ [W]\ ]\ ]\ \ldots,\ldots]\ \ldots]]]\ \ldots]]$

We can now state L.S.A. 8C, as follows:

L.S.A. 8C. Let $r_i^m(a_{i1},\ldots,a_{im_i})p,q$ be the i^{th} sentence in order of occurrence in a sequence $S = S_1,\ldots,S_t$ of sentences such that S contains all occurrences of referent expressions at which any referential links (non-distributed or distributed) issuing from any expression of $r^m(a_1,\ldots,a_m)p,q$ terminates. Then:

$f\ [r_i^m(a_{i1},\ldots,a_{im_i})p,q] = \{\{(f[r_i^m], \delta)\}\}$, if J_S [for some chain function g on $(B_{i1},\ldots,B_{im_i})$, and δ is the trace of g through $(B_{i1},\ldots,B_{im_i})$ with respect to q and $\delta = f\ [r_i m_i] \cap$
$\Big[\bigcup f\langle x_\nu;\ y_\nu\rangle_{\nu<\beta_S}\ [r_i^m(a_{i1},\ldots,a_{im_i})p,q)\ (a_{\mu_{i1}m_{i1}})]$
$\times\ldots\times \bigcup f\langle x_\nu;\ y_\nu\rangle_{\nu<\beta_S}\ [r_i^m(a_{i1},\ldots,a_{im_i})p,q)\ (a_{\mu_{im_i}m_{im_i}})]\Big]$,
and

$f[r_i^m(a_{i1},\ldots,a_{im_i})p,q] = \phi$, otherwise.

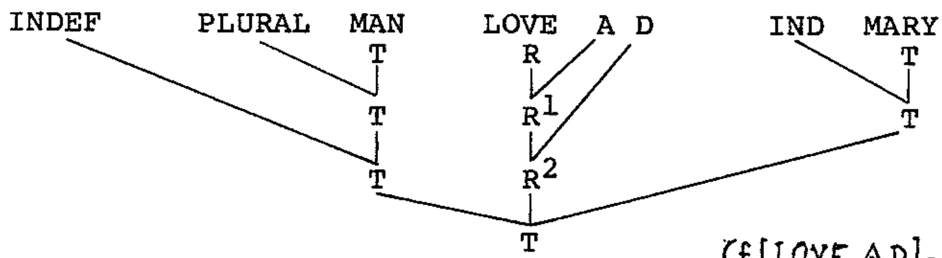
Some examples follow which illustrate the semantic treatment of some common types of non-distributed and distributed referencing. Examples (1) - (6) involve referentiality within single sentences; examples (7) and (8) involve referentiality across multiple sentences.

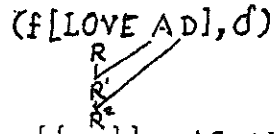
Example 1.

Let $S = \langle S_1' \rangle$, where S_1' (exhibited below) is the syntactic representation of the dominant normal reading of S_1:

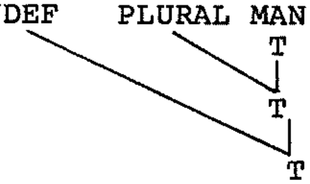
(S_1) Some men love Mary

(S_1')

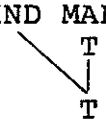




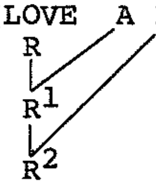
Let (D,f) be an interpretation. Then $f[S_1'] = \{\{\Lambda\}\}$, if there are non-empty subsets $B_1 \subseteq$ f(INDEF PLURAL MAN) and



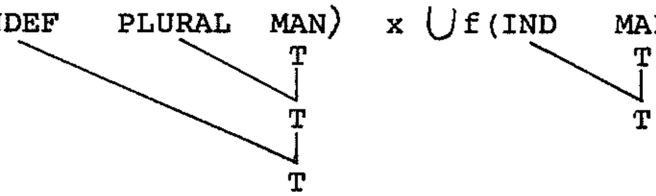
and $B_2 \subseteq$ f(IND MARY)



and there is a chain function g on (B_1, B_2) such that δ is the trace of g through (B_1, B_2) and $\delta =$ f (LOVE A D) $\cap$



[$\bigcup$f(INDEF PLURAL MAN) x $\bigcup$f(IND MARY)];



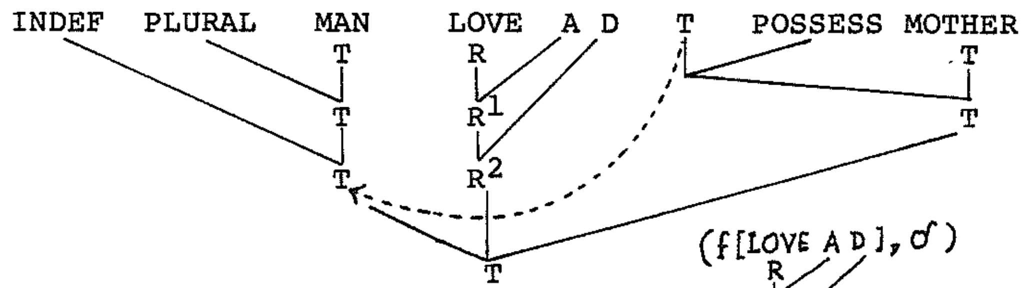
and $f[S_1'] = \phi$, otherwise.

Example 2.

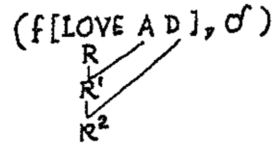
Let $S = \langle S_1' \rangle$, where S_1' (exhibited below) is the syntactic representation of the dominant normal reading of S_1:

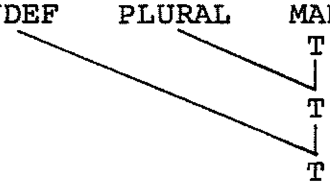
(S_1) Some men love their mother

(S_1')

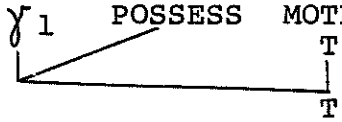


Let (D,f) be an interpretation, and let γ_1 be an expression of SYN_L^{TR} not occurring in S_1'. Then $f(S_2) = \{\{(f[\text{LOVE A D}], \delta)\}\}$, if there is a set $B_1 \subseteq f$(INDEF PLURAL MAN) and an $x_1 \in B_1$ such that, for

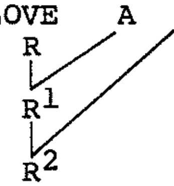




all $y_1 \in x_1$, there is a set $B_2 \subseteq f^{\langle x_1; y_1 \rangle}$ (γ_1 POSSESS MOTHER)



and there is some chain function g on (B_1, B_2) such that δ is the trace of g through (B_1, B_2) and $\delta = f^{\langle x_1; y_1 \rangle}$ (LOVE A D) $\cap$



$[\bigcup f^{\langle x_1 ; y_1 \rangle}$ [INDEF PLURAL MAN] x

T
T
T

$\bigcup f^{\langle x_1 ; y_1 \rangle}$ [T POSSESS MOTHER]] , and

T
T

f (S_2) = ϕ , otherwise.

We note that the occurrence T# of "T" adjacent to "POSSESS" in (S_1') is a thing-expression which is distributively referentially linked to g (T#) = INDEF PLURAL MAN

T
T
T

so that, by the definition of f_S $^{<w_V ; u_V>_{V<s_o}}$ (γ_i) on page , we have that "γ_1" in γ_1 POSSESS MOTHER receives under the

T
T

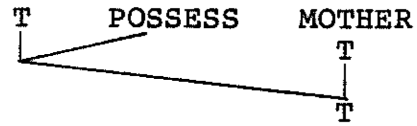
interpretation $f^{\langle x_1 ; y_1 \rangle}$ the denotation $\{\{y_1\}\}$; (where y_1 is an arbitrary but specific element of x_1 so that $f^{\langle x_1 ; y_1 \rangle}$

(γ_1 POSSESS MOTHER)

T
T

= $f \Big| \mathrm{T}/\!\{\{y_1\}\}$ (T POSSESS MOTHER)

T
T

(recalling that $f \Big| \mathrm{T}/\!\{\{y_1\}\}$ is that interpretation which is like f everywhere except that $f \Big| \mathrm{T}/\!\{\{y_1\}\}$ assigns $\{\{y_1\}\}$ to the to the first occurrence of T in 

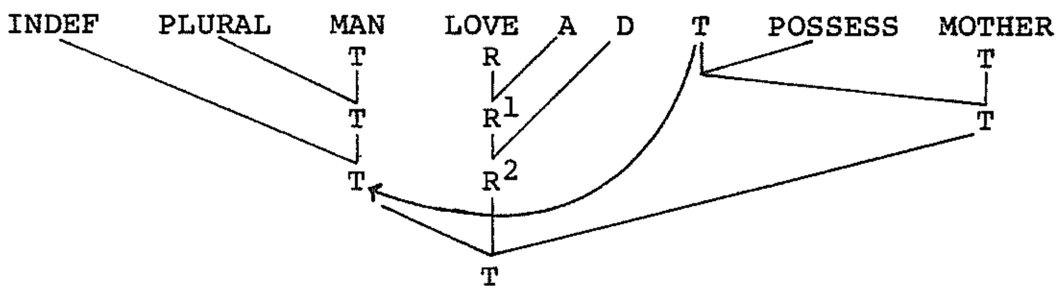
)

Example 3.

Let $S = S_1''$, where S_1'' (exhibited below) is the syntactic representation of another possible normal reading of S_1 of example 1:

(S_1) Some men love their mother

(whose meaning can be expressed by the English sentence, "Each of some men love all of their mothers")

(S_1'') 

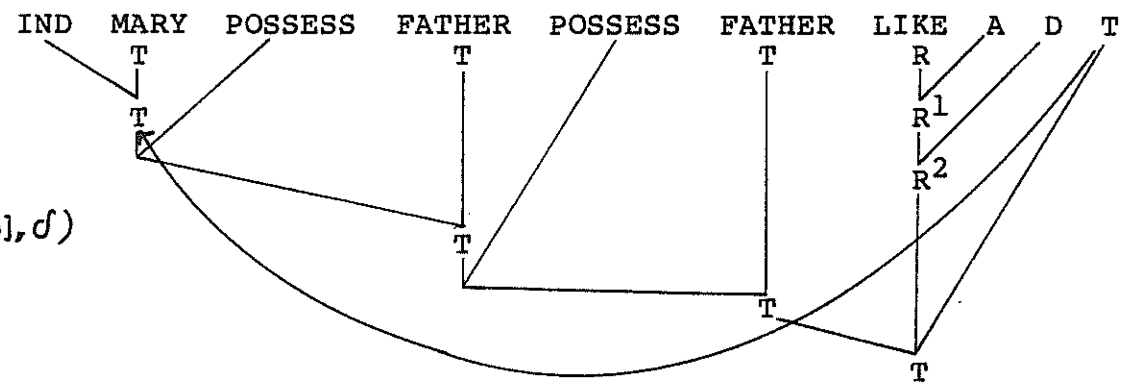
Letting again, γ_1 be an expression of SYN_L^{TR} not occurring in S_1'', then the denotation of S_1 is essentially the same as the foregoing except that, here, $f^{<x_v;\ y_v>}\ v<2\ (\gamma_1) = \{\{y_1\}\}$ whereas, in (3), $f^{<x_v;\ y_v>}\ v<2\ (\gamma_1) = \{x_1\}$

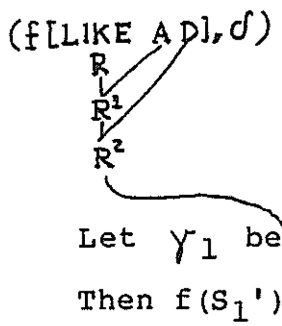
Example 4

Let (D,f) be an interpretation. Let S = S_1', where S_1' (exhibited below) is the syntactic representation of the dominant normal reading of S_1:

(S_1) Mary's father's father likes her

(S_1')

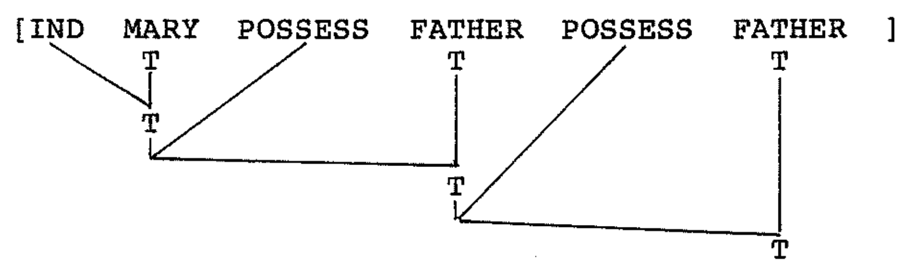




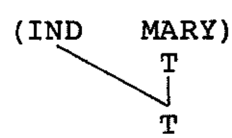
Let γ_1 be the first expression of SYN_L^{TR} not occurring in S_1'.

Then $f(S_1') = \{\{\Lambda\}\}$, if there is a non-empty set

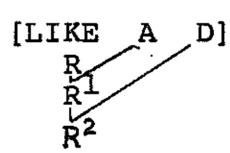
$B_1 \subseteq f$



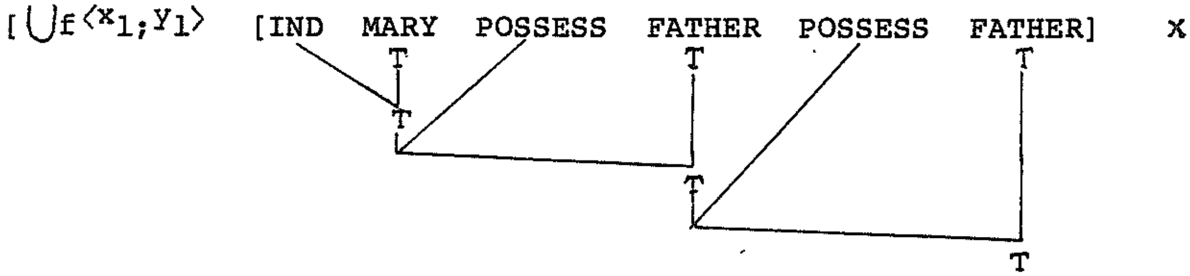
such that here is as $x_1 \in f$



such that for all $y_1 \in x_1$ there is a non-empty set $B_2 \subseteq f^{\langle x_1;y_1\rangle}(\gamma_1) = \{x_1\}$, and there is a chain function g through (B_1, B_2) and $\delta = f^{\langle x_1,y_1\rangle}$



$\cap$



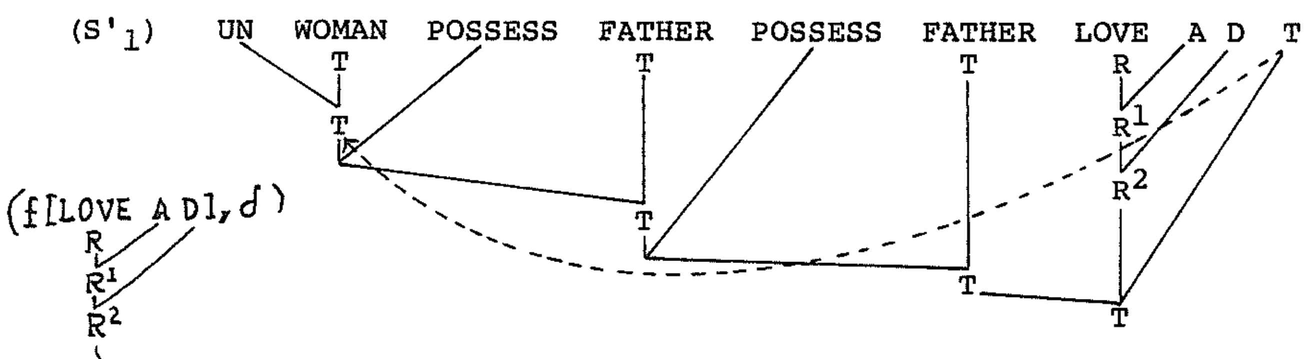
$\bigcup f^{\langle x_1;y_1\rangle}$ [γ_1]],

and $f[S^1{}_1] = \phi$, otherwise.

Example 5

Let (D,f) be an interpretation, and let $S = S_1$, where S'_1 (exhibited below) is the syntactic representation of the dominant normal reading of

(S_1) Every woman's father's father loves her

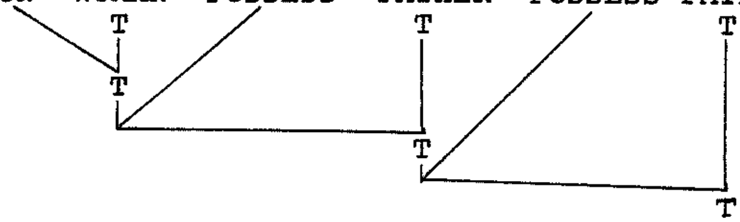


Let γ_1 be the first expression of SYN^{TR}_L not occurring in S'_1. Then

$f(S'_1) = \{\{\Lambda\}\}$, if there is an $x_{11} \in f$ [UN WOMAN] such that

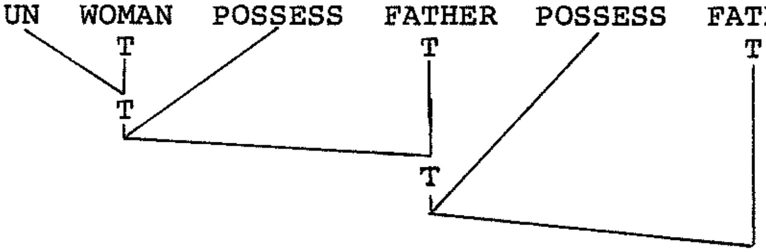
for all $y_{11} \in x_{11}$ there is an x_{12} and a non-empty set

$B_1 \subseteq f$[UN WOMAN POSSESS FATHER POSSESS FATHER]



such that for all $y_{12} \in x_{12}$ there is a set $B_2 \subseteq f^{\langle x_{12};y_{12}\rangle}(\gamma_1)$ and there is some chain function g on (B_1,B_2) such that δ is the trace of g through (B_1,B_2) and $\delta = f^{\langle x_1;y_1\rangle}$ (LOVE A D) $\cap$

[[$\bigcup f^{\langle x_1;y_1\rangle}$ [UN WOMAN POSSESS FATHER POSSESS FATHER]



x $\bigcup f^{\langle x_1;y_1\rangle}$ $[\gamma_1]$] ;

and $f(S'_1) = \phi$, otherwise.

Example 6

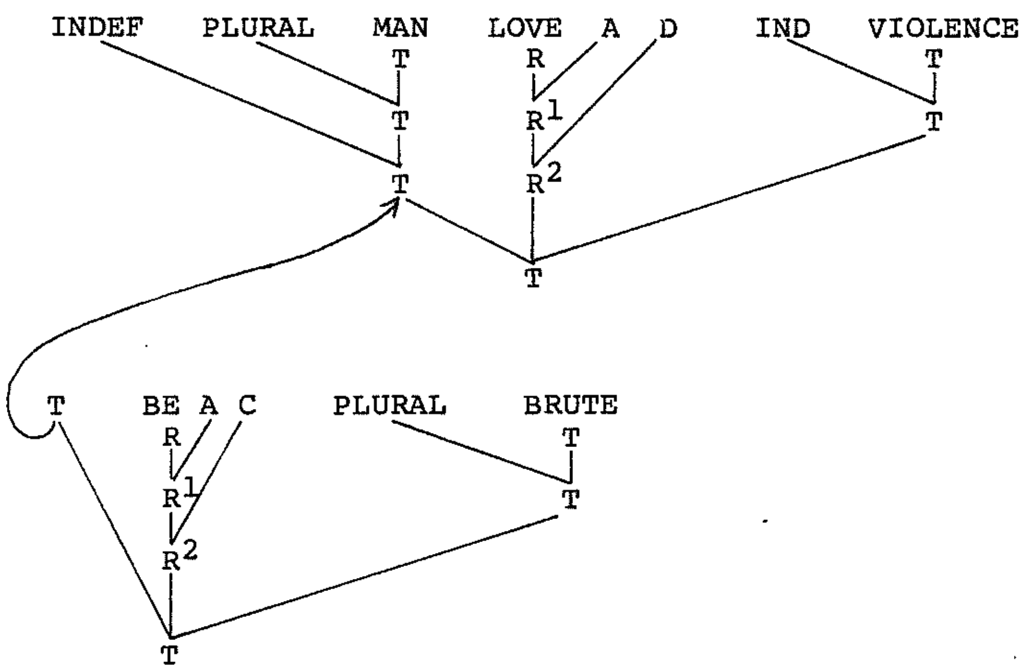
Let (D,f) be an interpretation, and let S = <S'$_1$, S'$_2$>, where S'$_1$,S'$_2$ (exhibited below) are the syntactic representations of the dominant normal readings of S_1,S_2 respectively:

(S_1) Some men love violence.

(S_2) They are brutes.

(S'$_1$)

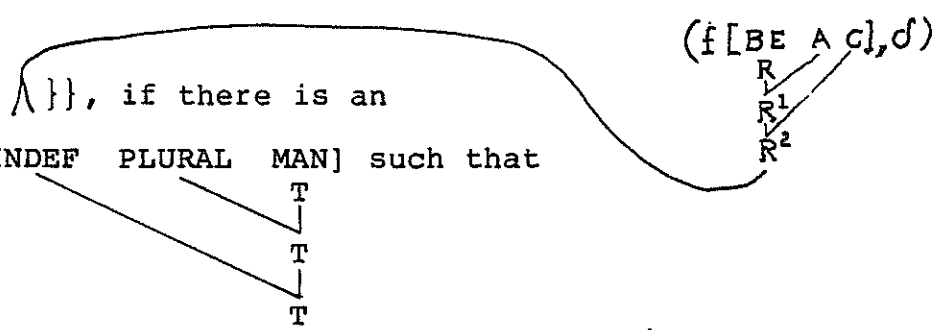
(S'$_2$)



Let γ_1 be an expression of SYN_L^{TR} not occurring in S_1 or S_2. Then

f[S'$_2$] = {{Λ}}, if there is an

$x_1 \in f^{\langle x_1;y_1\rangle}$ [INDEF PLURAL MAN] such that



for all $y_1 \in x_1$ there is an $x_2 \in f$[IND VIOLENCE]

T
|
T

such that for all $y_2 \in x_2$ there is an $x_3 \in$ some $B_1 \subseteq f^{\langle x_v ; y_v \rangle}{}_{v<3}$ (γ_1) such that for all $y_3 \in x_3$ there is an $x_4 \in$ some $B_2 \subseteq f^{\langle x_v ; y_v \rangle}{}_{v<4}$ [PLURAL BRUTE] such that

T
|
T

$\delta = f^{\langle x_v ; y_v \rangle}{}_{v<4}$ [BE A C] $\cap$

R
|
R^1
|
R^2

$[\bigcup f^{\langle x_v ; y_v \rangle}{}_{v<4} [\gamma_1] \times$

$\bigcup f^{\langle x_v ; y_v \rangle}{}_{v<4}$ [PLURAL BRUTE]] ; and

T
|
T

$f[S'_2] = \phi$, otherwise.

Example 7

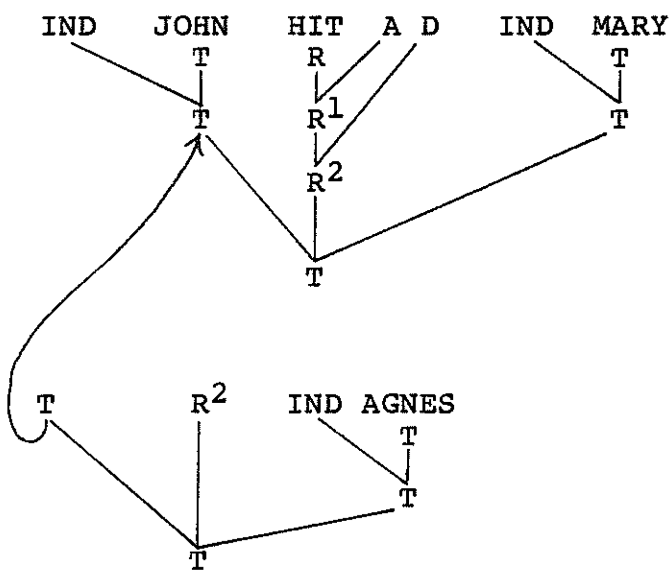
Let (D,f) be an interpretation, and let $S = \langle S'_1, S'_2 \rangle$, where S'_1, S'_2 (exhibited below) are the syntactic representations of the dominant normal readings of S_1, S_2 respectively:

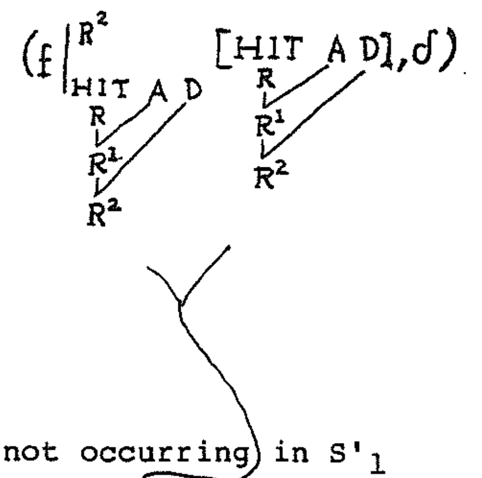
(S_1) John hit Mary.

(S_2) He did that to Agnes.

(S'_1)

(S'_2)





Let γ_1, γ_2 be any two expressions of SYN^{TR}_L not occurring in S'_1 or S'_2. Then the denotation $f[S'_2]$ of S'_2 is = {{ ✓ }}, if there is $x_1 \in$ f(IND JOHN) such that for all $y_1 \in x_1$ there is
T
T

an $x_2 \in$ f(IND MARY)
T
T

such that for all $y_2 \in x_2$ and $(y_1, y_2) \in$ f(HIT A D] there is $x_3 \in B_1 \subseteq$
R
R1
R_2

$f^{\langle x_v; y_v\rangle_{v<2}}(\gamma_1)$ such that for all $y_3 \in x_3$ there is $x_4 \in B_2 \subseteq f^{\langle x_v; y_v\rangle_{v<3}}$ (IND AGNES) such that for all

T

T

$y_4 \in x_4$, $(y_3, y_4) \in f^{\langle x_v; y_v\rangle_{v<3}}(\gamma_2)$, if there are any such non-empty subsets, and = ϕ otherwise.

The above 7 cases are of a fairly common sort, and illustrate some possible referential patterns covered by L.S.A.(8.).[77]

Note 77. L.S.A. 8.C is formulated to cover only those referential patterns which appear intuitively reasonable. In particular, it does not cover the following referential patterns (but could be extended to do so, as remarked below): (i) a modifier expression or relation-expression that is forward or backward linked by a dotted line; (ii) a thing-expression that is forward referentially linked by a dotted line; and (iii) cases involving "circular referencing". Cases of type (i) are those where the referenced expression of an ineliminable pronoun either is a modifier or relation-expression, and perhaps would occur only in languages where normal readings would require determiners over relations or modifiers. Cases of type (ii) are those where the referenced expression of an ineliminable pronoun is a thing-expression but follows that pronoun. Cases of this type do not appear to occur in normal readings of sentences; where a pronoun, say, precedes its distributed reference in the natural language word-string, its most plausible syntactic representation apparently requires that that pronoun follow its referrent in that representation. If known to fail for some TR languages, then this case could be handled by generalizing on the notion of relative-place order, enabling it thereby to apply to arbitrary expressions within a sequence of sentences, such that the order of any given subexpression of a containing expression relative to another subexpression of that containing expression would be given by a relative-order permutation on the occurrence order of those expressions. A referential link would then be defined as "forward" or "backward" only relative to this permutation.

Certain instances of case (ii) can already be handled by relativizing "forward" and "backward" to the relative place ordering, namely those instances in which the referenced and referencing expressions occurred in different major thing-expressions of the same sentence. While a reasonable generalization of the relative-place order could be introduced, it would introduce an added complexity which may be unnecessary; accordingly, we continue to invoke the simplifying assumption that we can ignore those instances of referencing mentioned in case (ii)

Turning to case (i), all word-strings I have considered that apparently involved such constructions could be assigned suitable normal readings that did not involve them (which is to say, that instances of case (i) are purely "surface" phenomena). If this turned out to be incorrect, that is, if there were indeed cases of ineliminable pronouns whose referenced expressions were relation-expressions of modifiers, we could handle this case by defining the denotations of relation-expressions and of modifiers in such a way that they were sets of subsets of the domain of discourse, hence had a structure analogous to that of the denotations of thing-expressions, hence that could have instances of ineliminable pronouns involving them handled in the same way as for thing-expressions. Again, in the interest of avoiding unnecessary generalizations, we invoke the simplifying assumption that we can ignore those instances of referencing mentioned in case (i). Case (iii) is somewhat anomalous; it is not at all clear that expressions incorporating such constructions are really understandable as involving circular referencing. As an example, consider the following apparent case of circular referencing:

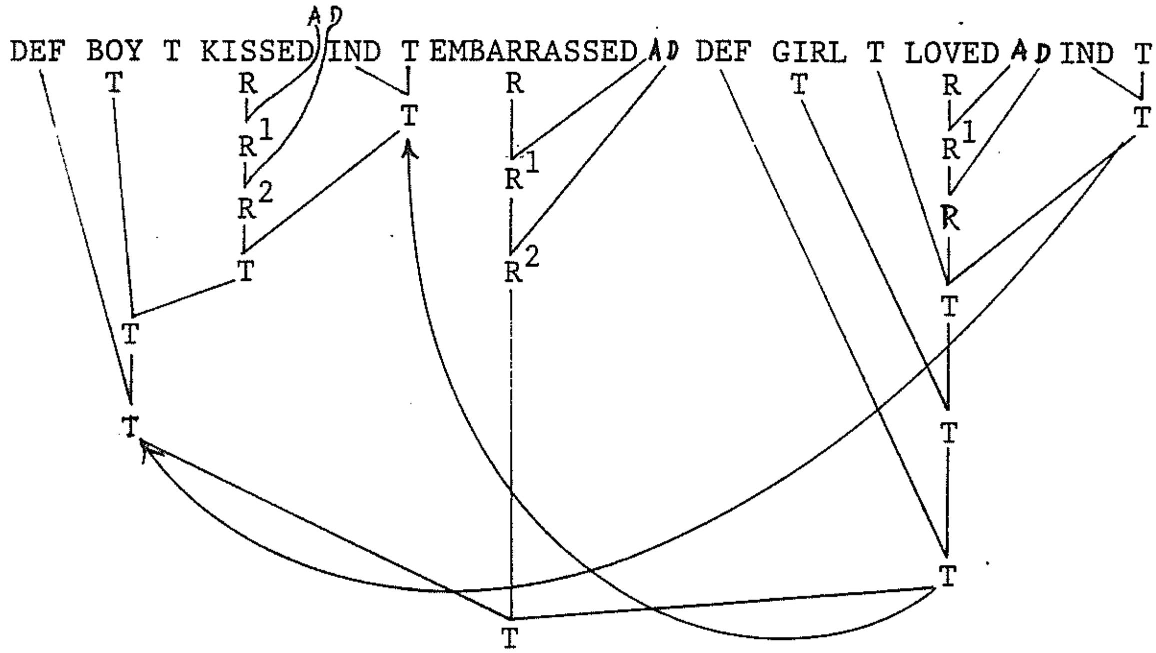
(1) The boy who kissed her embarrassed the girl who loved him.

The apparent circularity is that which links the pronoun "her" to the phrase "the girl who loved him", and links the pronoun "him" to the phrase "the boy who kissed her", thereby rendering the denotations of these two phrases dependent on each other. That is, the circularity derives from the apparent dependency of the denotation of "the boy who kissed her" on the denotation of "her", which in turn is dependent on the denotation of "the girl who loved him", and this last is dependent on the denotation of "him", which is dependent on the denotation of the original phrase "the boy who kissed her" In my opinion there can be no normal reading that strictly parallels such a referential circularity. Of course, (1) <u>does</u> have normal readings that do not involve referential circularity. Perhaps the most normal reading of (1) is that in which "the girl who loved him" is understood as denoting someone other than the denotation of "her", as where the embarrassment of the situation for the girl was prompted by the fact that the boy she loved kissed someone <u>else</u>. However, normal readings are also possible where the girl <u>that</u> is kissed is indeed the girl who loved him, but without involving the circular reference remarked on above. Such a reading of (1), shown below, is such that no part of the phrase "the boy who kissed her" has its denotation dependent on the denotation of any other word-string in (1), the precise referential pattern being exhibited below.

Note 77, continued

(1) The boy who kissed her embarrassed the girl who loved him.

(1')



If understanding (1) in the way corresponding to this reading, one would first address the phrase "the boy who kissed her", leaving the precise denotation of "her" open, until one reaches "him", then understand the denotation of "him" to be that of the first phrase "the boy who kissed her", and then derivatively understand the denotation of the entire second phrase to be that of "her". Under this reading, (1) is equivalent to (2) below, under its dominant normal reading:

(2) The boy who kissed the girl who loved him embarrassed her.

——— end of note 77.

If S* is a sentence within a discourse S^o, the referential closure of S* in S^o may be the entire discourse S^o considered as a single sentence, or may extend beyond the discourse S^o to comprise a sequence $S_1,\ldots,S_q$ of sentences. Thus either $S_1 = S_q = S^o$ or S^o is one of $S_1,\ldots,S_q$. The definition (8.1) applies in either case.[77.1]

Note 77.1. We could, alternatively, develop (8.1) for a single sentence e without reference to a sequence $S_1\ldots S_q$ of sentences which is referentially closed and contains e as one of its terms. This alternative would involve: (a) treating eliminable pronouns wholly as surface phenomena, i.e., as co-referentiality signals at the word-string level which were then syntactically represented at the representational level, (i.e., within SYN^{TR}_{L}) precisely by those expressions which syntactically represent their referent noun phrases; (b) treating two sentences, one of which contains an ineliminable pronoun and the other of which contains its referent noun phrase, as distinct only at the word-string level but not at the representational level, where they would be represented together as one sentence formed by conjoining the formerly distinct sentences. This expedient is tantamount to regarding the presence of ineliminable pronouns as surface signals that there are not two sentences, but one.

We do not employ this alternative because, even though it would result in a simpler formulation of (8.1), it would not yield sufficiently homologous readings; in particular, it would render syntactic representations of word-strings too dissimilar from those word-strings.

Now that we have introduced the denotations of sentences, we can introduce the following basic logical semantic axiom which interprets relative clauses relativized to thing expressions, relation expressions, and modifiers:

L.S.A.(9). Let $A(Q)$ be a sentence-as-modifier of SYN_L^{TR} containing the major expression Q, and let γ be an expression of SYN_L^{TR} not occurring in $A(Q)$. Then

(i) $f[A|^{\gamma}_{T}]\ [f[T]] =$

$\{x \subseteq PD \mid f|^{\gamma}_{x}[A(\gamma)]\ [f[T]] \neq \phi\}$, if γ is a thing-expression

(ii) $f[A|^{\gamma}_{R^m}]\ [f[R^m]] = \{x \subseteq D^m \mid f|^{\gamma}_{x}\ [\underset{T}{A(\gamma)}] \neq \phi\}$, if γ is an m-place relation-expression

(iii) $f[A|^{\gamma}_{*}]\ [f[*]] = \{x \in f[*] \mid f|^{\gamma}_{x}\ [\underset{T}{A(\gamma)}] \neq \phi\}$,

if γ is a modifier-expression

(iv) $f[A(\underset{T}{R^m})]\ [f[R^m]] = f[A(R^m)][f[R^m]]$

$f\left[\begin{smallmatrix} b \\ R^m \\ T \end{smallmatrix}\right] = \{\{f[\begin{smallmatrix} b \\ R^m \end{smallmatrix}]\}\}$

(v) $f[A(\overset{*}{T})]\ [f[*]] = f[A(*)]\ [f[*]]$

$f\left[\begin{smallmatrix} b \\ * \\ T \end{smallmatrix}\right] = \{\{f[\begin{smallmatrix} b \\ * \end{smallmatrix}]\}\}$

(vi) $f[\overset{*}{T}] = \{\{f[*]\}\}$

Discussion. (i) interprets relative clauses where thing expressions are relativized upon; (ii) interprets relative clauses where relation expressions are relativized upon; (iii) interprets relative clauses where modifiers are relativized upon; (iv) - (vi) introduce some alternative notations which simplify formulations. The reader might refer to Section 3.3.5 for examples of constructions requiring L.S.A (9) for their interpretation.

2.3.2.2 <u>Logical Semantic Axioms for Relation-Expressions of SYN_L^{TR}</u>

The axioms L.S.A.(1) -(9) specify the semantic structure of sentences of SYN_L^{TR} without regard for the internal semantic structure of thing and relation-expressions that enter into sentences. In this section we are concerned to formulate axioms (L.S.A.(10)-(13), to follow) that specify the internal semantic structure of relation-expressions of SYN_L^{TR} and, in the following section (2.3.2.3), we formulate axioms (L.S.A.(14) - (17)) that specify the internal semantic structure of thing-expressions.

<u>Permutation of the Domains of a Relation</u>

We first specify the semantic basis for inter-relating simple permutation variants of sentences, such as:

(i) John gave the book to Mary

(ii) John gave Mary the book

(iii) The book was given Mary by John

(iv) The book was given to Mary by John

(v) The book was by John given to Mary

Our semantic axioms should enable us to prove that under the dominant normal readings[78] of each of (i)-(v), that (i)-(v) are semantically equivalent. An essential logical semantic axiom for proving this is the following permutation axiom, which asserts that the domains of a relation can be permuted by permuting the

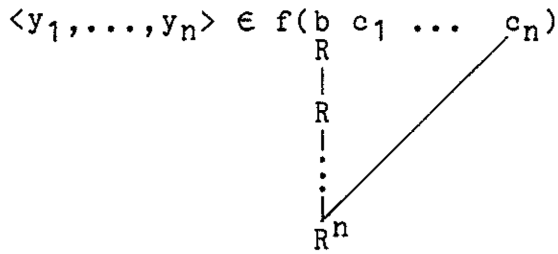
---

Note 78. Sentence (v) might possibly be regarded as a sentence which has no normal reading, which is to say, a sentence which is not grammatical. I prefer to take a permissive stance relative to such issues, and, accordingly, regard (v) as possessing at least one normal reading, albeit a marginal one.

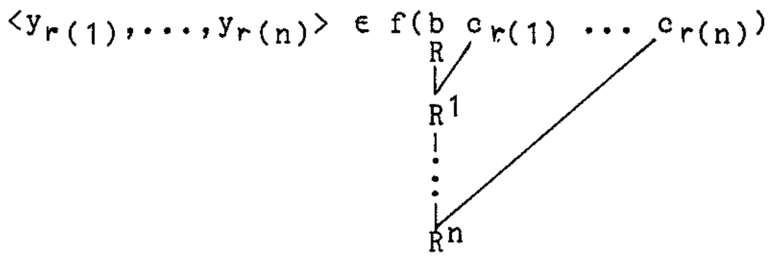
case morphemes on the relation-expression that denotes that relation.[79]

L.S.A.10. <u>Permutation Axiom</u>

Let $c_1, \ldots, c_n$ be modifiers. Let r be a permutation on the set $\{1,\ldots,n\}$. Let b be a modifier. Then for all $y_1, \ldots, y_n \in D$, we have:



if and only if



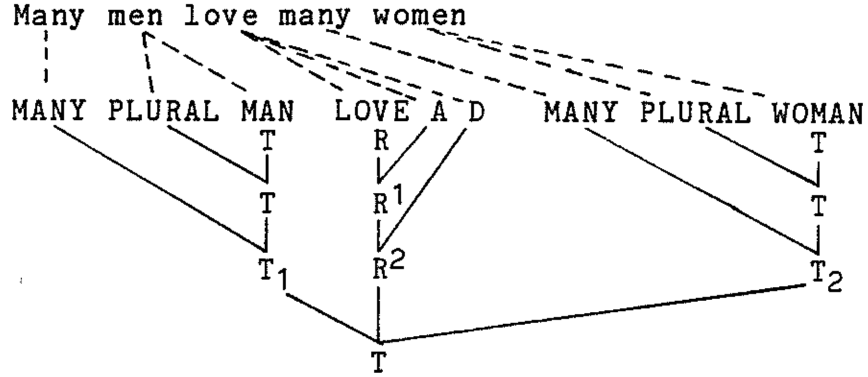
Note 79. This axiom, in conjunction with the axioms on determiners, to follow, permits only the right sorts of permutations, insofar as the condition described in Axiom 10, while necessary, is not by itself sufficient to permit arbitrary permutations; the permutations that are permitted are ultimately determined by the determiners that govern the permuted noun phrases.

Note 79, continued.

In this regard, consider:

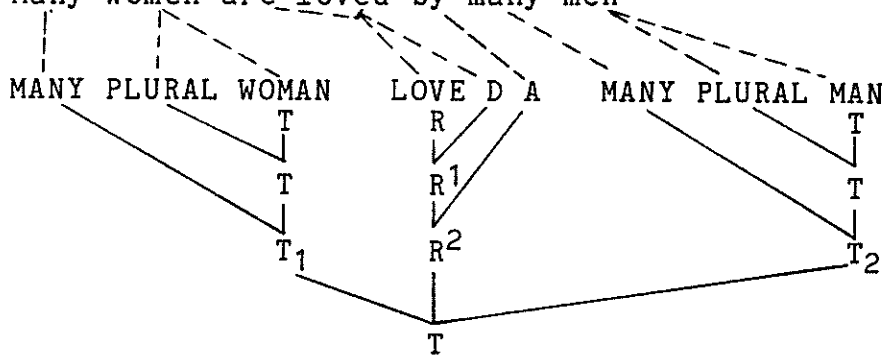
(1) Many men love many women

(1')

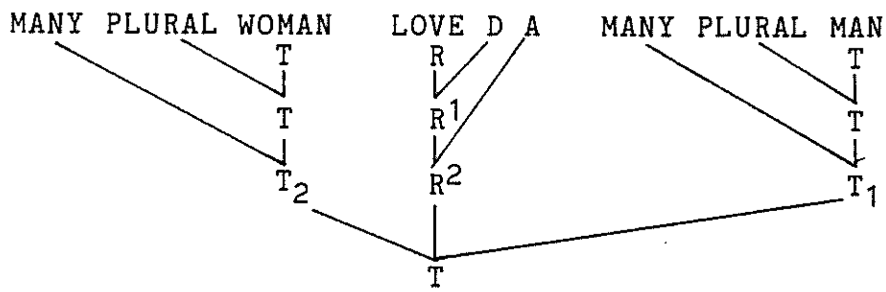


(2) Many women are loved by many men

(2')



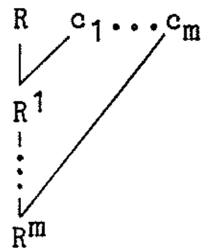
(2'')



Note that: (1) _is not_ equivalent to (2) under the readings (1') of (1) and (2') of (2), but _is_ equivalent to (2) under the readings (1') of (1) and (2'') of (2). This exemplifies a general relationship between the relative place and relative scope orderings of natural language sentences which can easily be proved, namely:

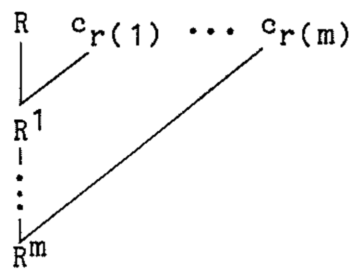
Note 79, continued.

Let

(i) 

$$\begin{pmatrix} {a_1}_{q(1)} & \cdots & {a_m}_{q(m)} \\ T_{p(1)} & & T_{p(m)} \end{pmatrix}$$

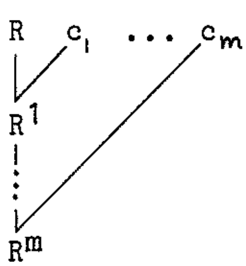
be a sentence of SYN_L^{TR} whose relative scope and relative-place orderings are respectively given by permutations p and q on {1,...,m}, and let r be a permutation on {1,...,m} (r corresponds to a change in relative-place ordering). Then the sentence (i) is equivalent to (ii):

(ii) 

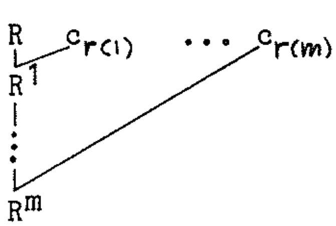
$$\begin{pmatrix} {a_1}_{r(q(1))} & \cdots & {a_m}_{r(q(m))} \\ T_{r(p(1))} & & T_{r(p(m))} \end{pmatrix}$$

under the logical semantic axioms of this chapter.

In the special case of languages like English for which the relative-place ordering is just the occurrence ordering, i.e., where the permutation q is the identity permutation, the equivalence between (i) and (ii) reduces to the equivalence between (i) and (ii') below:

(i') 

$$\begin{pmatrix} a_1 & \cdots & a_m \\ T_{p(1)} & & T_{p(m)} \end{pmatrix}$$

(ii) 

$$\begin{pmatrix} a_{r(1)} & \cdots & a_{r(m)} \\ T_{r(p(1))} & & T_{r(p(m))} \end{pmatrix}$$

Note 79, continued.

The import of this relationship between the relative-place and relative-scope orderings, as it pertains to the special case where an active sentence is passivized, that the apparent occasional loss of equivalence, as exemplified in going from (1) to (2), derives from changing the relative-place ordering without effecting a corresponding change in the relative-scope ordering. That is, if one were to effect a corresponding change in the relative-scope ordering by applying the same permutation that effects the change in the relative-place ordering (which produced, for example, the above passivization) to the indexes on thing-expressions determining their relative-scope ordering, then equivalence would be preserved.

The intuitively perceived inequivalence between (1) and (2) probably derives from the fact that readings of English sentences in which the relative-scope and relative-place orderings are identical to the occurrence ordering (such as (1') and (2') above) have a higher degree of normality than do those readings in which this is not the case (such as in (2'') above), and so constitute the more natural or preferred reading (with respect to usual contexts-of-utterance). Thus the reading (2') of (2) would be the preferred reading of (2), and (2'') the less preferred reading of (2) and, relative to the readings (1') and (2') of (1) and (2) respectively (which, indeed, are the dominant readings of these sentences), (1) and (2) are inequivalent.

Truncation of a Relation

We also need simple ways to obtain each of (ii) through (viii) from (i) below:

(i) John threw a ball to Henry for fun

(ii) John threw a ball to Henry

(iii) John threw a ball

(iv) John threw for fun

(v) John threw

(vi) John threw to Henry for fun

(vii) John threw to Henry

Our semantic axioms will enable us to prove that, under dominant normal readings of each of (i)-(viii), each of (ii)-(viii) follow from (i).[80] This result will follow from the logical semantic axiom L.S.A.(11), which defines the set-theoretic structure of many-place relations by imposing a regularity condition on the way that larger-termed relations are built out of smaller ones, namely by the accretion of successive argument places. We will state L.S.A.(11) immediately after the following auxiliary definition:

If R is an m-place relation, and $1 \leq j \leq m$, then

$$D^j(R) = \{\langle x_1,\ldots,x_j\rangle : \text{for some } x_{j+1},\ldots,x_m,\ \langle x_1,\ldots,x_j,x_{j+1},\ldots,x_m\rangle \in R\}$$

In words, $D^j(R)$ is <u>the restriction of the relation R to its first j domains</u>. We can then state L.S.A.(11) succinctly as follows:

Note 80. There are also normal readings of (i) - (vii) under which (v), say, would not follow from (i).

Truncation Axiom

L.S.A.(11) Let a,b be modifiers, and let n be a non-negative integer. Then

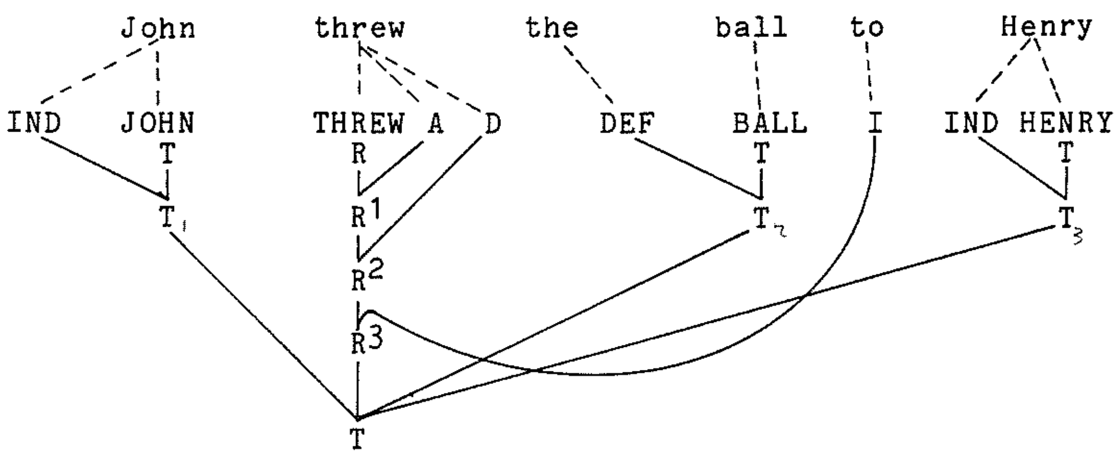
$$D^n(f(a_{R^n}\ b)) \subseteq f(a_{R^n})$$

$$R^{n+1}$$

Relativization of a Relation

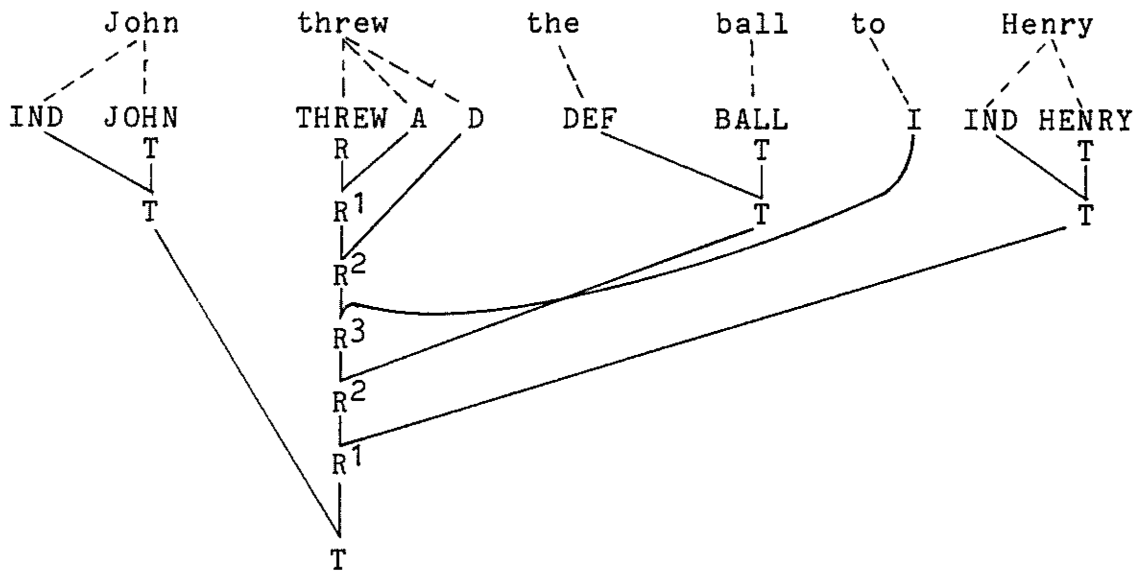
We had earlier (Section 2.3.1.1.2.1.2) introduced the notion of a relativized relation modifier as a subexpression of the differentiated relative construction, wherein the relativized relation modifier is applied to a thing expression. We now employ the relativized relation modifier as a subexpression of another construction called the relativized relation construction, wherein the relativized relation modifier is applied to a relation label. The relativized relation construction pares down the place number of a relation-expression by adjoining one or more thing-expressions to that relation-expression.

Consider the following examples:



"Threw" is variously an intransitive verb and a transitive verb; "threw to" is a 3-place relation, but furthermore "threw a ball to" should be extractable as a 2-place relation, and "threw a ball to Henry" should be extractable as a 1-place relation, and, finally, the whole sentence should be extractable as a 0-place relation.

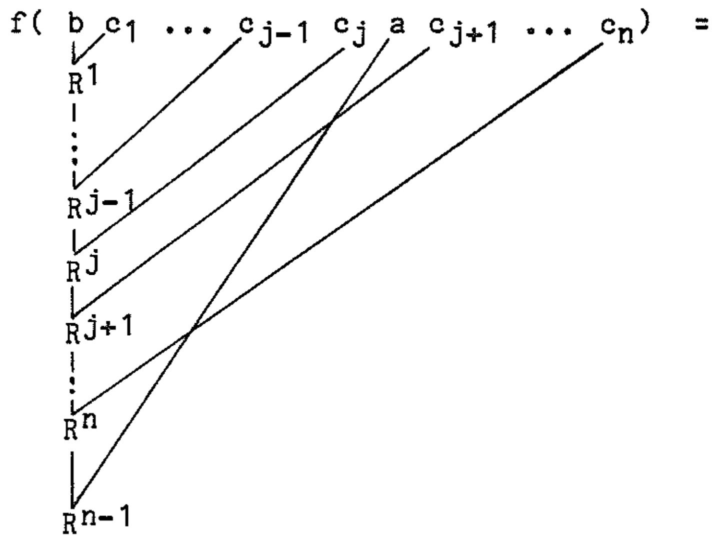
The following analysis reflects these further possibilities:



The above syntactic representation incorporates a succession of relativized relations. The following semantic axiom provides an interpretation for relativized relations.

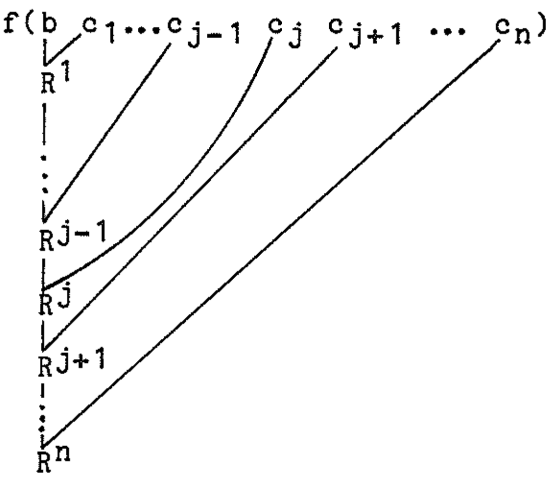
L.S.A.12. <u>Relativized Relation Axiom</u>

Let $1 \leqslant j \leqslant n$. Let $c_1, \ldots, c_n$ be modifiers. Let a be a thing-expression and let b be a relation-expression. Then:

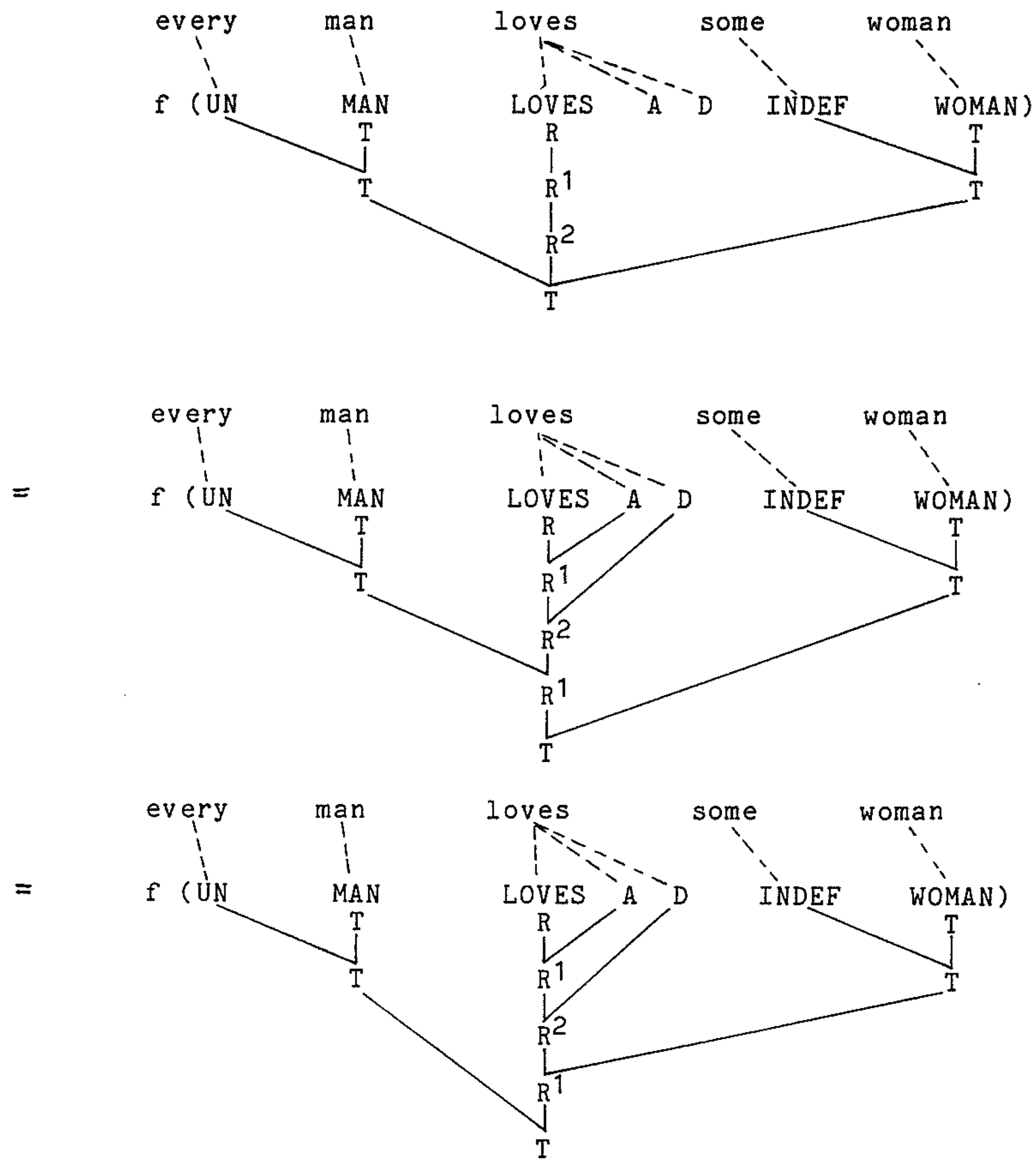


$\{\langle k_1,\ldots,k_{j-1},\ k_{j+1},\ldots,\ k_n\rangle \in D^{n-1}$: there is a $z \in f(a)$ such that for all $w \in z$,

$\langle k_1,\ldots,k_{j-1},\ w,\ k_{j+1},\ldots,k_n\rangle \in f(b\ c_1 \ldots c_{j-1}\ c_j\ c_{j+1} \ldots c_n)\}$

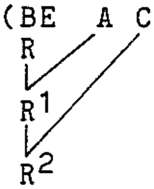


It is a consequence of the relativized relation axiom that all relativizations of a given sentence are semantically equivalent. For example, the following identities hold:



We next give a special logical semantic axiom for a single relation morpheme:

L.S.A.(13) f(BE A C) = {<x,y>: x=y}



2.3.2.3 Logical Semantic Axioms for Determiners of SYN_L^{TR}: Determiners and their Classification

Determiners play a key role in the semantic structure of TR-languages. In this section we formally characterize determiners, partition them into four types paralleling the intuitive discussion of Section 2.3.1.1.2.1.1, and formulate logical semantic axioms for them.

The specific structure of the denotation of thing-expressions influences the truth of sentences which contain them: determiners have the semantic function of fixing that specific structure. That is, what determiners "determine" is the internal semantic structure of thing-expressions: the way that they do this is spelled out by the semantic axioms that interpret them, to be detailed below.

Recall that in Section 2.3.1.1.2.1.1 we had provided an intuitive semantic motivation for a fourfold division of the thing-expressions of SYN_L^{TR} and a corresponding fourfold division of the determiners of SYN_L^{TR}.

We can now give a precise set-theoretic characterization of this fourfold division, and of the notion of determiner itself, defined ultimately in terms of an analogous fourfold division among sets defined as follows:

Definition

Let K be a set. Then

(i) K is definite if and only if for some x, $K = \{x\}$.

(ii) K is lower-bounded if and only if K is not definite and, for all x, if $x \in K$ and $x \le y \le \bigcup K$, then $y \in K$.

(iii) K is <u>upper-bounded</u> if and only if K is not definite, K is not lower-bounded, and, for all x, if $x \in K$ and $y \le x$, then $y \in K$.

(iv) K is <u>doubly-bounded</u> if and only if K is not definite, K is not lower-bounded, K is not upper-bounded, and, for some lower-bounded set K_1 and, for some upper-bounded set K_2, $K = K_1 \cap K_2$.

<u>Definition</u>

Let s be a semantic theory $\langle F,V,R \rangle$, let e be a thing-expression of SYN_L^{TR} and let $\langle D,f \rangle \in F$. Then e is <u>definite in</u>, <u>lower-bounded in</u>, upper-bounded in, or <u>doubly-bounded in</u> $\langle D,f \rangle$ according as, respectively, f(e) is definite, lower-bounded, upper-bounded, or doubly bounded.

Let s be a semantic theory $\langle F,V,R \rangle$ and let e be a modifier of SYN_L^{TR}. Then

(i) e is a <u>definite determiner relative to s</u>, a <u>lower-bounding determiner relative to s</u>, an <u>upper-bounding determiner relative to s</u>, or a <u>doubly bounding determiner relative to s</u> according as, respectively, for all thing-expressions b in SYN_L^{TR}, $f\left(\begin{array}{cc} a & b \\ \backslash & T \\ \multicolumn{2}{c}{T} \end{array}\right)$ and $f\left(\begin{array}{cc} b & \cdot a \\ T & / \\ \multicolumn{2}{c}{T} \end{array}\right)$ are definite, lower-bounded, upper-bounded, or doubly bounded in every interpretation $\langle D,f \rangle \in F$.

(ii) e is a <u>determiner relative to s</u> if and only if e is a definite determiner relative to s, a lower-bounding determiner relative to s, an upper-bounding determiner relative to s, or a doubly bounding determiner relative to s.

Note: We will often refer to an expression of $SYN_{L}^{T\;R}$ simply as being a *determiner*, a *definite determiner*, a *lower-bounding determiner*, an *upper-bounding determiner*, or as a *doubly-bounding determiner* without reference to a semantic theory s: when doing so, the intent is that s be understood as any semantic theory on which all the semantic logical axioms of this chapter hold.

Let $s = \langle F,V,R \rangle \in INT_{L}^{T\;R}$. Some determiners of $SYN_{L}^{T\;R}$ are interpreted in the same way in all interpretations $\langle D,f \rangle \in F$; others are not. Determiners of the former kind are said to be *fixed in s*; determiners of the latter kind are said to be *variable in s*. More precisely, a determiner a of $SYN_{L}^{T\;R}$ is *fixed in s* if and only if for all interpretations $\langle D,f \rangle$ and $\langle D',f' \rangle$ in the set F, and for all thing and relation-expressions $b_1, \ldots, b_n$ in $SYN_{L}^{T\;R}$, if for each $1 \leq i \leq n$, $f(b_i) = f'(b_i)$, then $f(a)[\langle f(b_1) \ldots f(b_n) \rangle] = f'(a)[\langle f'(b_1) \ldots f'(b_n) \rangle]$; a determiner a of $SYN_{L}^{T\;R}$ is *variable in s* if and only if a is not fixed in s. The difference between fixed and variable determiners is thus a difference in the determinateness of their interpretations in the sense that, in application to given arguments, their function values are fully specified: the interpretations of fixed modifiers are fully determinate, whereas the interpretations of variable determiners are not - that is, in application to given arguments, their function values are only partially determined.

A Logical Semantic Axiom for Fixed Determiners

L.S.A.(14) Let a, b, c be modifiers. Then:

(14.1) $f(UN)[f(\overset{a}{T})] = \{\bigcup f(\overset{a}{T})\}$

(14.2) $f(INDEF)[f(\overset{a}{T})] = \{x : x \subseteq \bigcup f(\overset{a}{T})\ \&\ x \neq \phi\}$

(14.3) $f(SING)[f(\overset{a}{T})] = \{\{x\} :\ x \in \bigcup f(\overset{a}{T})\}$

(14.4) $f(BLB)[f(\overset{a}{T})] = \{x \subseteq \bigcup f(\overset{a}{T}) : Ey \in f(\overset{a}{T})$ such that $y \subseteq x\}$

(14.5) $f(BUB)[f(\overset{a}{T})] = \{x \subseteq \bigcup f(\overset{a}{T}) : Ey \in f(\overset{a}{T})$ such that $x \subseteq y\}$

(14.6) $f(n)[f(\overset{a}{T})] = \{\{x_1, \ldots, x_n\}$: for all $1 \leq i, j \leq n$,

$x_i \neq x_j\ \&\ x_i, x_j \in \bigcup f(\overset{a}{T})\}$

(14.7) $f(MID)[f(\overset{a}{T})] = \{x \in P\bigcup f(\overset{a}{T})$: x is equinumerous

with $\bigcup f(\overset{a}{T}) - x\}$

(14.8) $f(GMID)[f(\overset{a}{T})] = \{x \in P\bigcup f(\overset{a}{T}) : Ey \in f(\text{MID}\ \overset{a}{T})$ such that $x \supsetneq y\}$ [tree: MID and $\overset{a}{T}$ under T]

(14.9) $f(LMID)[f(\overset{a}{T})] = \{x \in P\bigcup f(\overset{a}{T}) : Ey \in f(\text{MID}\ \overset{a}{T})$ such that $x \subsetneq y\}$ [tree: MID and $\overset{a}{T}$ under T]

(14.10) $f(NULL)[f(\overset{a}{T})] = \{\phi\}$

(14.11) $f(PLURAL)[f(\overset{a}{T})] = \{x \subseteq \bigcup f(\overset{a}{T}) : Ey, z$ such that $y \neq z\ \&\ y, z \in x\}$

(14.12) $f(TC)[f(\overset{a}{T})] = \{\bigcup f(\overset{a}{T}) - u : u \in f(\overset{a}{T})\}$

(14.13) $f(QC)[f(\overset{a}{T})] = \{u \subseteq \bigcup f(\overset{a}{T}) : u \notin f(\overset{a}{T})\}$

(14.14) $f(AC)[f(\overset{a}{T})] = \{x \subseteq D : x \neq \phi\ \&\ x \cap \bigcup f(\overset{a}{T}) = \phi\}$

(14.15) $f(EXCT)[f(\overset{a}{T})] = f(\overset{a}{T}) \cap f(BUB)[f(\overset{a}{T})]$

(14.16) $f(B\text{-}COMP)[\langle f(\overset{a}{T}), f(\overset{b}{T})\rangle] = f(\overset{a}{T}) \cap f(\text{AC}\ \overset{b}{T})$ [tree: AC and $\overset{b}{T}$ under T]

(14.17) $f(CONJ)[\langle f(\overset{a}{T}), f(\overset{b}{T})\rangle] =$

$\{x : Ey \in f(\overset{a}{T})\ Ez \in f(\overset{b}{T})$ such that $x = y \cup z\}$

? See pg 243

(14.18) $f(\text{INCDISJ})[\langle f(a_T), f(b_T)\rangle] =$

$\{x: Ey \in f(a_T) \text{ or } Ez \in f(b_T) \text{ such that } x \leq y \cup z\}$

(14.19) $f(\text{EXCDISJ})[\langle f(a_T), f(b_T)\rangle] = f(a_T) \cup f(b_T)$

(14.20) $f(\text{PWCONJ})[\langle f(a_T), f(b_T)\rangle] =$

$\{x \cap y: x \in f(a_T) \ \& \ y \in f(b_T)\}$

(14.21) $f[c_{R^2}\ b_T]\ [f(a_T)] = \{\bigcup\{g(x): x \in A\}: A \in f(b_T)$ and

g is a function whose domain is A and whose range is included in $\bigcup f(a_T)$, and which is such that, for all $y \in A$, for all $w \in g(y)$, $\langle w, y\rangle \in f(c_{R^2})\}$.

insert (14.211) from next page (206.6)

(14.22) $f(\text{S-CONJ})[\langle f(a_T), f(b_T)\rangle] =$

$\{y \subseteq \bigcup f(a_T) \times \bigcup f(b_T): \text{for all } u \in f(a_T) \times f(b_T),\ u \cap y \neq \emptyset\}$

Discussion of L.S.A.14: (14.1) defines the structure of the "universal quantifier" UN. (14.2) defines the structure of the "existential quantifier" INDEF. (14.1) and (14.2), in conjunction with the logical semantic axiom L.S.A.13, defining the semantic structure of $BE_R\ A_{R^1}\ C_{R^2}$, have the consequence that (1) and

(14.211) $f[\underset{R}{c}_{m+1}\ \underset{T}{b}_1, \dots, \underset{T}{b}_m]\ [f(\underset{T}{a})] = \{\bigcup\{g(x) : x \in A\}$: for some

$A_1 \in f(\underset{T}{b}_1), \dots, A_m \in f(\underset{T}{b}_m)$, $A = A_1 \times \dots \times A_m$ and g is a function whose domain is A and whose range is included in $\bigcup f(\underset{T}{a})$, and which is such that, for all $\langle y_1, \dots, y_m \rangle \in A$, for all $w \in g(y)$, $\langle w, y_1, \dots, y_m \rangle \in f(\underset{R}{c}_{m+1})\}$

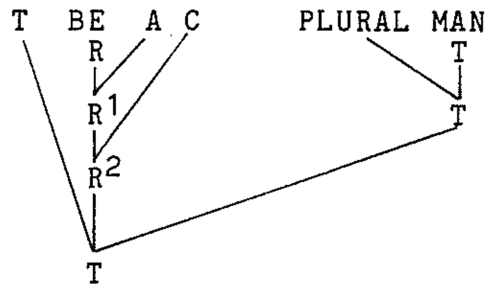
(2) together entail (3) below, under their respective dominant normal readings (1'), (2') and (3') following:

(1) There are men

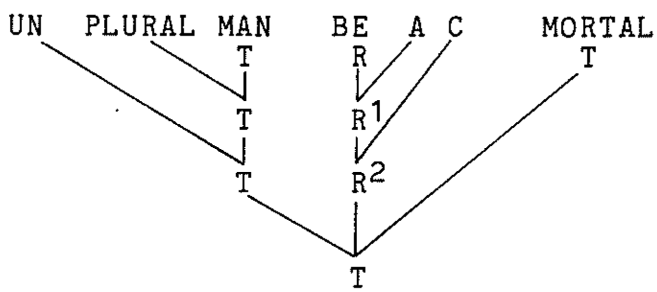
(2) All men are mortal

(3) Some men are mortal

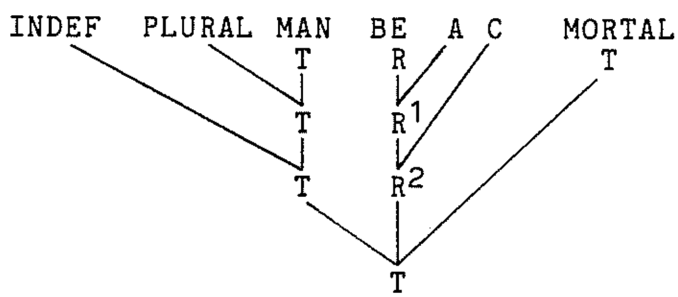
(1')



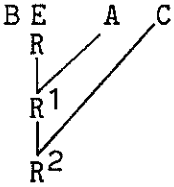
(2')



(3')



(14.3) defines the structure of the "singular determiner" SING which is useful in conjunction with the definite determiner DEF, whose structure is defined below under L.S.A.(15) to distinguish the normal reading of, for example, "the man" from "the men." (14.4) defines the structure of the lower-bounding determiner BLB, whose English analogue is "at least," while (14.5) defines the structure of the upper-bounding determiner BUB, whose English analogues include "at most" and "only." (14.6) defines the structure of the "numeric" determiners "1", "2", "3", etc. whose English analogues are "one", "two", "three", etc., in the sense of "exactly one", "exactly two", "exactly three", etc. (14.7) defines the structure of the "mid-point" determiner, whose English analogue is "half" in the sense of "exactly half". (14.8) defines the structure of the "greater than mid-point" determiner, whose English analogue is "more than half" in the sense of "more than exactly half". For example, (14.2) and (14.8), in conjunction with L.S.A.(13) defining the semantic structure of BE A C and L.S.A.(15) defining the



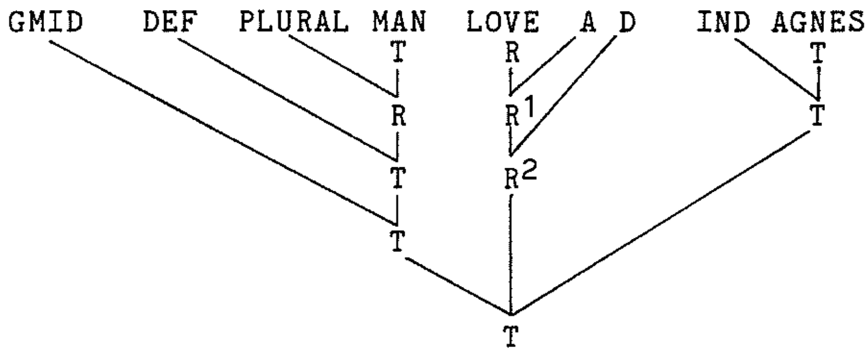
semantic structure of DEF, have the consequence that (4) and (5) below entail (6) under their respective dominant normal readings (4'), (5'), and (6') following:

(4) More than half the men love Agnes

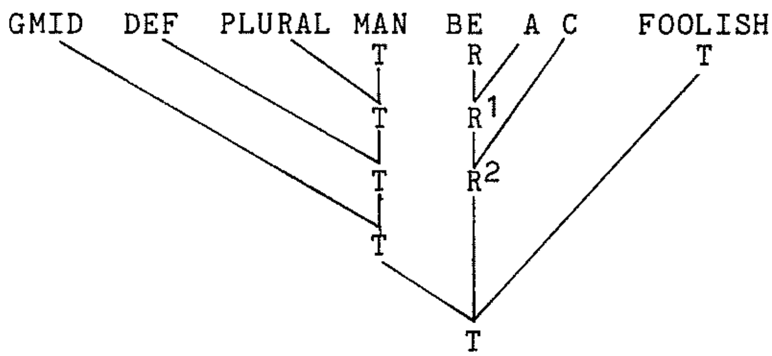
(5) More than half the men are foolish

(6) Some man loves Agnes and is foolish

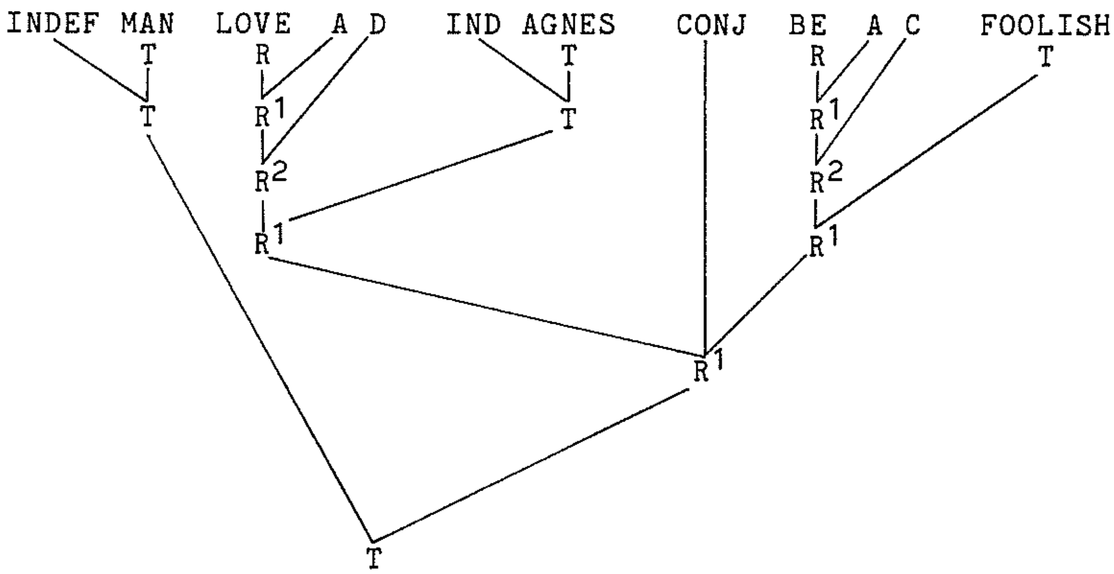
(4')



(5')



(6')



(14.9) defines the structure of the "less than mid-point" determiner, whose English analogue is "less than half" in the sense of "less than exactly half". (14.10) defines the structure of the null determiner NULL, whose English analogues are "no" and "not any". (14.11) defines the structure of the plurality determiner PLURAL, whose English analogues are a terminal "s" or "es" on certain nouns, or a vowel change in other nouns, e.g., men, geese. (14.12) defines the structure of the true complementor TC, whose English analogue is "not". (14.13) defines the structure of the quasi-complementor QC, whose English analogue is "all but". (14.14) defines the structure of the absolute true complementor AC, whose English analogues include "not" and "non-".

Some immediate consequences of (14.12), (14.13), and (14.14) are listed below. Let a be a thing-expression. Then:

(i) if f(a) is a lower-bounded set, then f(TC a) and f(QC a) are upper-bounded sets

```
f(TC   a)        f(QC   a)
   \___|            \___|
       T                T
```

(ii) if f(a) is an upper-bounded set, then f(TC a) and f(QC a) are lower-bounded sets.

```
f(TC   a)        f(QC   a)
   \___|            \___|
       T                T
```

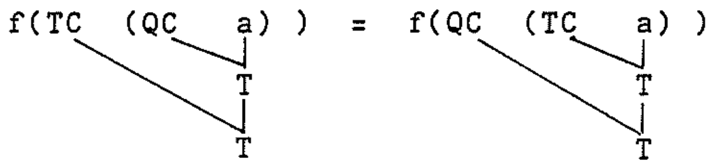
(iii) f(TC TC a) = f(a)

```
f(TC   TC   a)  =  f(a)
   \     \__|
    \       T
     \______|
            T
```

(iv) f(QC QC a) = f(a)

```
f(QC   QC   a)  =  f(a)
   \     \__|
    \       T
     \______|
            T
```

(v) If f(a) is a lower-bounded set or an upper-bounded set, then



(14.15) defines the structure of the exactness determiner EXCT, whose English analogue is "exactly". The absolute complementor AC is needed to provide homologous normal readings of phrases like

(7) Not John

in sentences like

(8) Not John loves Mary

having the sense of

(9) Something other than John loves Mary

On the other hand, in order to provide homologous normal readings of phrases like

(10) All men but John

in the sense of

(10.1) All men but not John

in sentences like

(11) All men but John love Mary

or

(12) Some men but not John love Mary

we require the binary absolute complement or B-COMP, whose defining logical semantic axiom is given by (14.16). (14.16) defines the binary complement function B-COMP, whose English analogues include "___ except ___", "___ but not ___".

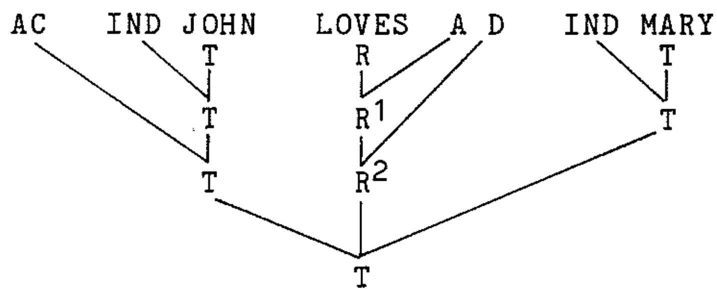
Under their dominant normal readings (8'), (9'), (10'), and (11') below, respectively, each of (8), (9), (10) and (11) entails

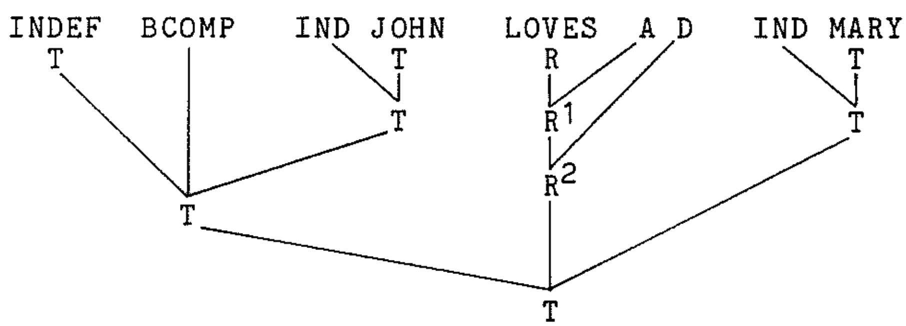
(13) John does not love Mary

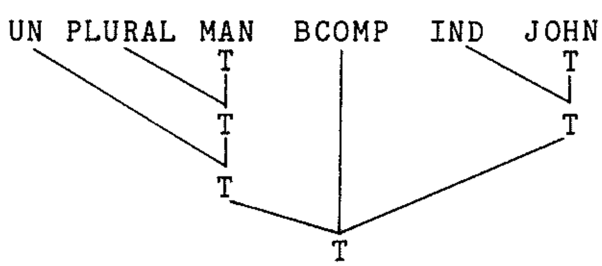
under both the readings of (13) that would be dominant, which we can paraphrase as

(13a) John fails to love Mary

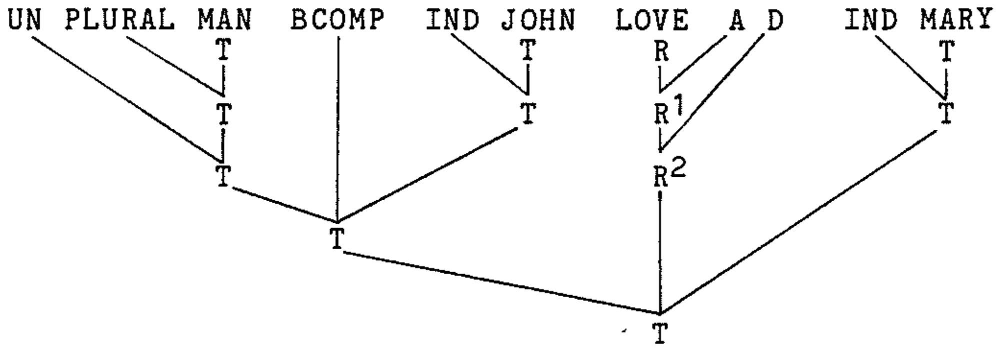
(13b) It is false that John loves Mary

(8')


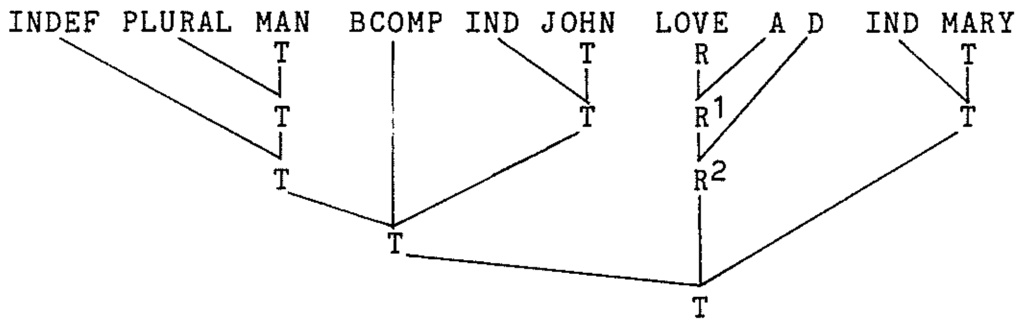
(9')


(10')


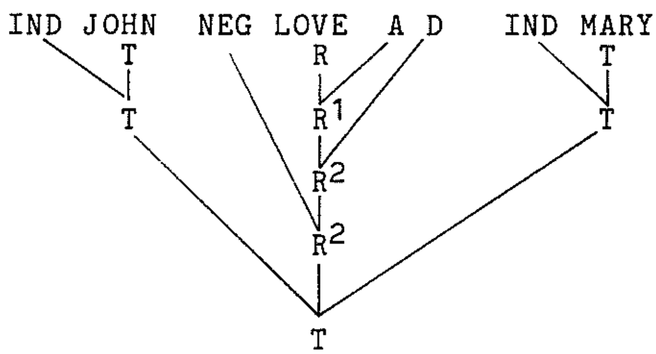
(11')


UN PLURAL MAN BCOMP IND JOHN LOVE A D IND MARY
T T R T
T T R1 T
T R2
T
T

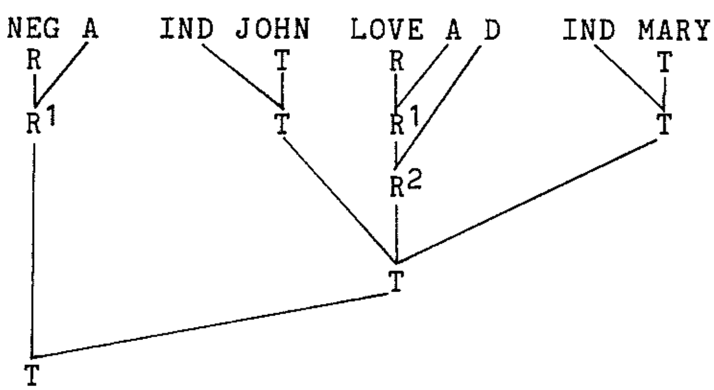
(12')


INDEF PLURAL MAN BCOMP IND JOHN LOVE A D IND MARY
T T R T
T T R1 T
T R2
T
T

(13a')


IND JOHN NEG LOVE A D IND MARY
T R T
T R1 T
R2
R2
T

(13b')


NEG A IND JOHN LOVE A D IND MARY
R T R T
R1 T R1 T
R2
T
T

It can easily be proved that for all modifiers a of $SYN_{L}^{T\,R}$, if $f(\frac{a}{T})$ is definite, lower-bounded, upper-bounded, or doubly-bounded, then, for all modifiers b of $SYN_{L}^{T\,R}$, $f(\text{B-COMP})[\langle \frac{a}{T}, \frac{b}{T} \rangle]$ is definite, lower-bounded, upper bounded, or doubly-bounded, respectively.

We next consider some instances of this fact. For example, (14) and (15) together entail each of (16) and (17) under the dominant normal readings of (14)-(17)

(14) John but not Henry likes Carol

(15) John but not Henry likes Agnes

(16) John but not Henry likes Carol and Agnes

(17) John but not Henry likes Carol or Agnes

which conforms to our intuitive criterion of a definite expression (see Section 2.3.1.1.2.1.1).

Consider further:

(18) and (19) below together entail (20) but not (21) under the dominant normal readings of (18)-(21):

(18) at least one man but not Henry likes Carol

(19) at least one man but not Henry likes Agnes

(20) at least one man but not Henry likes Carol and Agnes

(21) at least one man but not Henry likes Carol or Agnes

which conforms to our intuitive criterion of a lower-bounded expression.

Consider further:

(22) and (23) below together entail (24) but not (25) under the dominant normal readings of (22)-(25)

(22) at most one man but not Henry likes Carol

(23) at most one man but not Henry likes Agnes

(24) at most one man but not Henry likes Carol and Agnes

(25) at most one man but not Henry likes Carol and Agnes

which also conforms to our intuitive criterion of an upper-bounded expression.

Consider finally:

(26) and (27) below together entail neither (28) nor (29), under the dominant normal readings of (26)-(29)

(26) exactly one man but not Henry likes Carol

(27) exactly one man but not Henry likes Agnes

(28) exactly one man but not Henry likes Carol and Agnes

(29) exactly one man but not Henry likes Carol or Agnes

which conforms to our intuitive criterion of a doubly-bounded expression.

As another consequence of L.S.A.(14), we have that (30) and (31) below inter-entail each other under their dominant normal readings

(30) exactly two men love Carol

(31) at least two men and at most two men love Carol

(14.17) defines the structure of the (binary) conjunction function CONJ, whose English analogues include "and", "both ___ and ___", "___ together with ___". (14.18) defines the structure of the inclusive disjunction operation INC_DISJ, whose English analogues include "or", and "and/or". (14.19) defines the structure of the exclusive disjunction operation EXCDISJ, whose English analogues include "or", and "either ___ or ___". (14.20) defines the semantic structure of the pairwise conjunction operation PWCONJ, whose English analogue is "and".

The following consequences of L.S.A.(14) can be proved which pertain to the binary modifiers, CONJ, INCDISJ, EXCDISJ.

Let a, b be modifiers of SYN_L^{TR}. Then

(i) if $f(\underset{T}{a})$, $f(\underset{T}{b})$ are both lower-bounded (both upper-bounded), then $f(CONJ)[\langle f(\underset{T}{a}), f(\underset{T}{b})\rangle]$ is lower-bounded (upper-bounded).

(ii) if $f(\underset{T}{a})$ is lower-bounded (upper-bounded) and $f(\underset{T}{b})$ is upper-bounded (lower-bounded), then $f(CONJ)[\langle f(\underset{T}{a}), f(\underset{T}{b})\rangle]$ is doubly-bounded.

Logical Semantic Axioms for Variable Determiners

Providing axioms for variable determiners is less direct than for fixed determiners. We need to distinguish between two kinds of axioms for variable determiners: absolute axioms and relative axioms. Absolute axioms impose a uniform semantic structure on the meanings of thing-expressions into which variable determiners enter syntactically, but of course do not completely determine that structure as they would for fixed determiners. Relative axioms inter-relate those semantic structures in terms of special binary relations defined on the set of thing-expressions, to be defined shortly. The following absolute logical semantic axiom introduces some variable determiners:

L.S.A. 15. Let a be a thing-expression. Then

(15.1) $f(DEF)[f(\underset{T}{a})] \in \{\{x\}: x \in f(\underset{T}{a})\}$

(15.2) $f(IND)[f(\underset{T}{a})] \in \{\{\{x\}\}: x \in \bigcup f(\underset{T}{a})\}$

(15.3) $f(MASS)[f(\overset{a}{T})] \in \{\{\{x\}\}: x \in D\}$

(15.4) For all $x \in f(WMLB)[f(\overset{a}{T})]$, for all $y \subseteq \bigcup f(\overset{a}{T})$, if $x \subseteq y$, then $y \in f(WMLB)[f(\overset{a}{T})]$

(15.5) For all $x \in f(MXLB)[f(\overset{a}{T}]$, for all $y \subseteq \bigcup f(\overset{a}{T})$, if $x \subseteq y$, then $y \in f(MXLB)[f(\overset{a}{T})]$

Discussion of L.S.A. (15) The individuator morpheme IND and the massing morpheme MASS each have the semantic function of associating a singleton set {x} with the denotation y of a thing-expression e to which they are applied,which, in general, depends on y. The difference is that when the individuator morpheme IND is applied to e, x is an element of y and, when the massing morpheme MASS is applied to e, x need not be an element of y. Thus, while both IND and MASS produce singletons, hence, in that sense, are both what we might call individuators, they differ in the relationship between those singletons and the denotations of the thing-expressions to which they are applied.

The intuitive logical difference between IND and DEF is that f(DEF) is a choice function acting on $f(\overset{a}{T})$, selecting an element of $f(\overset{a}{T})$, and forming a singleton set whose only element is that selected element of $f(\overset{a}{T})$, whereas f(IND) is a choice function acting on $\bigcup f\overset{a}{T})$, and forming a singleton set whose only element is a singleton set whose only element is that selected element of $\bigcup f(\overset{a}{T})$. Thus f(IND) will always form an "individual", that is, a set whose union is an element of the domain of discourse, whereas f(DEF) may or may not, depending on the internal structure of the set f($\underset{T}{a}$). In particular, f(DEF), in application to f(PLURAL $\begin{array}{c} b \\ T \\ | \\ T \end{array}$),

will not form an individual, but rather a set whose element is a set of "f($\underset{T}{b}$)'s".

The intuitive logical difference between MASS and DEF is that the latter formalizes what might be called the "distributed sense" of a noun phrase, while the former formalizes the "non-distributed" sense of a noun phrase. Roughly put, the distributed sense of a noun phrase is that sense in which the predicate is regarded as applying to each entity alike referred to by the noun phrase, whereas the non-distributed sense of a noun phrase is that sense in which the predicate is regarded as applying, not to each entity referred to by the noun phrase, but to the "heap", i.e., the "mass" of those entities considered-as-a-whole, i.e., as a single unit.

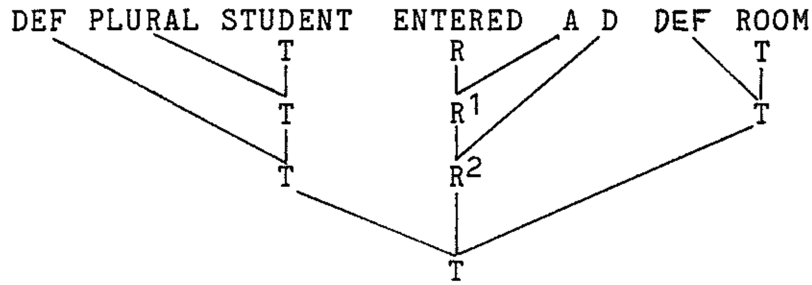
Let us consider some examples.

(1) The students entered the room

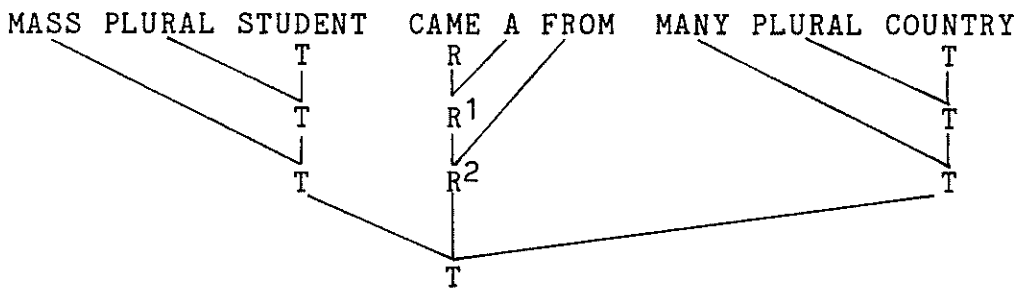
(2) The students came from many countries

The phrase "the students" in (1) has the distributed sense, i.e., the sense that means that each student entered the room, whereas the phrase "the students" in (2) has the non-distributed sense, i.e., the sense that means, not that each student came from many different countries, but that the "heap" or "mass" of students considered-as-a-whole came from different countries. The indicated dominant readings of (1) and (2) are respectively (1') and (2') below:

(1')



(2')

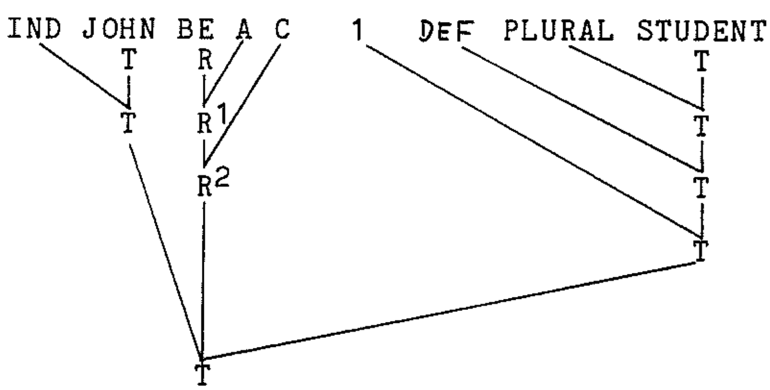


To appreciate the difference between (1) and (2) as syntactically represented by (1') and (2'), respectively, we note that

(3) John is one of the students

under the reading (3') of (3)

(3')

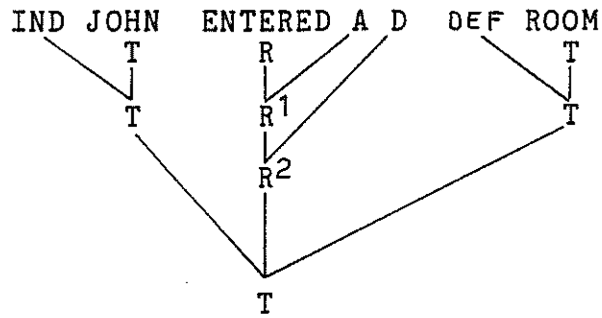


together with (1) under the reading (1') entails

(4) John entered the room

under the reading (4') of (4)

(4')

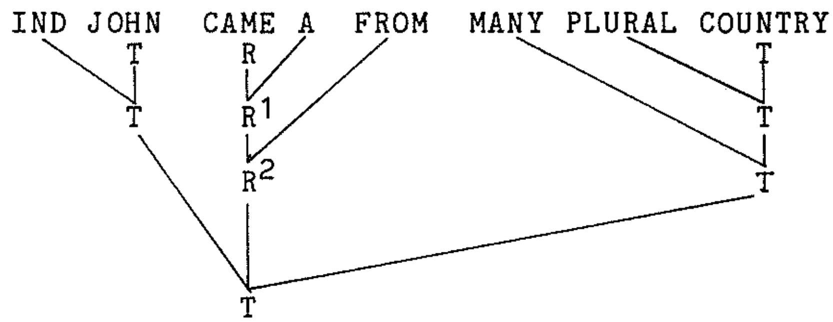


On the other hand, (3) under the reading (3') together with (2) under the reading (2') does <u>not</u> entail

(5) John came from many countries

under the reading (5') of (5):

(5)



Finally, (15.4) and (15.5) assert that WMLB[$f(\overset{a}{T})$] and MXPLB[$f(\overset{a}{T})$] are lower-bounded sets of subsets of $f(\overset{a}{T})$.

We now wish to introduce a <u>relative</u> logical semantic axiom for variable determiners. In order to do this, we first need to introduce some special binary relations on sets and on modifiers:

Let A, B be sets. Then:

(i) $A \ll B$ if and only if for all $y \in B$, there is an $x \in A$ such that $x \subseteq y$.

(ii) $A <^{o} B$ if and only if for all $x \in A$, there is a $y \in B$ such that $x \subseteq y$.

(iii) $A <^{*} B$ if and only if $A \ll B$ and $A <^{o} B$

(iv) $A \lhd B$ if and only if for all $x \in A$ and for all $y \in B$, $x \subseteq y$

Let a, b be modifiers of $SYN_{L}^{T} R$. Then

(i) $a \ll_{\langle D,f \rangle} b$ if and only if for all $A \subseteq SD$ such that $f(a)(A) \neq \emptyset$, $f(b)(A) \neq \emptyset$, and $f(a)(A) <^{o} f(b)(A)$

(ii) $a <^{o}_{\langle D,f \rangle} b$ if and only if for all $A \subseteq SD$ such that $f(a)(A) \neq \emptyset$, $f(b)(A) \neq \emptyset$, and $f(a)(A) <^{o} f(b)(A)$.

(iii) $a <^{*}_{\langle D,f \rangle} b$ if and only if $a \ll_{\langle D,f \rangle} b$ and $a <^{o}_{\langle D,f \rangle} b$.

(iv) $a \ll_{\langle D,f \rangle} b$ if and only if for all $A \subseteq SD$, $f(a)(A) \subseteq f(b)(A)$

(v) $a \subseteq_{\langle D,f \rangle} b$ if and only if for all $A \subseteq SD$, $f(a)(A) \subseteq f(b)(A)$

The following are simple consequences of the above definitions and earlier logical semantic axioms:

Let a and b be modifiers of $SYN_{L}^{T} R$. Then:

(i) If $a \subseteq_{\langle D,f \rangle} b$ or $a \lhd_{\langle D,f \rangle} b$, then $a \ll_{\langle D,f \rangle} b$

(ii) The relations $\ll$, $<^{o}$, $<^{*}$, $\lhd$, $\ll_{\langle D,f \rangle}$, $<^{o}_{\langle D,f \rangle}$, $<^{*}_{\langle D,f \rangle}$, and $\subseteq_{\langle D,f \rangle}$ are transitive and $\ll$, $<^{o}$, $<^{*}$, $\ll_{\langle D,f \rangle}$, $<^{o}_{\langle D,f \rangle}$, and $\subseteq_{\langle D,f \rangle}$ are reflexive as well.

(iii) If a is a lower-limited determiner in $\langle D,f \rangle$, then for all thing-expressions c, $f(UN)[f(c)] \subseteq f(a)[f(c)] \subseteq f(INDEF)[f(c)]$, and $f(INDEF)[f(c)] \ll f(a)\{f(c)\} \ll f(UN)\{f(c)\}$.

(iv) If a is a definite determiner in $\langle D,f \rangle$, then $a \subseteq_{\langle D,f \rangle} INDEF$ and $INDEF \ll_{\langle D,f \rangle} a$.

(v) For all thing-expressions c, $f(a)[f(c)] \lhd f(UN)[f(c)]$, and $f(a)[f(c)] \ll f(UN)[f(c)]$.

(vi) For all positive integers m, n, if $n \ll_{\langle D,f\rangle} m$, then $n <^{*}_{\langle D,f\rangle} m$.

(vii) For all thing-expressions c, d, if $f(c) \ll f(d)$, then $f(BLB)[f(c)] \ll f(BLB)[f(d)]$.

We now formulate a relative logical semantic axiom for some variable determiners.

L.S.A.(16)

(10.1) INDEF $\ll_{\langle D,f\rangle}$ WMLB $<^{*}_{\langle D,f\rangle}$ UN

(10.1) INDEF $\ll_{\langle D,f\rangle}$ LMID $<^{*}_{\langle D,f\rangle}$ MID $<^{*}_{\langle D,f\rangle}$ GMID $<^{*}_{\langle D,f\rangle}$ MXPLB-UN $<^{*}_{\langle D,f\rangle}$ UN

(10.3) INDEF $<^{*}_{\langle D,f\rangle}$ SMLB $<^{*}_{\langle D,f\rangle}$ UN

The following are some sample consequences of semantic axioms (14), (15), and (16); let a be a thing-expression in SYN^{R}_{L}. Then:

(i) f(WMLB PLURAL a) $\ll$

```
f(WMLB      PLURAL   a)  ≪
    \__________\_____|
                \____|
                     T
                     |
                     T
```

```
f(SMLB      PLURAL   a)   ⊆   f(a)  ⊲
                     T
                     T
```

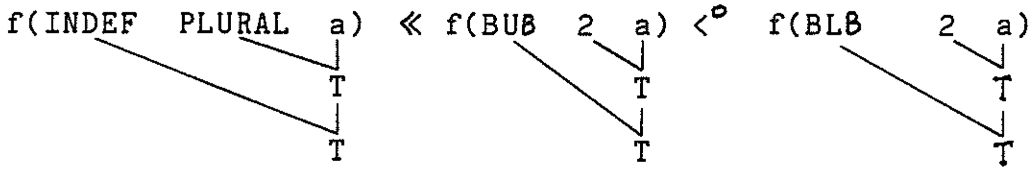
```
f(UN        PLURAL   a)
                     T
                     T
```

(ii)

```
f(UN     PLURAL   a)  <°
                  T
                  T
```

```
f(BLB   2   a) <°   f(INDEF  PLURAL   a) <*   f(PLURAL   a)
            T                         T                  T
            T                         T
```

(iii)



Extension of Semantic Axioms to External Determiners

It will be noted that in clauses (14.1)-(14.13), (14.15)-(14.23) and (15.1)-(15.3) of logical semantic axioms (14) and (15), the effect of each defined determiner is relativized to the elements of the sets on which that determiner operates, rather than to the domain as a whole. We distinguish the former as internal determiners and the latter as external determiners. We consider some examples which illustrate the difference:

The sentence

(1) At most two men love Mary

has at least two normal readings which we can express as (2) and (3):

(3) There are at most two things that love Mary and they are men.

(2) Among the things that love Mary, there are at most two men.

Thus, if Mary had a dog that loved her, and if Bill was the only man that loved her, then (2) would be true while (3) would be false. We distinguish the first of these two senses of "at most" as the internal sense and the second as the external sense, where the internal sense of "at most" is that in which "at most" is understood as a limitation relative to the class of men, and the external sense of "at most" is that in which "at most" is understood as a limitation relative to the class of all things, i.e., the entire domain of discourse.

Within SYN_L^{TR}, we express the internal sense of "at most" by the logical morpheme BUB, defined above by semantic axiom (14.5), and we express the external sense of "at most" by introducing a new logical morpheme ßUB-E (for ßUB-External), which we interpret as follows:

If a is a modifier of SYN_L^{TR}, then

$f(\text{ßUB-E})[f(\overset{a}{T})] = \{y \subseteq D: \text{for some } w \in D \text{ such that } w \cap P(\bigcup f(\overset{a}{T})) = \phi$ & for some $z \in f(\overset{a}{T})$ such that for all $v \in f(\overset{a}{T})$, (if $z \subseteq v$, then $z = v$), $y \subseteq z \cup w\}$

Each of the logical morphemes defined by logical semantic axiom (14) can, like ßUB above, be associated with a corresponding external version.[81] This association is formulated in a simple and uniform way by the logical semantic axiom (17) to follow later. But before stating this axiom, we provide some further special cases and examples to motivate it.

English phrases such as "all but at most two men" are expressed in $SYN_{English}^{TR}$ by the application of the complementor TC, (which syntactically represents "all but") to the syntactic representation within $SYN_{English}^{TR}$ of the phrase "at most two men". The internal sense of TC is given above by axiom (14.12). The external sense of TC, (which, as remarked above, we write as TC-E, for TC-external) is interpreted as follows:

If a is a modifier of SYN_L^{TR}, then

$f(\text{TC-E})[f(\overset{a}{T})] = \{x-y: x \in f(\overset{UN}{T}) \text{ \& } y \in f(\overset{a}{T})\}$

For the sake of comparison, we can express the interpretation of the internal sense of TC (given by (14.12)) in the equivalent form:

$f(\text{TC})[f(\overset{a}{T})] = \{x-y: x \in \{\bigcup f(\overset{a}{T})\} \text{ \& } y \in f(\overset{a}{T})\}$

Note 81. With the exception of the logical morpheme AC which is already external as defined in L.S.A. (14.14).

Further perspective on the general character of TC and TC-E, and of the relationship between them is afforded by the observation that both TC and TC-E can be obtained as special cases of a more general binary logical morpheme, which we write as EXCL, and interpret as follows:

Let $u, v \in PD$. Then

$f(EXCL)[\langle u,v \rangle] = \{x-y : x \in u \ \& \ y \in v\}$

We call EXCL the <u>element-wise difference</u>[82] <u>of the sets u and v</u>. The following consequences of these various preceding definitions can be easily proved:

Let a be a modifier of SYN_L^{TR}. Then

(i) $f(TC\text{-}E)[f(\overset{a}{T})] = f(EXCL)[\langle f(\overset{UN}{T}), f(\overset{a}{T}) \rangle]$

(ii) Letting $\overset{b}{T}$ be the "head" thing-expression of $\overset{a}{T}$, then

$f(TC)[f(\overset{a}{T})] = f(EXCL)[\langle f(\underset{T}{UN \ \underset{T}{b}}), f(\overset{a}{T}) \rangle]$

(iii) $f(TC)[f(UB\text{-}E)[f(\overset{a}{T})]] = f(TC\text{-}E)[f(UB)[f(\overset{a}{T})]]$

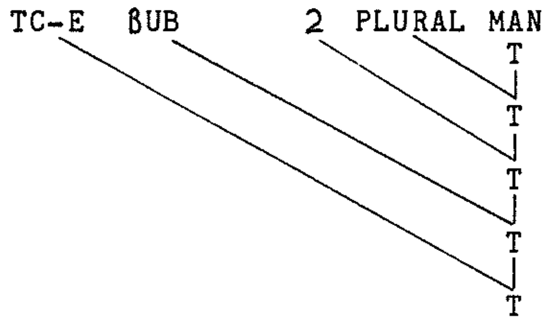
Thus the English noun phrase

(1) All but at most two men

can be represented in the external sense by

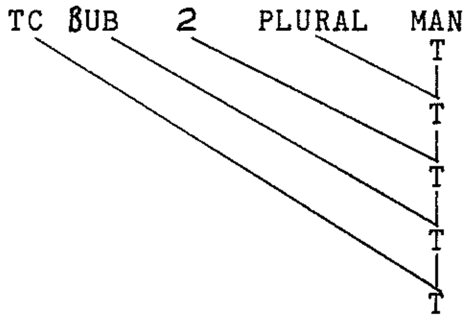
Note 82. Compare this with the analogous notions of element-wise intersection and union, defined earlier.

(1')



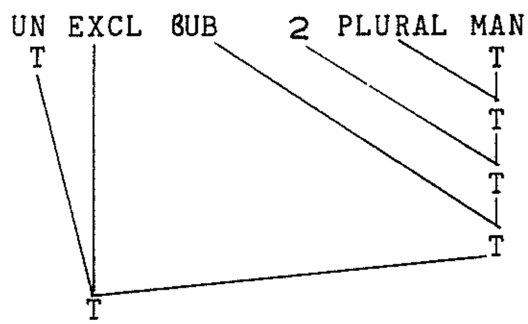
and in the internal sense by

(1'')



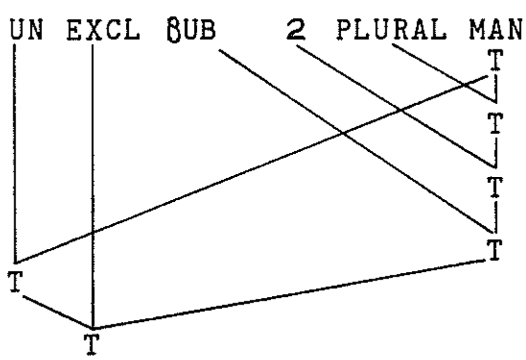
Equivalently[83] (1) can be represented in the external sense by

(1''')



and in the internal sense by

(1'''')



Note 83. By this I mean, as usual, that f(1') = f(1''') and that f(1'') = f(1''''), for all <D,f> satisfying the logical semantic axioms.

Let us consider another example which strongly parallels the case with "at most" illustrated in sentence (1) above: The sentence

(4) Exactly two men love Mary

has at least two normal readings, which we can express as

(6) There are exactly two things that love Mary and they are men.

and

(5) Among the things that love Mary there are exactly two men.

Thus, if Mary had a dog that loved her, and if Bill and John were the only men who loved her, then (5) would be true while (6) would be false. As with the discussion of "at most" with respect to sentence (1), we distinguish here two senses of "exactly" as the internal sense (which it has in (5)) and the external sense, (which it has in (6)), where the internal sense of "exactly" is understood as a limitation to the class of men, and the external sense of "exactly" is understood as a limitation relative to the class of all things. Within SYN_L^{TR}, we express the internal sense of "exactly" by the logical morpheme EXCT, whose interpretation is defined earlier by axiom (14.15), and we express the external sense of "exactly" by introducing the logical morpheme EXCT-E, whose interpretation can be given as:

Let a be a modifier of SYN_L^{TR}. Then

$$f(\text{EXCT-E})[f(\underset{T}{a})] = f(\underset{T}{a}) \cap f(\text{UB-E})[f(\underset{T}{a})]$$

The external versions of given logical modifiers, unary, binary, and, in general, n-ary, need not be assigned piecemeal, but can be assigned generally to arbitrary logical modifiers, as is done by the following logical semantic axiom.

L.S.A.17. Let $a_1, \ldots a_n$ be modifiers and let X be an n-adic logical modifier. Then

$$f(X\text{-}E)[\langle f(^{a}1)_T \ldots f(^{a}n)_T\rangle] =$$

$$\{y \subseteq D: \text{ for some } w \subseteq D - \bigcup[f(X)[\langle f(^{a}1)_T, \ldots, f(^{a}n)_T\rangle], \text{ there is a}$$

$$z \in f(X)[\langle f(^{a})_T, \ldots, f(^{a}n)_T\rangle] \text{ such that } y = w \cup z\}$$

In the case that n = 1, we have:

$$f(X\text{-}E)[f(^{a}_T)] = \{y \subseteq D: \text{ for some } w \subseteq D - \bigcup f(X)[f(^{a}_T)], \text{ there is a}$$

$$z \in f(X)[f(^{a}_T)] \text{ such that } y = w \cup z\}$$

Let us consider some examples illustrating the interpretations which syntactic representations of some English noun phrases would receive under their dominant normal readings by virtue of these definitions:

The external sense of the English word

(4) John

is represented in $SYN^{TR}_{English}$ as

(4') $\text{IND-E}\ \text{JOHN}_{T}$ ${}_{T}$

and is accorded the interpretation

(4'') $f(\text{IND-E JOHN}_{T})_{T} = \{y \subseteq D: \text{ for some } w \subseteq D - \bigcup f(\text{IND})[f(\text{JOHN}_T)], \text{ there is a } z \in f(\text{IND})[f(\text{JOHN}_T)] \text{ such that } y = w \cup z\}.$

The English noun phrase

(5) All but John

is represented in $SYN^{TR}_{English}$ as

(5') TC IND-E JOHN

T
T
T

and is accorded the interpretation

(5'') f(TC IND-E JOHN) = {x-y:x∈f($^{UN}_{T}$) & y∈f(IND JOHN)}

T T
T T
T

= {x-y: x∈{D} & y⊆D & f(IND) [f($JOHN_T$)]∈y}

= {x-y: x = D & y⊆D & f(IND) [($JOHN_T$)]∈y}

= {D-y: f(IND) [f($JOHN_T$)]∈y & y⊆D}

The external sense of the English noun phrase

(6) At most John

is represented in $SYN^{TR}_{English}$ as

(6') SUB IND-E JOHN (or equivalently, as SUB-E IND JOHN)

T T
T T
T T

and is accorded the interpretation

(6'') f (UB IND-E JOHN) = {y⊆D: for some z∈f(IND JOHN), & for all
T T
T T
T v∈f(IND JOHN)(if z⊆v, then z=v)
T
T

and y⊆z}

={y⊆D: for some z ∈ {y⊆D: f(IND) [f(JOHN)]∈y}, &
T

for all v∈{y⊆D: f(IND) [f(JOHN)]∈y}, (if z⊆v, then z = v) & y⊆z}
T

= {y⊆D: y⊆{f(IND)[f(JOHN)]}}
T

= {y⊆D: y = ∅ or y = {f(IND)[f(JOHN)]}}
T

The external sense of the English noun phrase

(7) all but exactly John

is represented in $SYN^{TR}_{English}$ as

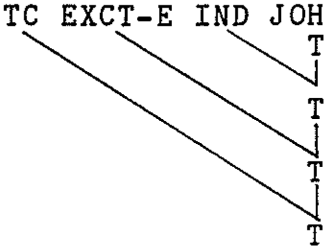
(7'') TC EXCT-E IND JOHN
T
T
T
T

(or equivalently, by TC-E EXCT IND JOHN
T
T
T
T

or by TC EXCT IND-E JOHN)
T
T
T
T

and is accorded the interpretation

(7'') f(TC EXCT-E IND JOHN) = {x-y: x=D & y={f(IND)[f(JOHN$_T$)]}}



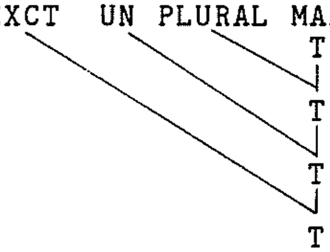
= D - {f(IND)f[(JOHN$_T$)]}

The internal sense of the English noun phrase

(8) Exactly all men

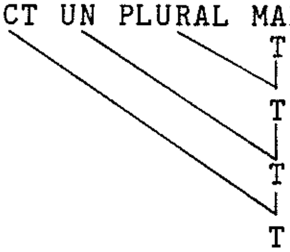
is represented in $SYN^{TR}_{English}$ as

(8') EXCT UN PLURAL MAN

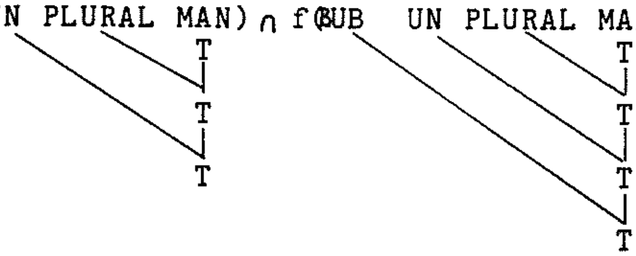


and is accorded the interpretation

(8'') f(EXCT UN PLURAL MAN)



= f(UN PLURAL MAN) ∩ f(SUB UN PLURAL MAN)



```
= {y⊆D:y⊇∪f(PLURAL MAN)} ∩
                       T
                       T

    {y⊆D: for some z ∈ f(UN PLURAL MAN), and for all
                                    T
                                    T
                                    T

v∈f(UN PLURAL MAN (if z⊆v, then z = v) and y⊆z}
              T
              T
              T

= {y:y⊇∪f(PLURAL MAN)} ∩ {y:y⊆∪f(PLURAL MAN)}
                   T                    T
                   T                    T

= {y:y = ∪f(PLURAL MAN)} = {∪f(PLURAL MAN)}
                   T                 T
                   T                 T
```

The internal sense of the English noun phrase

(9) Exactly John

is represented in $SYN^{TR}_{English}$ as

```
(9')  EXCT IND JOHN
                T
                T
                T
```

and is accorded the interpretation

```
(9'') f(EXCT IND JOHN) = f(IND JOHN) ∩ f (SUB  IND JOHN) =
                  T               T                T
                  T               T                T
                  T                                T
```

$\{y \subseteq D: f(IND)[f(JOHN_T)] \in y\} \cap \{y \subseteq D: y = \phi \text{ or } y = \{f(IND)[f(JOHN_T)]\}\}$

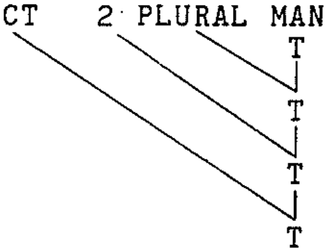
$= \{y: f(IND)[f(JOHN_T)] \in y\} \cap \{\phi, \{f(IND)[f(JOHN_T)]\}\} = \{f(IND)[f(JOHN_T)]\}$

The external sense of the English noun phrase

(10) Exactly two men

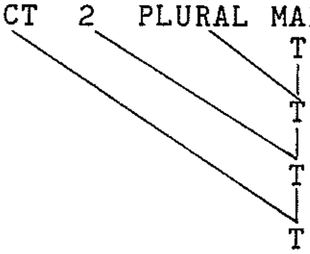
is represented in $SYN^{TR}_{English}$ as

(10') EXCT 2 PLURAL MAN

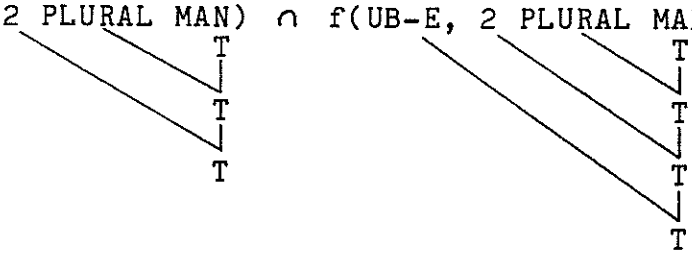


and is accorded the interpretation

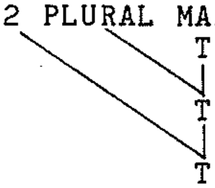
(10'') f(EXCT 2 PLURAL MAN) =



f(2 PLURAL MAN) $\cap$ f(UB-E, 2 PLURAL MAN) =



{y⊆D: for some w such that w $\cap$ f(2 PLURAL MAN) = Ø, and for some



$x_1, x_2 \in \bigcup f($2 PLURAL MAN$)$ such that $x_1 \neq x_2$

T
T
T

and $y = \{x_1, x_2\} \cup w\} \cap$

$\{y \subseteq D$: for some z such that $y \leq z$ and such that

$z \in \{y \subseteq D$: for some $x_1, x_2 \in \bigcup f($2 PLURAL MAN$)$

T
T
T

such that $x_1 \neq x_2$ and $y \supseteq \{x_1, x_2\}\}$ and

z is also such that for all $v \in \{y \subseteq D$: for some $x_1, x_2 \in \bigcup f($2 PLURAL MAN$)$

T
T
T

such that $x_1 \neq x_2$ and $y \supseteq \{x_1, x_2\}$,

then, for such a z, if $z \leq v$, then $z = v.\}$

= (since the pairs $\{x_1, x_2\}$ are precisely the minimal elements of the set in question) $\{\{x_1, x_2\}: x_1 \neq x_2$ and $x_1, x_2 \in \bigcup f$ (2 PLURAL MAN)$\}$.

T
T
T

On the other hand, the <u>external</u> sense of the English noun phrase

(11) Exactly two men

is represented in SYN$^{TR}_{E}$nglish as

(11') EXCT-E 2 PLURAL MAN

and is accorded the interpretation

(11'') f(EXCT-E 2PLURAL MAN) =

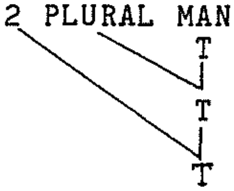
f(2 PLURAL MAN) $\cap$ f(UB-E 2PLURAL MAN) =

(letting A = $\{y \subseteq D$: for some w such that $w \cap$ f(2 PLURAL MAN) = $\emptyset$,

and for some $x_1, x_2 \in \bigcup$f(2 PLURAL MAN) such that

$x_1 \neq x_2$ and $y = \{x_1, x_2\} \cup w\}$) =

$A \cap \{y \subseteq D$: for some $w \subseteq D$: such that $w \cap \bigcup f(2\ \mathrm{PLURAL\ MAN}) = \emptyset$,



and for some $z \in A$[for all $v \in A$, (if $z \leq v$, then $z = v$)

and $y \leq z \cup w\}$

$= A$.

2.3.2.4 Logical Semantic Axioms for Modifiers and Relations on Sentences of SYN_L^{TR}

Logical Modifiers on Sentences

In this section we define the interpretations of two types of syntactic representations for each of the sentential connectives: conjunction, inclusive-disjunction, exclusive-disjunction, negation, implication, and equivalence, and for the modal operators possibility and necessity. The first type of representation will be as a logical relation on sentences; the second as a modifier on sentences.

Essentially, the difference between these two types of representations is this: a syntactic representation of a connective or of a modal operator as a modifier, when applied to sentences-as-assertions does not produce sentences, but rather produces thing-expressions of a more general kind; on the other hand, a syntactic representation of a connective or a modal operator as a relation, when applied to sentences-as-assertions, does produce sentences. Nonetheless, there is a simple relationship that holds between the interpretations of such logical expressions as modifiers and as relations, which will be stated below.

Logical Relation Expressions on Sentences

Sentential connectives and modal operators on sentences can be syntactically represented as relation-expressions on sentences and interpreted by logical semantic axioms (18) and (19) (to follow), or as modifiers on sentences, and interpreted later by logical semantic axioms (20) and (21).

Let r, r_1, r_2 be the major relation expressions of the sentences a, a_1, a_2 respectively in L.S.A 18, 19, and 20.

L.S.A.(18.1) $f(\text{INCDISJ}_{R}\ A_{R^1}\ D_{R^2}) = \{(f[r_1], P_{(D,f)}[a_1]) \times (f[r_2], P_{(D,f)}[a_2]) \mid a_1, a_2$ are sentences of L', and $f[a_1] \neq \phi$ or $f[a_2] \neq \phi\}$

L.S.A.(18.2 $f(\text{CONJ}_{R}\ A_{R^1}\ D_{R^2}) = \{(f[r_1], P_{(D,f)}[a_1]) \times (f[r_2], P_{(D,f)}[a_2]) \mid a_1, a_2$ are sentences of L', and $f[a_1] \neq \phi$ and $f[a_2] \neq \phi\}$

L.S.A.(18.3) $f(\text{NEG}_{R}\ A_{R^1}) = \{(f[r], P_{(D,f)}[a]) \mid a$ is a sentence of L', and $f[a] = \phi\}$

L.S.A.(18.4) $f(\text{IMPL}_{R}\ A_{R^1}\ D_{R^2}) = \{(f[r_1], P_{(D,f}[a_1]) \times (f[r_2], P_{(D,f)}[a_2]) \mid a_1, a_2$ are sentences of L', and $f[a_1] = \phi$ or $f[a_2] \neq \phi\}$

L.S.A.(18.5) $f(\text{EQUIV}_{R}\ A_{R^1}\ D_{R^2}) = \{(f[r_1], P_{(D,f)}[a_1]) \times (f[r_2], P_{(D,f)}[a_2]) \mid$ a_1, a_2 are sentences of L', and $f[a_1] \neq \phi$ if and only if $f[a_2] \neq \phi\}$

L.S.A.(18.6) $f(\text{EXC DISJ}_{R}\ A_{R^1}\ D_{R^2}] = \{(f[r_1], P_{(D,f)}[a_1]) \times (f[r_2], P_{(D,f)}[a_2]) \mid$ a_1, a_2 are sentences of L', and $f[a_1] \neq \phi$ if and only if $f[a_2] = \phi\}$

Parallel to the case with sentential connectives, we can also syntactically represent modal operators as relation-expressions.

The interpretation of modal operators as relation-expressions is given in L.S.A.(19) below: the interpretation of modal operators as modifiers is given later in this section.

L.S.A.(19) Let a be a modifier that is a sentence. Let $H \in R$.[84] Then:

(19.1) $$f(\mathrm{POSS}_H A) = \{(f[r], P_{(D',f')}[a]) \mid f'[a] \neq \phi \text{, for some } (D',f') \in F$$

R

R^1

such that $(D',f')\,H\,(D,f)\}$

(19.2) $$f(\mathrm{NEC}_H A) = \{(f[r], P_{(D',f')}[a]) \mid f'[a] \neq \phi \text{, for all } (D',f') \in F$$

R

R^1

such that $(D',f')\,H\,(D,f)\}$

Logical Modifiers on Sentences

Sentential connectives and modal operators on sentences can also be syntactically represented as modifiers on sentences and

Note 84. Kripke [6] has shown that various definitions of "possibility", hence of "necessity", can be obtained by varying the algebraic structure on H. The set R of binary relations on interpretations contains, as elements, those binary relations H obtained by such variation of structure. The structures which Kripke explicitly examines are those where H is, respectively, reflexive, reflexive and symmetric, and reflexive, symmetric and transitive.

interpreted by logical semantic axioms (21) and (22) below.[85] Such representation as modifiers also makes it possible to easily distinguish between two senses of exclusive disjunction (L.S.A. (20.3.1) and (20.3.2) below, which we distinguish as "strong" and "weak", as illustrated in the following example:

(1) John will visit his friends or his relatives

In the strong sense of "or", John will visit either his friends who are not also his relatives or his relatives who are not also his friends, whereas in the weak sense of "or", John will visit his friends, who may or may not -- coincidentally -- also be his relatives, or will visit his relatives, who may or may not -- coincidentally -- also be his friends. In a dominant normal reading of (1), John's friends would be distinct from his relatives, so that the strong and weak senses of "or" as described here would coincide. However, there would still be possible readings of (1) which, while not dominant, would nonetheless be normal in which John's friends were not necessarily distinct from his relatives.

We represent the strong exclusive disjunction by the representational morphemes SEXCDISJ and weak exclusive disjunction by the representational morpheme WEXCDISJ, and interpret them respectively by L.S.A. (20.3.1) and (20.3.2).

Note 85. Clauses (21.4)-(21.6) make essential reference to a definition (of the NC transform) which is given later on page 279.

L.S.A.(20). Let a,b be sentences of SYN^{TR} in SAM-form (see pages 40 and 41). Then:

(20.1) $f(CONJ)[<f(\frac{a}{T}),f(\frac{b}{T})>] = \{x \cup y : x \in f(\frac{a}{T})\ \&\ y \in f(\frac{b}{T})\}$

(20.2) $f(INCDISJ)[[<f(\frac{a}{T}),f(\frac{b}{T})>] = f(\frac{a}{T}) \cup f(\frac{b}{T})$

(20.31) $f(SEXCDISJ)[<f(\frac{a}{T}),f(\frac{b}{T})>) = \{x - \bigcup f(\frac{b}{T}) : x \in f(\frac{a}{T})\} \cup \{x - \bigcup f(\frac{a}{T}) : \quad x \in f($

(20.32) $f(WEXCDISJ)[<f(\frac{a}{T}), f(\frac{b}{T})>] = \{x : x \in f(\frac{a}{T}) \cup f(\frac{b}{T})\ \&\ x \notin f(\frac{a}{T}) \cap f(\frac{b}{T})\}$

(20.4) $f(NEG)[f(\frac{a}{T})] = \begin{cases} \Phi, & \text{if } f(\frac{a}{T}) \neq \Phi \\ P(D,f)(\frac{a}{T})), & \text{if } f(\frac{a}{T}) = \Phi \end{cases}$ (f[r],

(20.5) $F(IMPL)[<f(\frac{a}{T}),f(\frac{b}{T}))>] = f(NEG(\frac{a}{T})) \cup f(\frac{b}{T}))$

(20.6) $f(EQUIV)[<f(\frac{a}{T})f(\frac{b}{T})>] = \{x \cup y : x \in f(IMPL)[<f(\frac{a}{T}),f(\frac{b}{T})>]\ \&\ y \in f(IMPL)[<f(\frac{a}{T}),f(\frac{b}{T})>]\}$

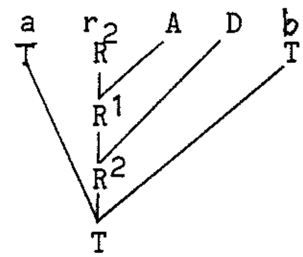
L.S.A.(21). Let a be a sentence that is a modifier. Let $H \in R$. Then:

(21.1) $f(POSS_H)[f(\frac{a}{T})] = \{k : k$ is an event particular of $f(\frac{a}{T})$ in some $(D',f') \in F$ such that $(D',f')H(D,f)\}$

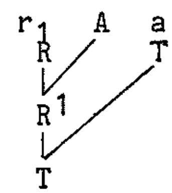
(21.2) $f(NEC_H)[f(\underset{T}{a})]=\{k{:}k$ is an event particular of $f(\underset{T}{a})$ in every $\langle D',f'\rangle \in F$ such that $\langle D',f'\rangle H \langle D,f\rangle\}$

The following relationship between the two treatments of sentential connectives can easily be proved:

Let a,b be sentences of SYN_L^{TR} in SAM-form. Then, letting r_1 = NEG, and letting r_2 = CONJ, INCDISJ, SEXC_DISJ, WEXCDISJ, IMPL, or EQUIV, then

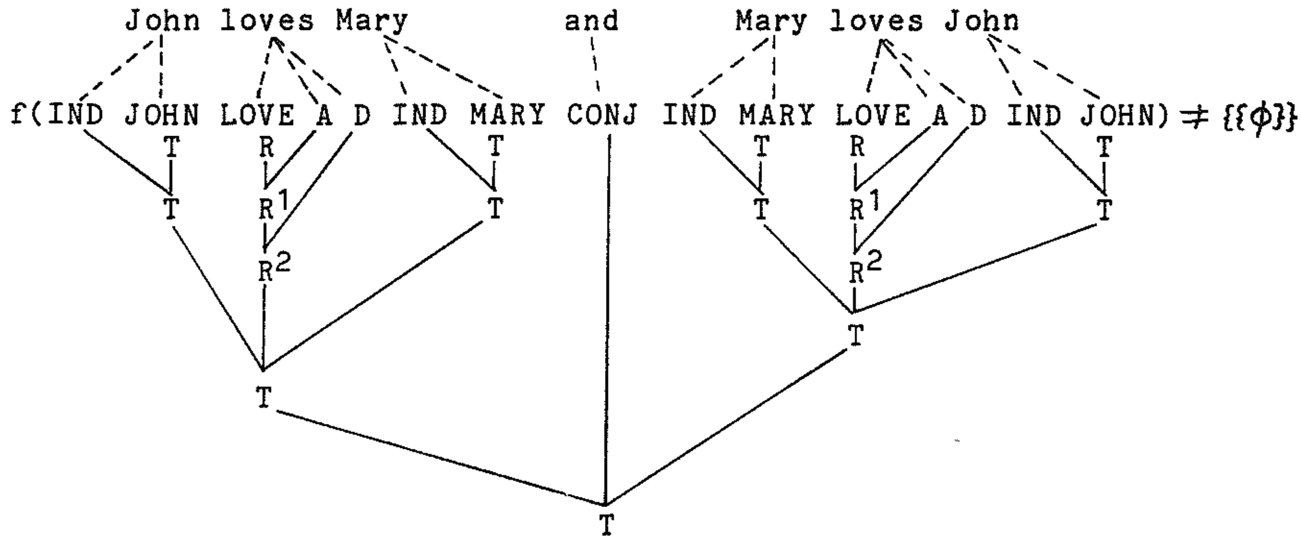


is true in $\langle D,f\rangle$ if and only if there is a $y \in f(r_2)[\langle (f(\underset{T}{a}),f(\underset{T}{b})\rangle]$ & $y \neq \{\{\emptyset\}\}$, and

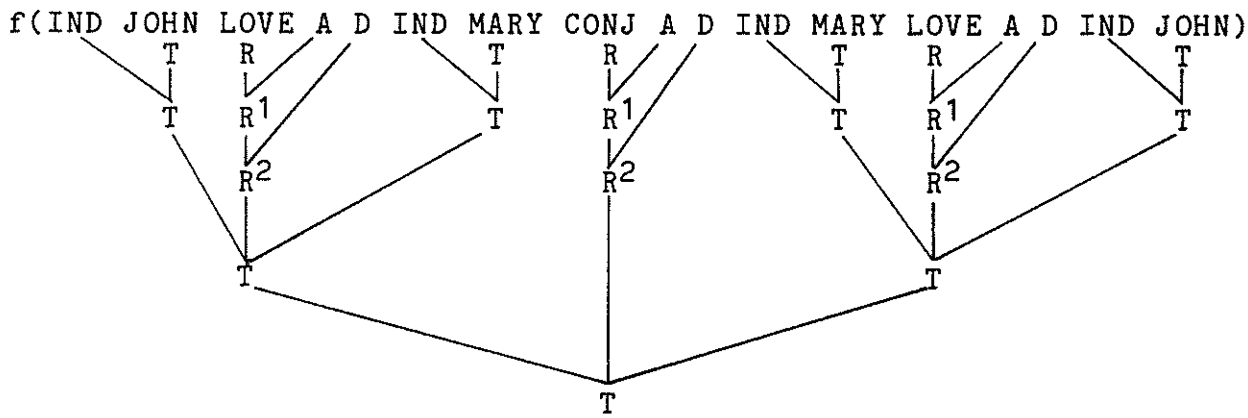


is true in $\langle D,f\rangle$ if and only if there is a $y \in f(r_1)[(NC(\underset{T}{a}))]$ & $y \neq \{\{\emptyset\}\})$

As an example of this relationship, the two following equivalences can be shown to hold on all interpretations $\langle D,f\rangle \in F$, where $\langle F,V,R\rangle$ is any semantic theory satisfying the logical semantic axioms (1)-(21).



if and only if



$\neq \{\{\emptyset\}\}$

The Comparative as a Logical Relation-Expression

The structure of comparatives is dictated by the kinds of intuitive entailments that comparatives enter into. That structure should, at the least, be conditioned by the requirement that it yield, for example, certain entailments among the following sentences:

(1) John is tall

(2) Bill is tall

(3) Bill is taller than John

(4) John is taller than Henry

(5) Bill is taller than Henry

(6) Henry is not taller than Bill

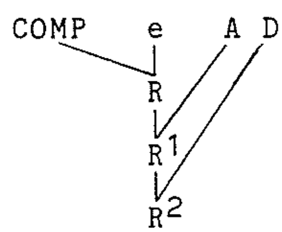
(7) Bill is not taller than Bill

(8) Bill is tall more than John is tall

The requisite intuitive entailments among (1)-(8) are: (1) and (3) should entail (2); (3) should entail neither (1) nor (2); (3) and (4) should entail (5); (5) should entail (6), but (6) should not entail (5); (3) and (8) should inter-entail each other.

We will formulate a semantic axiom below that yields the above entailments under the dominant normal readings of (1)-(8). We make the following initial observation: Let <D,f> be an interpretation. By L.S.A.(4), $f(TALL_T)$ is a set of subsets of D. Let X be a set of ordered pairs of elements of D that provisionally interpret some expression of $SYN^{TR}_{English}$ that has "is taller than" as its English analogue. Then the above entailment requirements mean, in part, that if x and y are elements of D such that $x \in \bigcup f(TALL_T)$ and $\langle x,y \rangle \in X$, then $y \in \bigcup f(TALL_T)$; that is, the X-images of elements of $\bigcup f(TALL)$ are elements of $\bigcup f(TALL)$ or, more simply put, $\bigcup f(TALL)$ is closed relative to X.

We want to have a set with the structure of X above be the denotation of a two-place relation-expression of SYN^{TR}_{L} which is constructed out of a thing-expression (intuitively expressing the absolute form upon which the comparative is based). For this purpose, we introduce a logical morpheme COMP which, syntactically, when applied to an arbitrary thing-expression e, converts it into the two-place relation expression:



Finally, the semantic axiom which enables us to induce the requisite entailments among (1)-(8) is the following:

L.S.A.(22). Let t be a tense of SYN_L^{TR} or the empty expression. Then

(i) If y ∈ ⋃f(a t) and if ⟨x,y⟩ ∈ f(COMP a A D t)

T

R

R^1

R^2

R^2

then x ∈ ⋃f(a t)

T

(ii) f(COMP a A D t) is antisymmetric,

R

R^1

R^2

R^2

transitive, and irreflexive.

(iii) f (COMP a A D)

R

R^1

R^2

= f (a COMP T A D)

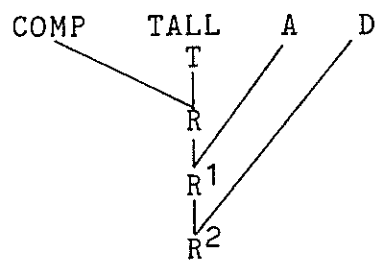
R

R^1

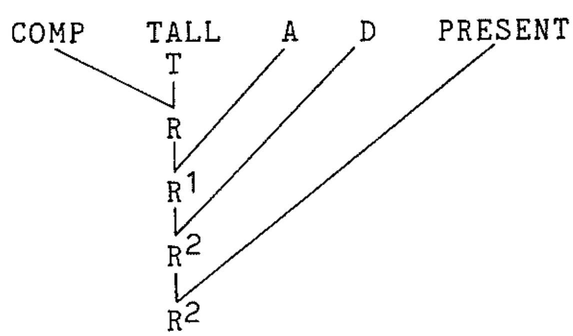
R^2

R^2

Note that the English analogue of the expression

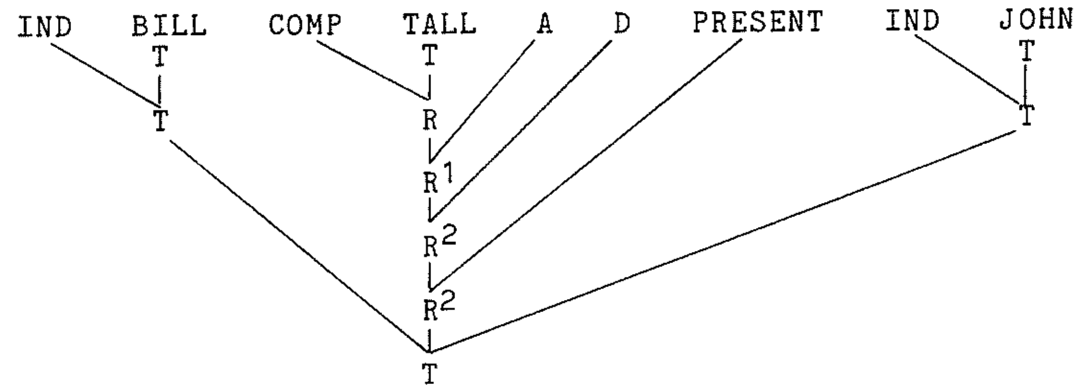


of $\mathrm{SYN}^{TR}_{English}$ is "taller than," rather than "is taller than." This leaves open the possibility of tensing "taller than" as we please, to form, say, "is taller than," "was taller than," "will be taller than," etc. For example, "is taller than" would be syntactically represented in $\mathrm{SYN}^{TR}_{English}$ by:



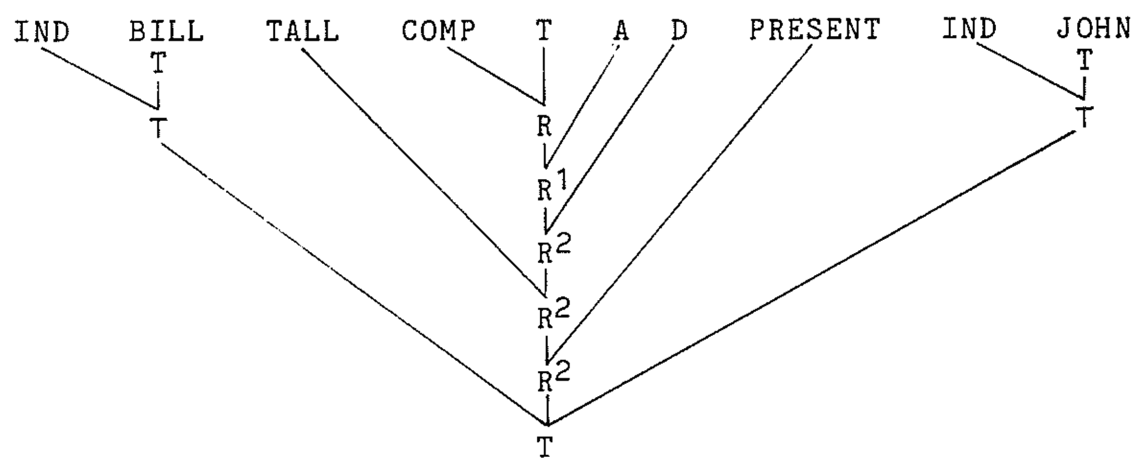
Thus under one of the normal readings of (3), (3) would receive the syntactic representation (3'):

(3')



Under one of the normal readings of (8), (8) would receive the syntactic representation

(8')

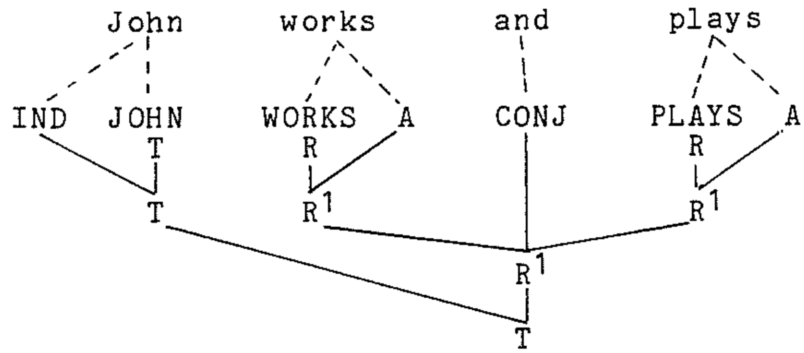


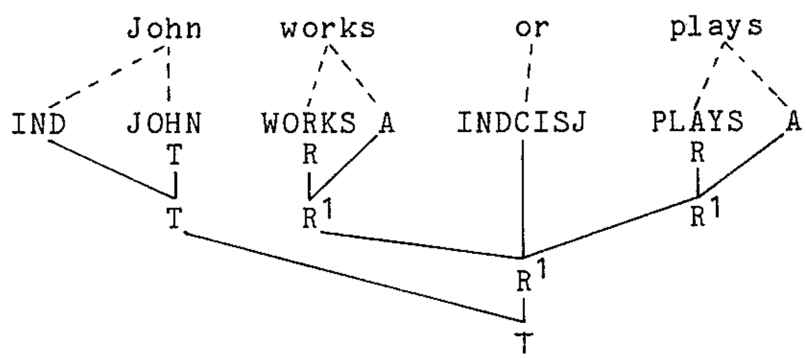
which, under L.S.A.22 (iii) above, is equivalent to (3').

In both (3') and (8') the copula "is" is represented as the present-tense morpheme PRESENT.

Simple Logical Modifiers on Relation-Expressions

In this section we formulate an axiom which defines the interpretations of several special logical modifiers on relation-expressions. We use such modifiers to form compound relation expressions of SYN_L^{TR} that are syntactic representations of word-strings like "John works and plays" or "John works or plays". The interpretation of such compound relation expressions turns on the semantic axioms governing the logical relation morphemes. In particular, CONJ and INCDISJ are two such morphemes that enter into normal readings of the above two word-strings, as in the following examples:





The following semantic axiom interprets the two modifiers CONJ and INCDISJ in their application to relation-expressions:

L.S.A.(23) Let a,b be modifiers. Then

(i) $f(\text{CONJ})[\langle a_{R^n}, b_{R^n}\rangle] = f(a_{R^n}) \cap f(b_{R^n})$

(ii) $f(\text{INCDISJ})[\langle a_{R^n}, b_{R^n}\rangle] = f(a_{R^n}) \cup f(b_{R^n})$

2.3.2.5 Some Equivalences

In this section I present some consequences of our logical semantic axioms pertaining to syntactic inter-relationships among semantically equivalent sentences of L' involving determiner structure. In order to describe these syntactic relationships in a general way, I employ the notion of a "transform" of an L'-sentence, which is another L'-sentence that is semantically equivalent to the previous sentence and which is obtainable from the given sentence by some effective, i.e., mechanically executable, procedure.

Description of the transform requires reference to the earlier given four-fold classification of thing-expressions of L' (see pages), which we summarize here for easy reference.

Recall: (i) a thing-expression of SYN_L^{TR} is definite, lower-bounded, upper-bounded, or doubly-bounded relative to a semantic theory s = <F,V,R> just in case it denotes, respectively, a definite set, a lower-bounded set, an upper-bounded set, or a doubly-bounded set in every interpretation $\langle D,f\rangle \in F$; (ii) a SYN_L^{TR}-sentence $r^m(a_1 \ \ldots \ a_m)\ q$ is non-upper-bounded relative to a semantic theory s just in case each of its major thing-expressions $a_1,\ldots,a_m$ is either a definite thing-expression or a lower-bounded thing-expression relative to s; (iii) a SYN_L^{TR}-sentence is upper-bounded relative to a semantic theory s just in case it is not non-upper-bounded relative to s, i.e., just in case at least one of its major thing-expressions is upper-bounded or doubly-bounded relative to s; and (iv) if s is a refinement of the minimal semantic theory s_o satisfying L.S.A.(1)-L.S.A.(31), then we drop the reference to s, and speak simply of definite, lower-bounded, upper-bounded, and doubly-bounded thing-expressions, and of upper-bounded and non-upper-bounded sentences, without relativization to s.

The essential rquirements for a transform are (i) it should convert, i.e., "transform," a SYN_L^{TR} sentence a into (a) SYN_L^{TR}-sentence T(a) by a syntactic procedure that is effective, i.e., mechanically executable. A transform T is said to preserve formal equivalence relative to a semantic theory s if and only if for all sentences a' of SYN_L^{TR}, a' is provably equivalent to T(a') relative to the logical semantic axioms of this chapter, in symbols, a' ≡ T(a)). A transform T is said to preserve intuitive equivalence relative to the semantic theory s if, in addition, for all sentences a of L, if a' is the syntactic component of a normal reading r_1 of a, and b' is the syntactic component of a normal reading r_2 of b, such that b' ≡ T(a'), and such that r_1 and r_2 have s as their common semantic theory, then a is intuitively equivalent to b if and only if a' is provably equivalent to b' under the logical semantic axioms defining s.[86]

Note 86. The question of whether any defined transform actually preserves equivalence in this sense cannot be proved in any rigorous sense, but is an empirical hypothesis which can be confirmed or disconfirmed. At this early point in the examination of such issues we can only attempt to render this hypothesis at least plausible by consideration of a variety of cases from English.

Another point is worth noting: For certain TR-languages L, there may be no word-string e such that a given SYN_L^{TR} sentence or its transform is a syntactic representation of a homologous normal reading of e, owing to the fact that L may be limited in logical expressiveness. Generally speaking, the logical expressiveness of SYN_L^{TR} is very rich, providing for all possible logically normal readings of word-strings of arbitrary TR-languages, so that any given TR language L will inevitably lack word-strings that express a certain expression e' of SYN_L^{TR} in the sense that there would exist a homologous normal reading whose syntactic component is e'; in particular, a given TR-language L may be such that a certain SYN_L^{TR}-sentence e' may be the syntactic representation of an L-sentence e within a homologous normal reading of e, whereas the T transform b' of a' may not be the syntactic representation of any L-sentence b within any normal reading of b. Thus, L may lack expressive power to the extent that such a requisite b were not a word-string of the language. Roughly put, it may well be the case that T(a') cannot be expressed in L. For that matter, if T is 1-1, hence has an inverse, designatable as T^{-1}, a given TR-language L may well be such that, while you can express b in L by b', say, you cannot express T^{-1}(b') in L by any word-string (that has a normal reading). Different TR-languages may well evolve only certain upper-bounded or only certain non-upper-bounded forms. The extent to which this occurs is largely unknown at present. In particular, it is unclear as to whether English is relatively rich or impoverished in logical expressiveness in this regard. On the other hand, there appear to be adequate English forms for expressing $T(a')$, for English-expressible a, and for expressing $T^{-1}(b')$, for English-expressible b', even though, as will be noted in the examples, some of those English-expressible assertions are somewhat stilted.

In the remainder of this section I discuss two transforms involving determiner structure. The first of these eliminates doubly bounded thing-expressions, and is called the Doubly Bounded Elimination Transform (DBET). The DBET Transform preserves formal equivalence relative to the semantic theory of this chapter, but does not preserve intuitive equivalence relative to the semantic theory. The second transform eliminates upper bounded thing-expressions, and is called the Upper Bounded Elimination Transform (UBET), and preserves both formal equivalence and intuitive equivalence relative to the semantic theory of this chapter.

Immediate Subtransforms of the BCT transform: The Doubly-Bounded Elimination Transform DBET, and the Upper Bound Elimination Transform UBET.

The BCT transform is to be defined in terms of two other transforms: (1) The doubly-bounded elimination transform (DBET), which replaces a sentence to which it is applied by an equivalent one which has no major doubly-bounded thing-expressions, thereby converting a SYN_L^{TR}-sentence containing at most definite, lower-bounded, upper-bounded, and doubly-bounded thing-expressions into a SYN_L^{TR}-sentence containing at most definite, lower-bounded, and upper-bounded major thing-expressions; and (2) The upper bounded elimination transform (UBET), which replaces a sentence containing at most definite, lower-bounded, and upper-bounded major thing-expressions to which it is applied by an equivalent one which has no major upper-bounded thing-expressions, thereby converting a SYN_L^{TR}-sentence containing no major doubly-bounded-thing expressions into one containing at most definite and lower-bounded thing-expressions. The BCT transform is then defined as the result of applying first the DBET transform to eliminate all major doubly bounded thing-expressions followed by applying the UBET transform to eliminate therefrom all major upper-bounded expressions, thereby converting a sentence of SYN_L^{TR} into an equivalent one containing only definite and lower-bounded major thing-expressions, which is precisely the form required by L.S.A.(8.1). In this manner all sentences of SYN_L^{TR} receive a denotation. That is, if e is an upper-bounded sentence of SYN_L^{TR}, then BCT converts e into a non-upper-bounded sentence BCT(e) = UBET (DBET (e)) of SYN_L^{TR}.

As remarked earlier, the BCT transform as defined here is not known to be total, that is, it is not known whether it applies to all upper-bounded sentences of SYN_L^{TR}. This limitation derives from the fact that the DBET transform, which enters into the construction of the BCT transform, is not known to be total.

Several points should be noted. The first is procedural; the second is theoretical:

(1) While the above procedure indirectly defines the denotation of an upper-bounded sentence of SYN_L^{TR} to be that directly assigned to its BCT transform, we could, alternatively, have proceeded directly by having an analogue of the UBET transform operate on the truth clause itself. This latter transform would then operate on the translations of SYN_L^{TR}-sentences into the semantic metalanguage rather than on the sentences of SYN_L^{TR} itself, and one could then derive as a consequence of this definition that each SYN_L^{TR} sentence and its BCT transform had the same denotation. Clearly nothing would be gained by the latter approach; indeed, the former, which we officially adopt, is much more perspicuous, since the syntactic structure of SYN_L^{TR}-sentences is precisely described whereas the syntactic structure of their translations into the meta-language of SYN_L^{TR} is not, owing to the fact that that meta-language is not formalized.

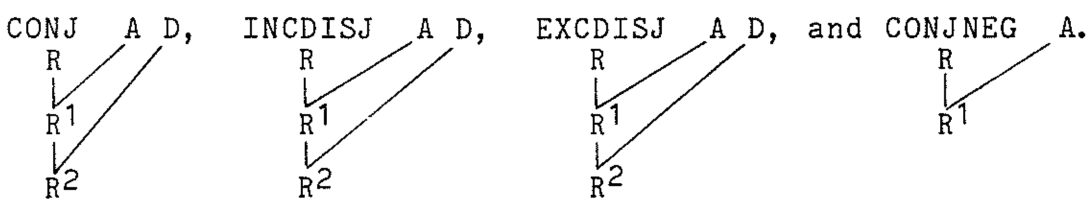
(2) For certain TR-languages L, there may be no word-string e such that a given SYN_L^{TR} sentence or its BCT transform is a syntactic representation of a homologous normal reading of e, owing

to the fact that L may be limited in logical expressiveness. Generally speaking, the logical expressiveness of SYN_L^{TR} is very rich, providing for all possible logically normal readings of word-strings of arbitrary TR-languages, so that any given TR language L will inevitably lack word-strings that express a certain expression e' of SYN_L^{TR} in the sense that there would exist a homologous normal reading whose syntactic component is e'; in particular, a given TR-language L may be such that a certain SYN_L^{TR}-sentence e' may be the syntactic representation of an L-sentence e within a homologous normal reading of e, whereas the BCT transform b' of a' may not be the syntactic representation of any L-sentence b within any normal reading of b. Thus, L may lack expressive power to the extent that such a requisite b were not a word-string of the language. Roughly put, it may well be the case that BCT(a') cannot be expressed in L. For that matter, since it will turn out that BCT is 1-1, hence has an inverse, designatable as BCT^{-1}, a given TR-language L may well be such that, while you can express b in L by b', say, you cannot express BCT^{-1}(b') in L by any word-string (that has a normal reading). Different TR-languages may well evolve only certain upper-bounded or only certain non-upper-bounded forms. The extent to which this occurs is largely unknown at present. In particular, it is unclear as to whether English is relatively rich or impoverished in logical expressiveness in this regard. On the other hand, there appear to be adequate English forms for expressing BCT(a'), for English-expressible a, and for expressing BCT^{-1}(b'), for English-expressible b', even though, as will be noted in the examples, some of those English-expressible assertions are somewhat stilted.

The DBET Transform

Distribution Properties of Binary Logical Modifiers

The elimination of doubly-bounded thing-expressions of sentences of SYN_L^{TR} is accomplished essentially by the conversion of doubly-bounded thing-expressions into thing-expressions involving only lower and upper-bounded sub-thing-expressions connected by a binary logical modifier, a process called <u>bound conversion</u>, followed by the re-casting of that binary logical modifier on those thing-expressions into a binary logical relation over associated sentences, a process called <u>modifier distribution</u> wherein the binary logical modifiers are supplanted by binary logical relations on sentences. The logical binary modifiers on thing-expressions discussed here are CONJ, INCDISJ, EXCDISJ, and CONJNEG. The binary logical relations on sentences which supplant them are, respectively,



We call a SYN_L^{TR}-sentence b obtained from a given SYN_L^{TR}-sentence a by replacing binary logical modifiers on thing-expressions by binary logical relations on sentences, <u>the modifier distribution of a</u>. In this subsection we discuss some of the general properties governing modifier distributions of SYN_L^{TR}-sentences.

In the following examples, (1.1), (2.1), and (3.1) are the respective modifier distributions of (1), (2), and (3):

(1) At least two men and at most two men love Mary

(1.1) At least two men love Mary and at most two men love Mary

(2) At least two men or at most two men love Mary

(2.1) At least two men love Mary or at most two men love Mary.

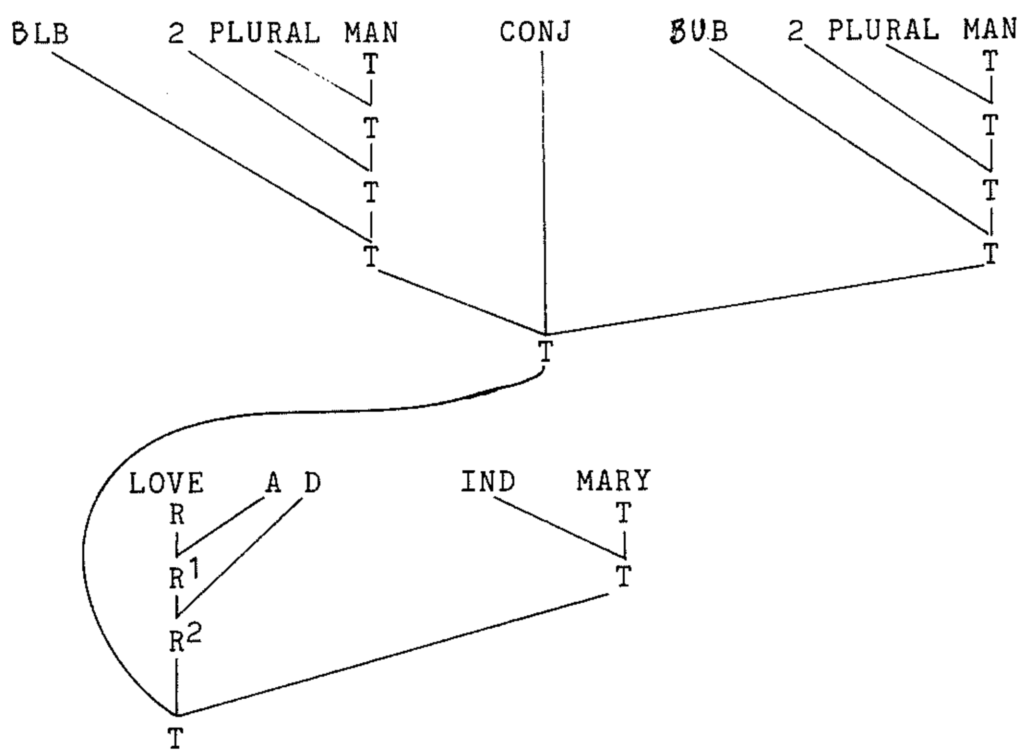
(3) At least two men but not John love Mary

(3.1) At least two men love Mary and it is false that John loves Mary.

The noun phrase "at least two men and at most two men" in (1) is an English analogue of a doubly-bounded thing-expression in SYN_L^{TR}; the noun phrase "at least two men or at most two men" in (2) is an English analogue of a lower-bounded thing-expression in SYN_L^{TR}, as is the noun phrase "at least two men but not John" in (3).

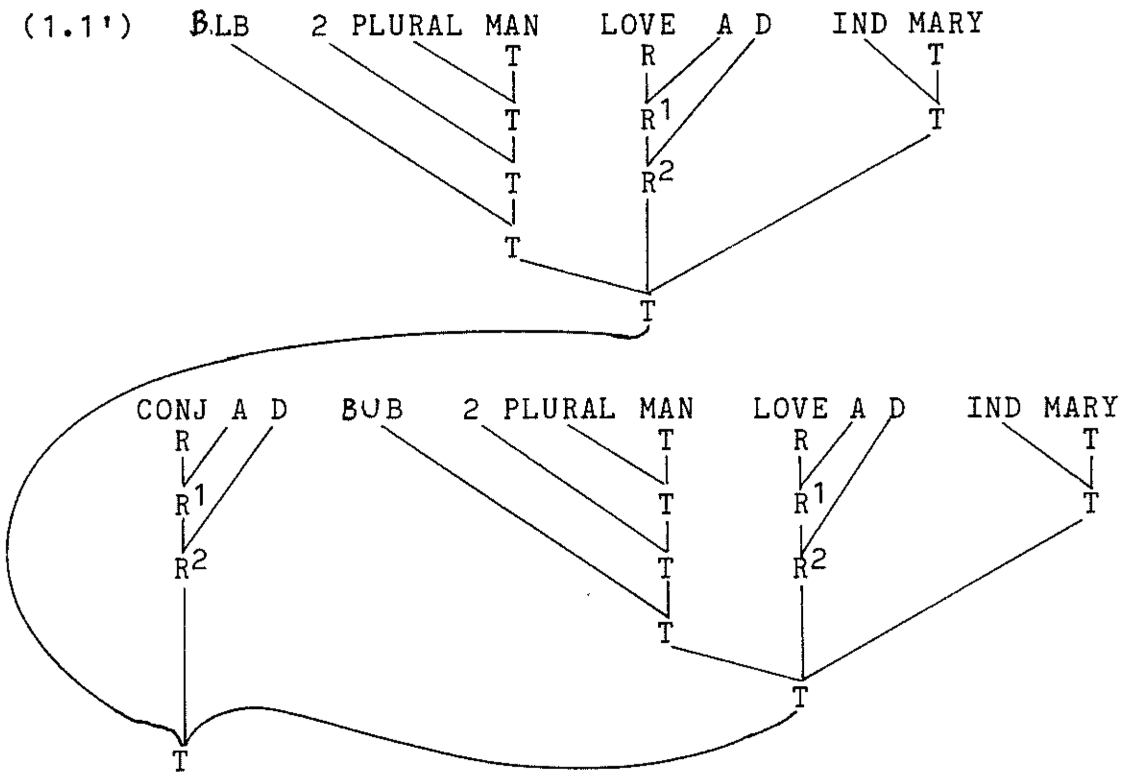
The syntactic representation of (1) is

(1')

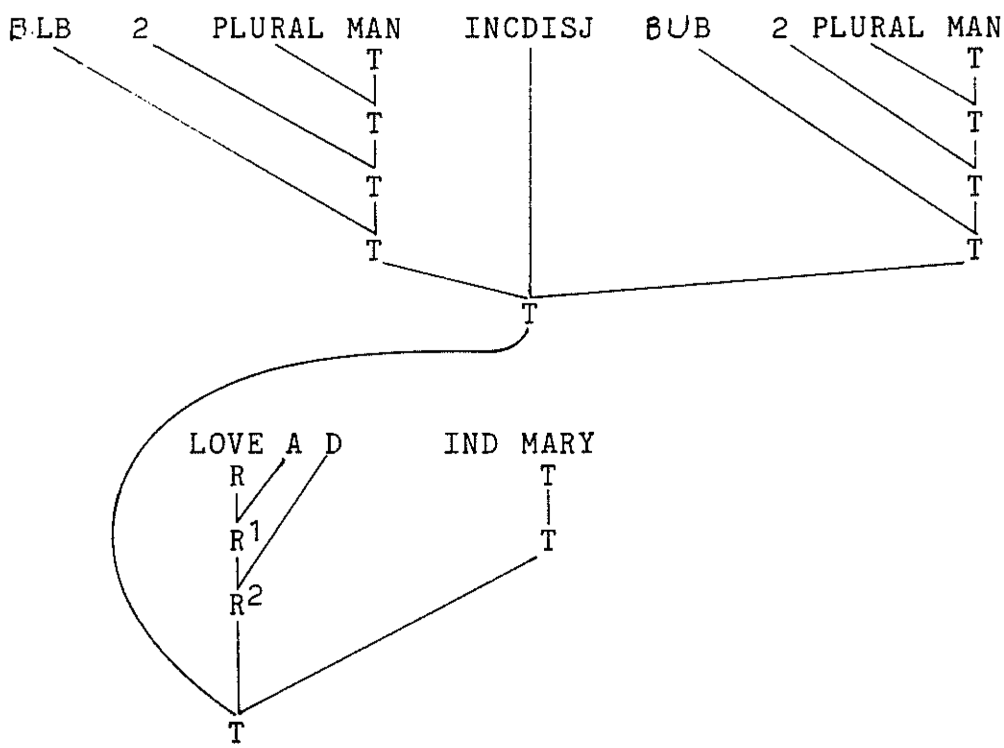


The syntactic representations of (1.1) and (2) are respectively:

(1.1')

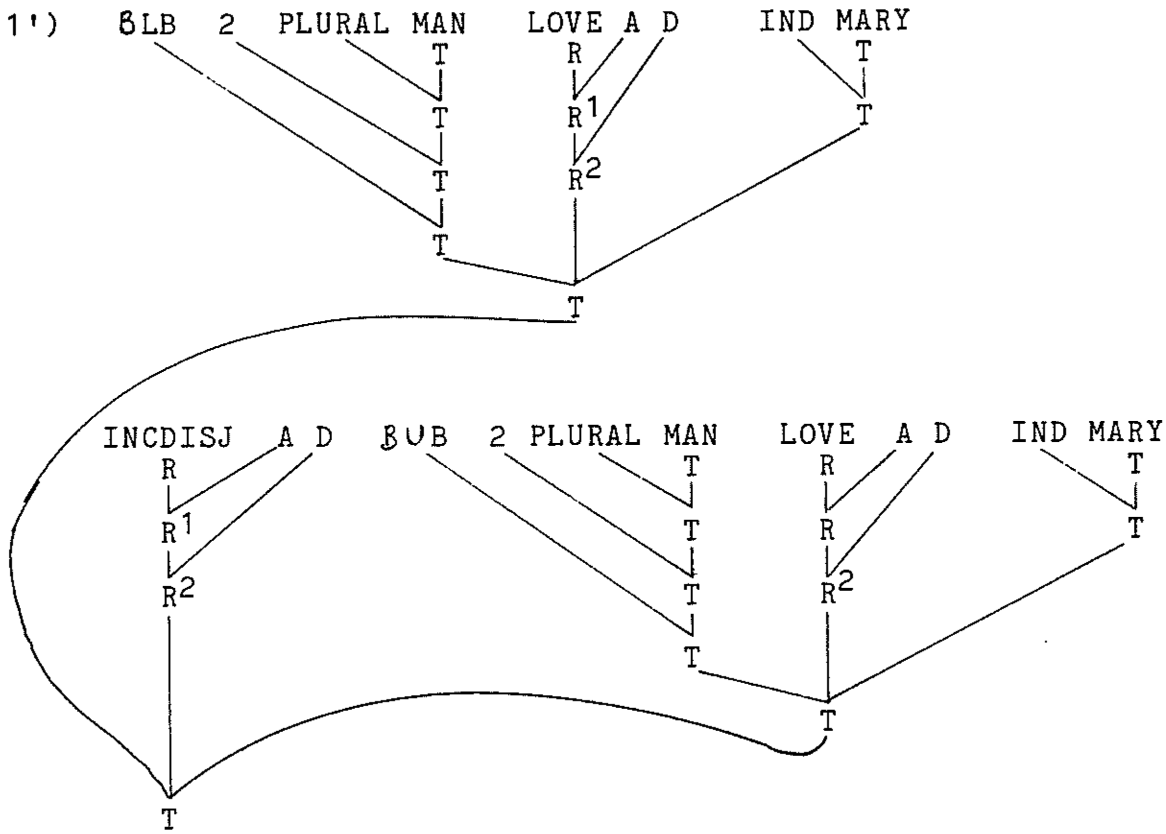


(2')

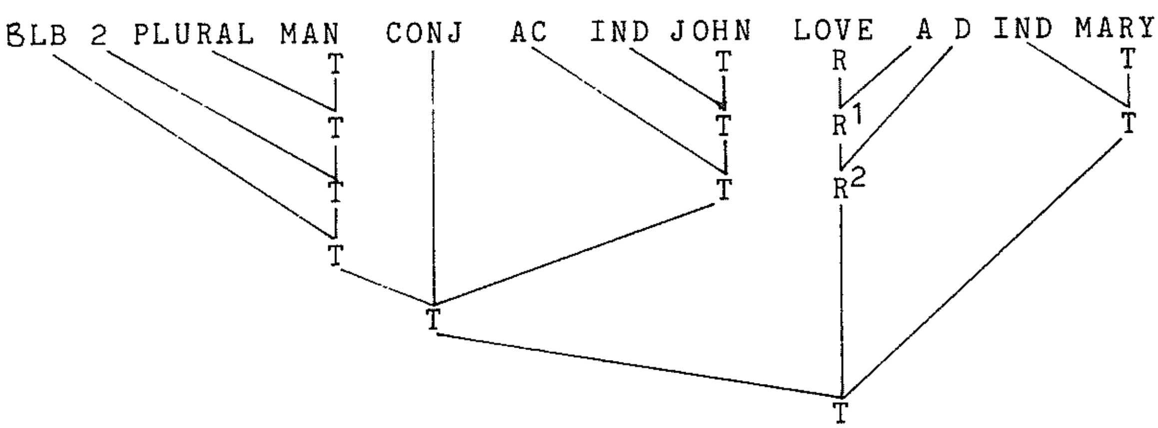


The syntactic representations of (2.1) and (3) are respectively:

(2.1')

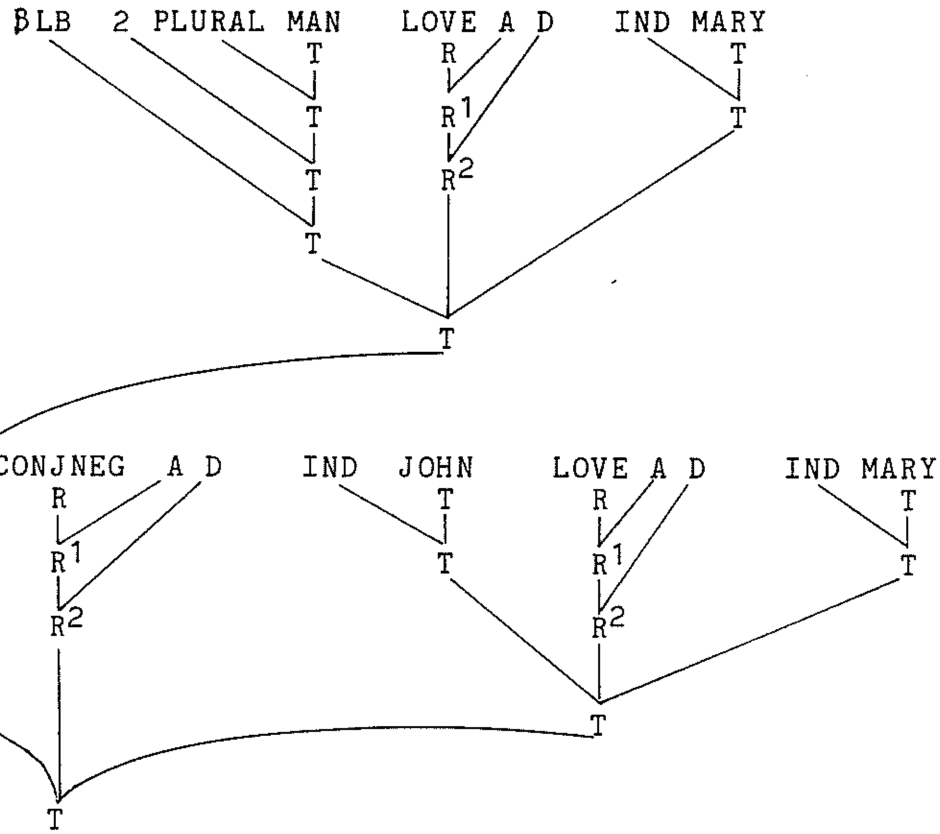


(3')



Finally, the syntactic representation of (3.1) is:

(3.1')



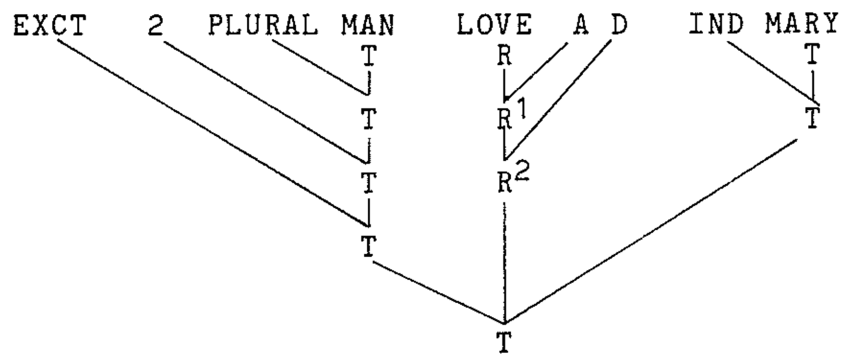
Under the semantic axioms (1)-(23), (1') and (1.1') are equivalent, (2') and (2.1') are equivalent, and (3') and (3.1') are equivalent.

Let us now turn to the consideration of bound conversion. The following sentence (4) becomes converted to (1) through <u>bound conversion</u>:

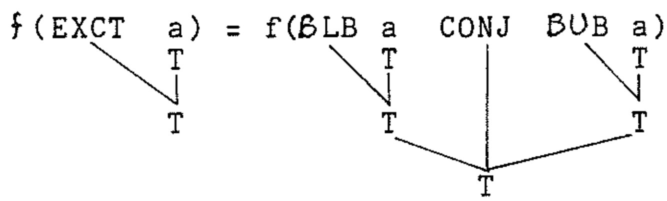
(4) Exactly two men love Mary

and has the dominant normal reading:

(4')



The following is a consequence of our logical axioms: for every modifier a,



By this consequence, we have that (1') and (4') are equivalent, hence (1'), (1.1') and (4) are equivalent.

In this connection, we can consider also an example involving another sort of doubly-bounded expression, namely that occurring in:

(5) Between 5 and 12 men love Mary

and the bounded conversion (6) of (5):

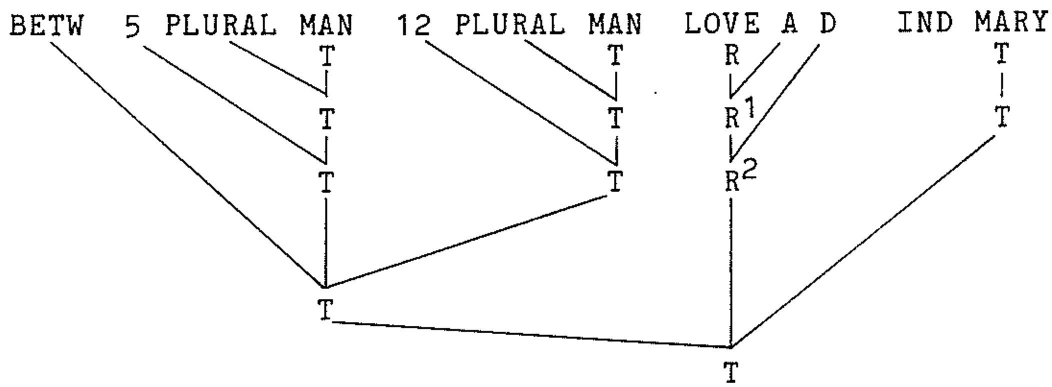
(6) at least 5 men and at most 12 men love Mary

and the modifier distribution (6.1) of (6):

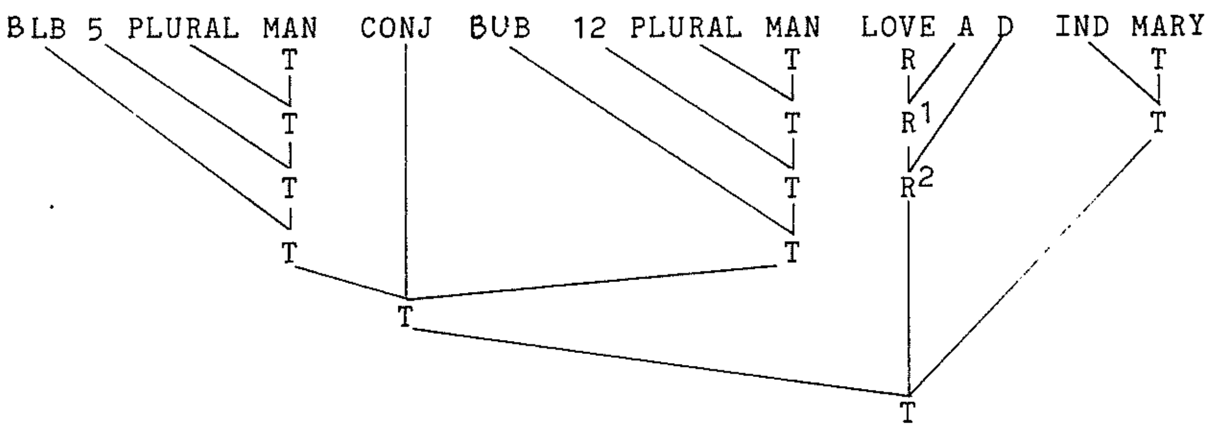
(6.1) at least 5 men love Mary and at most 12 men love Mary.

which have the respective syntactic representation in their dominant normal readings:

(5')

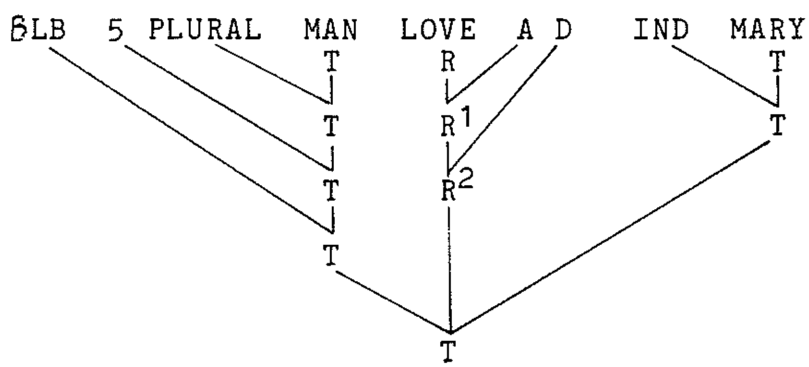


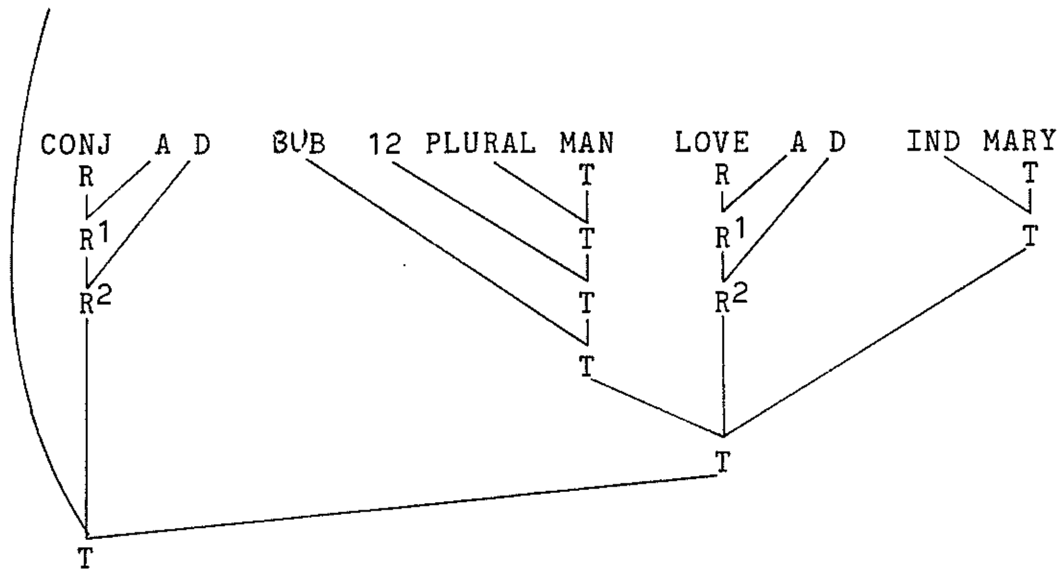
(6')



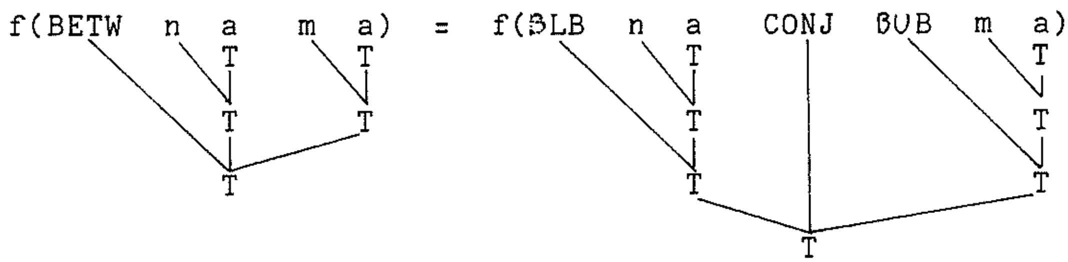
which has the same denotation as

(6.1')





Consider L.S.A. :



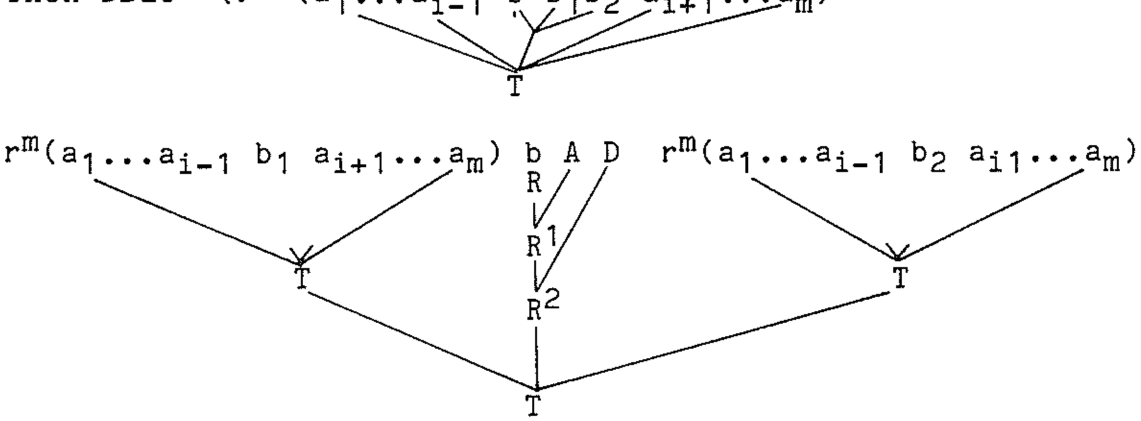
By this axiom, (5') and (6') are equivalent and, by the other axioms of this chapter, (6') and (6.1') are also equivalent; thus, (5'), (6'), and (6.1') are all equivalent.

A First Approximation DBET° to the DBET Transform

Let us attempt to approximate the DBET transform by the transform DBET° defined as follows:

Let b be a binary logical modifier on thing-expressions. Let $a_1,.., a_{i-1}, a_{i+1},...a_m, b_1,b_2$ be thing-expressions.

Then DBET° (r^m ($a_1...a_{i-1}$ b b_1b_2 $a_{i+1}...a_m$) =



It can easily be seen that, applying this definition of DBET° to the earlier sentences (1'), (1.1'), (2'), (2.1'), (3'), (3.1'), (5'), and (5.1'), we have: (1.1') = DBET°(1'), (2.1') = DBET°(2'), (3.1') = DBET°(3'), and (5.1') = DBET°(5'). Moreover, our intent that DBET° be equivalence preserving is fulfilled. However, DBET° is still too permissive as can be seen in the next series of examples.[90]

In the following series of examples from English, we will, as usual, write, where a and b are English sentences, the expression a≡b to mean that a and b inter-entail each other

---

Note 70. One could readily utilize DBET° to handle phrases like "John and Mary or Bill," "John and Mary or Bill but not Henry," etc. by iterating the application of DBET°.

under their dominant normal readings, and write $a \supset b$ to mean that a entails b under their dominant normal readings. Accordingly $a \equiv b$ just in case $a \supset b$ and $b \supset a$. Also, we write $a \not\equiv b$ and $a \not\supset b$ to mean, respectively, that it is false that $a \equiv b$, and that it is false that $a \supset b$.

The following sets of examples are intended to illustrate the cause of failures of modifier distribution and, thereby, the inadequacy of $DBET^{o}$.

Consider:

(1) Exactly two men love Mary

(1.1) At least and at most two men love Mary

(1.2) At least two men and at most two men love Mary

(1.3) At least two men love Mary and
at most two men love Mary

(2) Mary loves exactly two men

(2.1 Mary loves at least and at most two men

(2.2) Mary loves at least two men and at most two men

(2.3) Mary loves at least two men and
Mary loves at most two men

(3) Exactly two men love many women

(3.1) At least and at most two men love many women

(3.2) At least two men and at most two men love many women

(3.3) At least two men love many women and
at most two men love many women

(4) Many women love exactly two men

(4.1) Many women love at least and at most two men

(4.2) Many women love at least two men and at most two men

(4.3) Many women love at least two men and
many women love at most two men

We note that (1)$\equiv$(1.1)$\equiv$(1.2)$\equiv$(1.3), (2)$\equiv$(2.1)$\equiv$(2.2)$\equiv$(2.3), and that (3)$\equiv$(3.1)$\equiv$(3.2)$\equiv$(3.3), whereas while (4)$\equiv$(4.1)$\equiv$(4.2), (4)$\not\equiv$(4.3); in particular (4)$\supset$(4.3) but (4.3)$\not\supset$(4).

It is at (1.3), (2.3), (3.3), and (4.3) where the logical modifiers on thing-expressions become converted to logical relations (i.e., connectives) on sentences, and it is at that point where the equivalences break down, if at all. The root of the break-down in the transition from (4.2) to (4.3), as opposed to the transition from (3.2) to (3.3), is the same as that underlying the failure of the operation of the permutation of determiner phrases to preserve equivalence among the sentences containing them.

Let us consider a more complex example:

(5) Exactly two women love exactly two men

(5.1) Exactly two women love at least and at most two men

(5.2) Exactly two women love at least two men and at most two men

(5.3) Exactly two women love at least two men and exactly two women love at most two men

(5.4) At least and at most two women love at least two men and at least and at most two women love at most two men

(5.5) At least two women and at most two women love at least two men and at least two women and at most two women love at most two men

(5.6) At least two women love at least two men and at most two women love at least two men and at least two women love at most two men and at most two women love at most two men.

Now (5)$\equiv$(5.1)$\equiv$(5.2), but (5)$\not\equiv$(5.3); indeed, (5)$\not\supset$(5.3) and (5.3)$\not\supset$(5). On the other hand, (5.3)$\equiv$(5.4)$\equiv$(5.5)$\equiv$(5.6). Thus the equivalences that break down here break down at exactly the same points as in the cases (1) - (4).

Consider further:

(6) At least two women love exactly two men

(6.1) At least two women love at least two men and at most two men

(6.2) At least two women love at least two men and at least two women love at most two men

(7) At most two women love exactly two men

(7.1) At most two women love at least two men and at most two men

(7.2) At most two women love at least two men and at most two women love at most two men

Here (6) $\equiv$ (6.1), (6) $\supset$ (6.2), but (6.2) $\not\supset$ (6), and (7) $\equiv$ (7.1), (7.2) $\supset$ (7), but (7) $\not\supset$ (7.2).

The following necessary and sufficient condition for distributivity of binary logical modifiers can be readily proved for the sentences of SYN_L^{TR}:

If e = $r^m(a_1 \ldots a_m)$ is a sentence of SYN_L^{TR}, then for all $1 \leq i \leq m$, a binary logical modifier ~~on~~ a_i is distributable if and only if a_i has the least index among the major thing-expressions $a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_m$ that are not definite thing-expressions.

In the preceding examples (1) - (3), the above condition was fulfilled and binary logical modifier CONJ was distributable in each of (1), (2), (3). In the examples (4) - (6), this condition was not fulfilled and the binary logical modifier CONJ was not distributable in any of (4), (5), (6). For example, in (4), the index of "at least two men and at most two men" in (4.2) is 2, and that of "many women" is 1, which is not a definite thing-expression yet has a lower index than "at least two men and at most two men."

In the above examples (1) - (6), the binary logical modifier used was CONJ, since it has a special role in relation to the determiner EXCT. The condition for distributivity applies as well to the case where the thing-expression which the binary logical modifiers enter into is not doubly-bounded, and to the case of other binary logical modifiers that correspond to logical binary relations, such as INCDISJ, EXCDISJ, and CONJNEG.

The next set of examples (8) - (11) deals with some cases of the binary logical modifier CONJ being distributed over lower-bounded and upper-bounded thing-expressions. The subsequent examples then following, i.e., (12) - (15), deal with the binary logical modifier INCDISJ.

(8) At least two men but not John love Mary

(8.1) At least two men love Mary but John does not love Mary

(9) Mary loves at least two men but not John

(9.1) Mary loves at least two men but Mary does not love John

(10) At least two women love at least two men but not John

(10.1) At least two women love at least two men but at least two women do not love John

(11) At most two women love at least two men but not John

(11.1) At most two women love at least two men but at most two women do not love John

Here, (8) (8.1), (9) (9.1), but (10) (10.1) and (11 (11.1), since (10.1) (10) and (11) (11.1).

(12) At least two men or Agnes love Mary

(12.1) At least two men love Mary or Agnes loves Mary

(13) Mary loves at least two men or Agnes

(13.1) Mary loves at least two men or Mary loves Agnes

(14) At least two women love at least two men or Agnes

(14.1) At least two women love at least two men or at least two women love Agnes

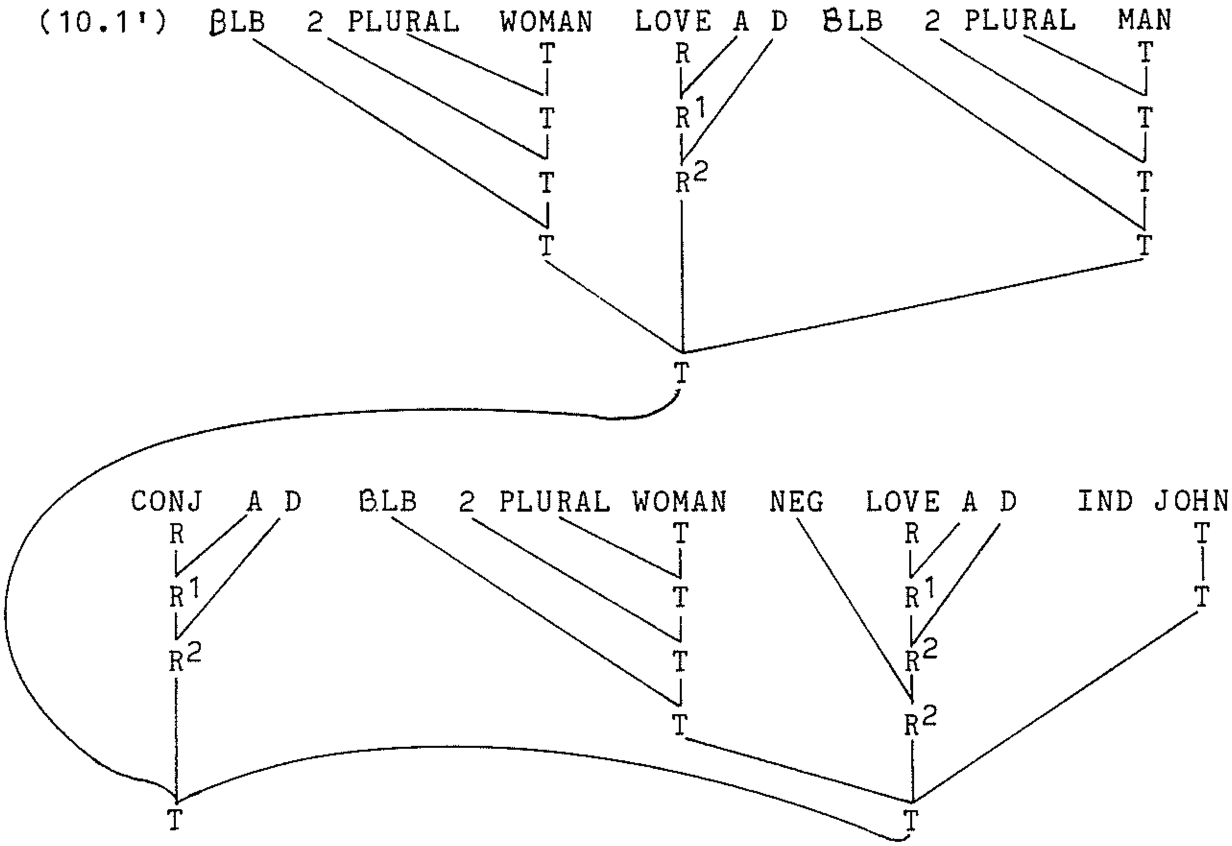
(15) At most two women love at least two men or Agnes

(15.1) At most two women love at least two men or at most two women love Agnes

Here, (12)$\equiv$(12.1), (13)$\equiv$(13.1), but (14)$\not\equiv$(14.1) and (15)$\equiv$ (15.1), since (14)$\not\supset$(14.1) and (15.1)$\not\supset$(15).

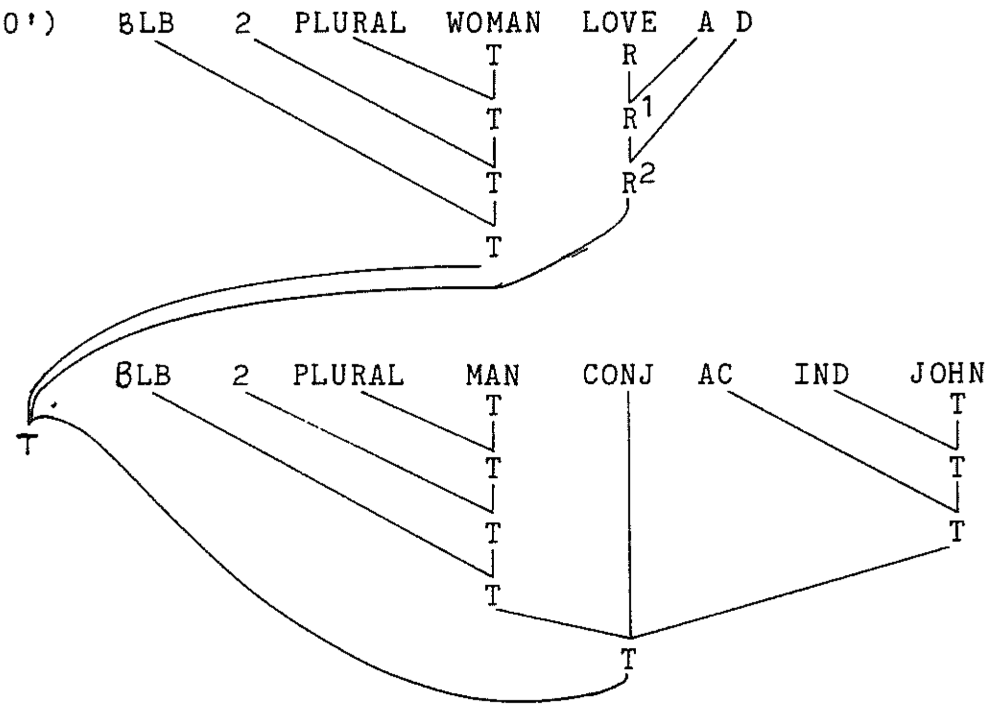
The above non-equivalences mean that under the dominant normal readings r_1 of (10), r_2 of 10.1, r_3 of (11), and r_4 of (11.1), (10) is not equivalent to (10.1) and (11) is not equivalent to (11.1). However, there are readings r'_2 of (10.1) and r'_4 of (11.1), (though possibly non-normal ones) under which, in conjunction with reading r_1 of (10) and r_3 of (11), (10) is equivalent to (10.1) and (11) is equivalent to (11.1). The syntactic component of such r'_2, for example, could be as follows:

(10.1')

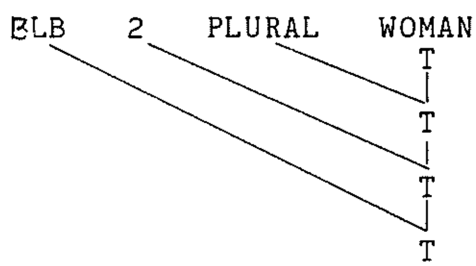


While this syntactic representation of (10.1) would not be the syntactic component of a <u>normal</u> reading of (10.1) whose semantic component satisfied the logical semantic axioms of Chapter 2, it would inter-entail (hence be equivalent to) the following syntactic representation of (10), which <u>would be</u> the syntactic component of the dominant normal reading of (10) whose semantic component satisfied the logical semantic axioms of Chapter 2:

(10')



The equivalence of (10) and (10.1) under the readings r_1 and r_2 whose respective syntactic components are (10') and (10.1') turns on the interpretation of the referential link in (10.1') that joins the two occurrences of



in (10.1').

Consider further:

(16) At least John and at most John and Bill love exactly two women

(16.1) At least John loves exactly two women and at most John and Bill love exactly two women

(16.2) At least John loves at least two women and at least John loves at most two women and at most John and Bill love at least two women and at most John and Bill love at most two women

(16.3) At least John and at most John and Bill love at least two women and at least John and at most John and Bill love at most two women.

(17) Exactly John loves exactly two women

(17.1) Exactly John loves at least two women and exactly John loves at most two women

(17.2) At least John loves at least two women
and at most John loves at least two women
and at least John loves at most two women
and at most John loves at most two women

"Exactly John" is a definite thing-expression, that is, it is syntactically represented as a definite thing-expression in $SYN^{TR}_{English}$ in the dominant normal readings of (17) and (17.1). This is, of course, consistent with our basic criterion for definite thing-expressions, which yields, for example, that (18.1) and (18.2) together entail each of (18.3) and (18.4):

(18.1) Exactly John loves Agnes

(18.2) Exactly John loves Carol

(18.3) Exactly John loves Carol and Agnes

(18.4) Exactly John loves Carol or Agnes

We remark on a further aspect of modifier distributions. Consider:

(19) At least two men and at most two women

The paired intersection interpretation of (19) would yield only the set of all sets of men who were also women.

There is also the "external" interpretation of "at least" where it is taken with respect to the entire domain of discourse rather than just with respect to the interpretation of its head noun, e.g. "men" above, which is the "internal" interpretation. Under an external interpretation which is discussed earlier in the preceding section. (19) denotes the set of all sets each consisting of at least two men, at most two women, and zero or more other elements of the domain of discourse; under an internal interpretation, (19) denotes the set of all sets consisting of at least two men and at most two women.

Let us examine this further.

(20) At least John and at most Bill love Mary

(21) John is Bill

(21) follows from (20) if "at least" is interpreted in either the internal or external senses.

On the other hand, in the case of:

(22) At least two men and at most two women love Agnes

(23) Nothing loves Agnes

(23) follows from (22) if "at least" is interpreted in the internal sense but not if "at least" is interpreted in the external sense.

Whether in a particular case "at least" is to be interpreted in the internal or external sense (syntactically represented by ßLB and ßLB-E, respectively) depends on a multitude of linguistic and non-linguistic factors, and would be specified in reading rules, that is, in those rules that assign normal readings to specific occurrences (i.e., tokens) of word-strings.

As further examples of cases where the external rather than internal interpretation of "at least" would yield the dominant normal reading of (24), (24.1), (25), (25.1) and (25.2), that is, those readings which yield (24) (24.1), and (25) (25.1) (25.2), we have the following:

(24) At least two men and at most three boys love Mary

(24.1) At least two men love Mary and at most three boys love Mary

(25) At least two men and at m ost three boys love all but two girls and almost all dogs

(25.1) At least two men love all but two girls and almost all dogs, and at most three boys love all but two girls and almost all dogs

(25.2) At least two men love all but two girls and at least two men love almost all dogs, and at most three boys love all but two girls and at most three boys love almost all dogs

On the other hand, under any (otherwise) normal reading of the following sentences in which "at least" were interpreted in the internal sense (24) (24.1), (25) (25.1) (25.2). For example, any lexically normal reading of (24), (24.1) (but not (24.1)) would be equivalent to:

(24.2) Nothing loves Mary

and, under any lexically normal reading of (25), (25) (but neither (25.1) nor (25.2)) would be equivalent to

(25.3) Nothing loves all but two girls and almost all dogs

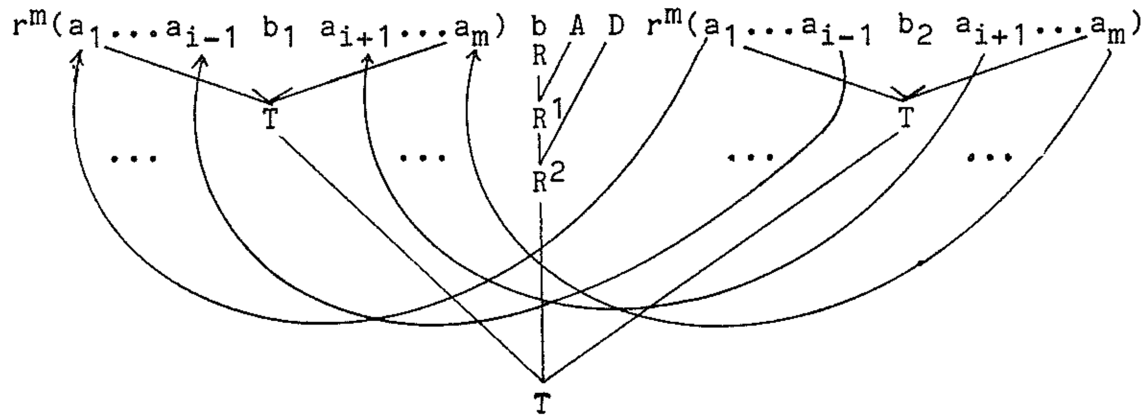
These results on the distribution of logical modifiers over sentences are as yet very fragmentary, and the special case dealing with doubly-bounded expressions is not very clear as yet. Preliminary results suggest that all doubly-bounded thing-expressions may be eliminable.

The failure of modifier distribution to preserve equivalence is explained within our framework by the fact that the different occurrences of given lower and upper bounding thing-expressions, while denoting the same set X of subsets of D, need not have the same particular subset of X chosen for each when existentially instantiated as required by the truth clauses embodied in L.S.A.(8.1) and (8.2). In order to assure this we define the DBET transform in such a way that identical instantiations are chosen, by employing referential links that join together the different occurrences of the same thing-expression generated when expanding binary logical modifiers to binary logical relations on sentences.

The DBET transform which supplants the unsatisfactory approximation $DBET^{o}$ is defined accordingly as follows:

Let b be a binary logical modifier on thing-expressions. Let $a_1, \ldots, a_{i-1}, a_{i+1}, \ldots, a_m, b_1, b_2$ be thing-expressions. Then

$$\mathrm{DBET}(r^m(a_1 \ldots a_{i-1}\, b\, b_1\, b_2\, a_{i+1} \ldots a_m)) =$$



The UBET Transform

The UBET transform is defined in terms of successive applications of two simple transforms called <u>the negation-conversion transform</u> and <u>the paired-conversion transform</u>, written respectively simply as the <u>NC</u> and <u>PC</u> transforms. Let us first describe the NC and PC transforms and illustrate their effects on English sentences.

The Negation-Conversion Transform NC

Let $b = r^m(a_1 \ldots a_m)$ be a sentence with major-relation expression r^n and major thing-expressions $a_1, \ldots, a_m$ in the order of their occurrence in b. Let, furthermore, a_j be the left-most thing-expression among $a_1, .., a_m$ which is either lower-limited or upper-limited. Then NC(b) is obtained from b by replacing a_j by $\underset{T}{\text{TC } a_j}$ if a_j is closed[91] and by $\underset{T}{\text{AC } a_j}$, if a_j is open, to obtain b' and then forming $\underset{T}{\text{NEG } b'}$. If there is no such a_j then NC(b) = b.

The Paired Conversion Transform PC_j

Let $b = r^m(a_1 \ldots a_m)$ be a sentence with major-relation expression r^m and major thing-expressions $a_1, \ldots, a_m$ in order of their occurrence in b. Now let a be any thing-expression among $a_1, \ldots, a_{m+n}$ which is either lower-bounded or upper-bounded, and let j be the least integer greater than i such that a_j is either

---

Note 91. A <u>closed</u> thing-expression is one whose left-most sub-expression is a determiner; an <u>open</u> thing-expressions is one that is not closed.

lower-bounded or upper-bounded. Then $PC_j(b)$ is obtained from b by replacing a_i by $\underset{T}{TC\ a_i}$ if a_i is closed, and by $\underset{T}{AC\ a}$, if a is open, and by replacing a_j by $\underset{T}{QC\ a_j}$. If there are no such a_i or a_j then PC_j (b) = b.

Some examples follow which illustrate the effect of the NC and PC_j transforms. We first note a few facts concerning these transforms:

(i) For every integer j, NC and PC_j are one-one functions and so have inverses, which we designate by NC^{-1} and PC_j^{-1}, respectively.

(ii) For every integer j, NC and PC_j will turn out to be equivalence preserving.

This cannot be rigorously proved but can only be argued for on intuitive grounds, largely by the consideration of examples, such as the following, whose derived patterns of equivalences are consequences of our assumption that the above transforms are equivalence preserving, as well as from certain assumptions regarding the nature of normal readings of the English word-strings occurring in these examples.

Each of the following 19 sets of English sentences consists of equivalent sentences; moreover, for any two sentences of each set, the syntactic representation of each, under its dominant normal reading, is obtainable from the syntactic representation of the other by application of one of the transforms NC, PC_j,

NC^{-1}, PC_j^{-1}. Also each set contains exactly one non-upper-bounded sentence.[92]

(1.1) At least five men love at least five women

(1.2) All but at least five men love fewer than five women

(1.3) It is false that fewer than five men love at least five women

(1.4) It is false that all but fewer than five men love fewer than five women

(1.5) All but at least five men fail to love at least five women

(1.6) At least five men fail to love fewer than five women

(1.7) It is false that fewer than five men fail to love fewer than five women

(1.8) It is false that all but fewer than five men fail to love at least five women

(2.1) At most five men love at most five women

(2.2) All but at most give men love more than five women

(2.3) It is false that more than five men love at most five women

(2.4) It is false that all but more than five men love more than five women

(2.5) All but at most five men fail to love at most five women

---

Note 92. Note also that each set is doubly partitionable into two subsets. Under the first partition the sentences of one subset have an unnegated verb and the sentences of the other subset have a negated verb. Under the second partition, the sentences of one subset are sententially unnegated and the sentences of the other subset are sententially negated.

(2.6) At most five men fail to love more than five women

(2.7) It is false that more than five men fail to love more than five women

(2.8) It is false that all but more than five men fail to love at most five women

(3.1) At most five men love at least five women

(3.2) All but at most five men love fewer than five women

(3.3) It is false that more than five men love at least five women

(3.4) It is false that all but more than five men love fewer than five women

(3.5) All but at most five men fail to love at least five women

(3.6) At most five men fail to love fewer than five women

(3.7) It is false that more than five men fail to love fewer than five women

(3.8) It is false that all but more than five men fail to love at least five women

(4.1) At least five men love at most five women

(4.2) All but at least five men love more than five women

(4.3) It is false that fewer than five men love at most five women

(4.4) It is false that all but fewer than five men love more than five women

(4.5) All but at least five men fail to love at most five women

(4.6) At least five men fail to love more than five women

(4.7) It is false that fewer than five men fail to love more than five women

(4.8) It is false that all but fewer than five men fail to love at most five women

(5.1) The man loves at least five women

(5.2) It is false that the man loves fewer than five women

(5.3) The man fails to love all but at least five women

(5.4) It is false that the man fails to love all but fewer than five women

(6.1) The man loves at most five women

(6.2) It is false that the man loves more than five women

(6.3) The man fails to love all but at most five women

(6.4) It is false that the man fails to love all but more than five women

(7.1) At least five men love the woman

(7.2) It is false that fewer than five men love the woman

(7.3) All but at least five men fail to love the woman

(7.4) It is false that fewer than five men fail to love the woman

(8.1) At most five men love the woman

(8.2) It is false that more than five men love the woman

(8.3) All but at most five men fail to love the woman

(8.4) It is false that all but more than five men fail to love the woman

(9.1) At most five men are honest

(9.2) All but at most five men are non-honest

(9.3) All but at most five men fail to be honest

(9.4) At most give men fail to be non-honest

(10.1) At least five men are honest

(10.2) All but at least five men are non-honest

(10.3) All but at least five men fail to be honest

(10.4) At least five men fail to be non-honest

(11.1) The man is honest

(11.2) It is false that the man is non-honest

(11.3) The man fails to be non-honest

(11.4) It is false that the man fails to be honest

(12.1) The man loves the woman

(12.2) It is false that the man fails to love the woman

(13.1) It is false that some books are difficult

(13.2) No books are difficult

(13.3) It is false that all but some books fail to be difficult

(13.4) All books fail to be difficult

(14.1) John's favorite book is non-difficult

(14.2) It is false that John's favorite book is difficult

(14.3) John's favorite book fails to be difficult

(14.4) It is false that John's favorite book fails to be non-difficult

(15.1) At most one man has a happy life

(15.2) All but at most one man has a non-happy life

(15.3) All but at most one man fails to have a happy life

(15.4) At most one man fails to have a non-happy life

(16.1) Some men are mortal

(16.2) Not all men are non-mortal

(16.3) It is false that all men are non-mortal

(16.4) All but some men fail to be mortal

(16.5) All but not all men fail to be non-mortal

(16.6) It is false that not all men fail to be mortal

(17.1) All drivers who break the law endanger some person

(17.2) No driver who breaks the law endangers no person

(17.3) No driver who breaks the law fails to endanger some person

(17.4) All drivers who break the law fail to endanger no person

(17.5) It is false that some driver who breaks the law endangers no person

(17.6) It is false that some driver who breaks the law fails to endanger no person

(18.1) At most a few politicians give the President some credit for understanding complex issues

(18.2) All but at most a few politicians give the President no credit for understanding complex issues

(18.3) It is false that more than a few politicians give the President some credit for understanding complex issue

(18.4) All but at most a few politicians fail to give the President some credit for understanding complex issues

(18.5) At most a few politicians fail to give the President no credit for understanding complex issues

(18.6) It is false that all but more than a few politicians fail to give the President some credit for understanding complex issues

(19.1) At most a few politicians give the democratic candidate at most an outside chance to win at least three states

(19.2) At most a few politicians give the democratic candidate all but at most an outside chance to win at least three states

(19.3) All but at most a few politicians give the democratic candidate more than an outside chance to win at least three states

(19.4) All but at most a few politicians fail to give the democratic candidate at most an outside chance to win at least three states

(19.5) All but at most a few politicians fail to give the democratic candidate at most an outside chance to win at least three states

(19.6) It is false that all but more than a few politicians fail to give the democratic candidate at most an outside chance to win at least three states.

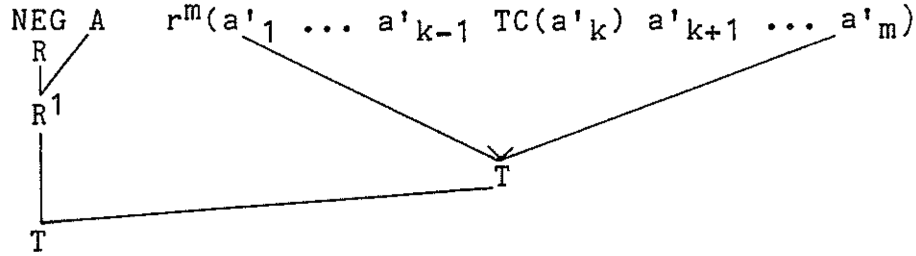
(19.7) At most a few politicians fail to give the democratic candidate more than an outside chance to win at least three states

The UBET transform is defined in terms of a certain succession of applications of the NC and PC transforms: its effect is to convert a sentence containing only definite, lower-bounded (LBE's) and upper-bounded thing-expressions (UBE's) into one containing only definite and lower-bounded thing-expressions. That is, the UBET transform converts sentences containing no major occurrences of doubly-bounded thing-expressions into non-upper-bounded sentences. Completion of the task of converting arbitrary sentences into non-upper-bounded sentences will be at hand once we can convert sentences containing major occurrences of doubly-bounded thing-expressions into sentences not containing them. The following transform effects this conversion.

The <u>UBET transform</u> is defined as follows:

Let $UBET^{o}(r^m(a_1 \dots a_m))$ be the result of replacing the right-most major UBE a_j, $1 \leq j \leq m$, in $r^m(a_1 \dots a_m)$ that is preceded by some UBE or LBE among $a_1, \dots, a_{j-1}$ by $TC(a_j)$, and replacing the right-most major UBE or LBE a_i (where $1 \leq i \leq m$) in $r^m(a_1 \dots a_m)$ to the left of a_j, by $QC(a_i)$. As can easily be verified, $UBET^{o}$ reduces the subscript j of the right-most UBE among $a_1, \dots, a_m$ by 1 or 2, according as a_i is an LBE or a UBE. Thus after at most j applications of $UBET^{o}$, one obtains an expression $r^m(a'_1 \dots a'_m)$. Let $r^m(a'_1 \dots a'_m)$ then be the sentence that is obtained after (at most j) iterated application of $UBET^{o}$ to $r^m(a_1 \dots a_m)$ which is such that either (i) none of $a'_1, \dots, a'_m$ are UBE's, in which case UBET $(r^m(a_1 \dots a_m))$ is defined as $r^m(a'_1 \dots a'_m)$, or (ii) exactly one thing-expression, say a'_k, among $a'_1, \dots, a'_m$ is a

UBE which is not preceded by a UBE, in which case UBET ($r^m(a_1 \ldots a_m)$ is defined as



It is clear that the UBET transform as defined above is intuitively equivalence-preserving, under the assumption that the NC and PC_i transforms are equivalence preserving. For, as can easily be verified, each application of $UBET^o$ is simply an application of the paired conversion transform PC_i, where the latter is applied to the right-most pair <a, a_j> such that a_j is a UBE, and since equivalence is transitive, the final application of $UBET^o$ yields a sentence intuitively equivalent to the original sentence $r^m(a_1 \ldots a_m)$. If case (i) holds, i.e., if no further major UBE's remain, then by the immediately preceding considerations, UBET($r^m(a_1 \ldots a_m)$) is intuitively equivalent to $r^m(a_1 \ldots a_m)$. If case (ii) holds, i.e., if exactly one major thing-expression is a UBE not preceded by any other UBE, then UBET ($r^m(a \ldots a_m)$) is obtained by following the preceding succession of PC_i transforms by a single application of the NC-transform. Under the assumption that the latter is equivalence preserving, it follows that the UBET transform is also equivalence preserving.

For simplicity, the above proof that the UBET transform is equivalence preserving deals with the special case of SYN_L^{TR} sentences which are such that the occurrence ordering of their major thing-expressions coincides with the relative scope ordering of those thing-expressions. It is clear, however, that the proof can be made completely general to apply to the general case where these two orderings do not coincide, by arguing on the order of the indeces on the thing-expressions rather than on their order of occurrence of those thing-expressions.

As indicated at the beginning of this section, the bounded conversion transform BCT of a given SYN_L^{TR}-sentence e is obtained by first applying DBET to e to obtain the SYN_L^{TR}-sentence DBET(e), and then applying UBET to the latter. Thus BCT is defined as follows on all SYN_L^{TR}-sentences e:

$$BCT(e) = UBET(DBET(e))$$

We can now state (8.2), which extends L.S.A.(8.1) to cover upper-bounded sentences of SYN_L^{TR} as well as non-upper-bounded sentences, as follows:

L.S.A.(8.2) Let e be a sentence of SYN_L^{TR}. Then

$$f(e)\ [f(T_n)] = f(BCT(e))\ [f(T_n)]$$

We seek in the following section 2.3.2.6 on Semantic Axioms for Temporal Relation, to employ the minimum amount of structure necessary to semantically interpret natural language sentences involving temporal expressions.

Toward this end we introduce a set D_T of times as a distinguished finite subset of the universe of discourse, thereby enabling a finite model of entailment in which the set of times is finite.

2.3.2.6 Semantic Axioms for Temporal Relations

In order to treat temporal relations we need to extend our formal framework slightly. The formulations of this section are tentative and deal with a small number of temporal constructions. In spite of these limitations, I include a discussion of such matters in this study in order to indicate one way in which temporal relations could be handled within our general framework.

Time is regarded as comprised of "points," called *time points*, which can be identified with ~~the set R^+ of non-negative real numbers.~~

As remarked earlier in Section 2.3.1.1.2.3, sentences of SYN_L^{TR} denote set-theoretic structures called *events* and each event is itself a set of sets of structures called *event particulars*. An event particular is defined solely in terms of the entities which enter into a given relation and not in terms of time; thus an event particular can occur at more than one time point r. Generally, an event particular x occurs at each time point on a set of time points called the *lifespan* of that event particular, which is symbolized by sp(x). The lifespan of a given event particular can be a singleton time point (i.e., a singleton set containing exactly one real number), an interval (of R^+)[92], the union of a set of intervals, the union of a set of time points and intervals, and so on. A *point instance* of an event particular x is a pair $\langle x,r \rangle$, where

---

Note 92. An interval can be *closed*, ($\{x:a\leq x\leq b\}$ for some $a,b\in D_T$); *open* ($\{x:a<x<b\}$, for some $a,b\in R^+$); or *half-open* ($\{x:a<x\leq b\}$, or $\{x:a\leq x<b\}$, $\{x:x\leq a\}$ or $\{x:a\leq x\}$, for some $a,b\in R^+$).

$r \in sp(x)$, that is, where r is a time-point in the lifespan of x or, more colloquially, where r is a time-point at which x "occurs." An *interval instance* of an event particular x is a set A of point instances $\langle x,r \rangle$ of x such that, for some closed interval $I \subseteq \mathbb{D}_T$, $r \in I$ if and only if $\langle x,r \rangle \in A$. An interval I which is related to an interval instance A of x in this way is called the *base* of the interval instance A. An interval instance A of an event particular x is *maximal for x* if and only if there is no other interval instance B of x which has a base that properly includes the base of A. Finally, an *instance* of an event particular x is a point instance or an interval instance of x, and an instance of an event particular is said to *occur* at every time point of that instance, and an event particular is said to *occur* at every time point at which some instance of that event particular occurs.

The preceding definitions are partly motivated by the following sort of intuition: when a person asserts a (declarative) sentence, it seems reasonable to regard him as asserting that a specific instance of an event particular has occurred, which event particular, in turn, belongs to the set of event particulars comprising the event denoted by that sentence. For example, a person asserting "John kissed Mary" can be regarded as asserting that a given instance of John's kissing Mary has occurred at an unspecified past time. In this case the event denoted by the specific sentence "John kissed Mary" is particularly simple, for it is a singleton set containing exactly one event particular: the case would be somewhat more complex if one were asserting "Some man kissed Mary," for then one would be asserting that some instance of some event particular belonging to the event denoted by that sentence has occurred. That is to say, a person asserting "Some man kissed Mary" can be regarded as asserting that a given instance of the event particular of a particular man kissing Mary has occurred, which event particular belongs to the union of the set of sets of event particulars comprising the event of some man kissing Mary; other event particulars in this set would consist of other particular men kissing Mary.

The constructions considered in this section are intended to illustrate the formal interpretation of monadic temporal and tense morphemes of SYN_L^{TR} and of binary temporal morphemes of SYN_L^{TR}. English analogues of <u>monadic</u> temporal morphemes of SYN_L^{TR} include "often," "always," "seldom," "periodically," etc. English analogues of monadic tense morphemes of SYN_L^{TR} include the tense indicating morphs of English. English analogues of <u>binary</u> temporal morphemes of SYN_L^{TR} include "before," "after," "when," "whenever," "until," etc.

For this purpose we introduce various further semantic axioms, the first of which is:

L.S.A.23. For all tense morphemes b of SYN_L^{TR} and for all monadic temporal morphemes q of SYN_L^{TR}, $f(\underset{T}{q\ \underset{T}{b}}) \overset{*}{<} f(\underset{T}{b}) \subseteq P(R^{+})_{D_T}$[93].

For example, if b = PAST and q = RARELY, then $f(\underset{T}{b})$ is the set of

Note 93. For the meaning of $<^*$, see page 221 .

all subsets of past times and f(q b) is the set of all subsets of

T

T

past times that contain "at most a few" times.

The notion of "lifespan" is not peculiar to event particulars, but can be applied to arbitrary elements of D. Accordingly, we extend the definition of "lifespan" to apply to arbitrary elements of D: for all $x \in D$, the <u>lifespan of x</u>, abbreviated as <u>sp(x)</u>, is a subset of R and, in the special case that $x \in R$, $sp(x) = \{x\}$; also, $sp(x) = \bigcup\{sp(y): y \in x\}$. In particular, the lifespan of an event is the union of the lifespans of its event particulars.

Letting <D,f> be an interpretation, we write P_f to designate <u>the set of event particulars under f</u>, that is, $P_f = \{x : \text{for some sentence S of } SYN_{L}^{TR},\ x \in \bigcup f(S)\}$.

We state these various relationships in the following semantic axiom:

L.S.A.24

(i) for all $x \in D$, $sp(x) \subseteq R^{T}_{D_T}$

(ii) if $x \in R^{T}_{D_T}$, $sp(x) = \{x\}$

(iii) for all $x \in D$, $sp(x) = \bigcup\{sp(y): y \in x\}$

Monadic Temporal Relations and the Temporal Morpheme TEMP

There are numerous monadic temporal adverbs of English that have the same intuitive meanings as certain English noun phrases with the word "time" as their head.

Monadic Temporal Adverbs	Corresponding Determiner Phrase
usually	at most times
rarely	at most a few times
occasionally	at some times
consistently	at many separate times
sometimes	at some time
sometime	at some time
frequently	at many closely separate times
periodically	at evenly separated times
always	at all times
never	at no times
almost never	at almost no times
hardly ever	at almost no times
continuously	at all times on an interval
almost continuously	at almost all times on an interval
henceforth	at all future times

We use the logical representational morpheme TEMP as that expression of SYN_L^{TR} that has the word "time" or times" as its English analogue. The logical semantic axiom that interprets TEMP is the following:

L.S.A.25. (i) $f_T(\text{TEMP}) = PR$

(ii) for all tense morphemes $b \in SYN_L^{TR}$ and for all monadic temporal morphemes $q \in SYN_L^{TR}$, there are intervals

$A, B \subseteq R^+$ such that f(b TEMP) = PA and f(b q TEMP) = PB
T   T
           T

(iii) for all $x \in \bigcup f$(PAST TEMP), for all
T
T

$y \in \bigcup f$(PRESENT TEMP), and for all $z \in \bigcup f$(FUTURE TEMP), $x < y < z$.
T   T
T   T

and $\bigcup f$(PAST TEMP) $\cup \bigcup f$(PRESENT TEMP) $\cup \bigcup f$(FUTURE TEMP) $= R^+$
T   T   T
T   T   T

(iv) for all sentences S of SYN_L^{TR}, f(TEMP S) = the
T
T

set of all subsets of non-negative real numbers j such that for some $x \in \bigcup f(S)$, $\langle x,j \rangle$ is a point instance of x.

Clause (i) means that TEMP is interpreted as the set of all subsets of R^+; clause (ii) means that, for every tense morpheme b, b TEMP is interpreted as the set of subsets of an interval of R^+, and for every
T

monadic temporal morpheme q, bq TEMP is also interpreted as
T
T

the set of subsets of an interval of R^+; clause (iii) means that PAST TEMP, PRESENT TEMP,
T   T
T   T

and FUTURE TEMP are interpreted as partitioning the positive real
T
T

numbers R^+ into three disjoint intervals such that the interval interpreting $[\text{PAST}\ \text{TEMP}_T]_T$ precedes the interval preceding $[\text{PRESENT}\ \text{TEMP}_T]_T$, which in turn precedes the interval interpreting $[\text{FUTURE}\ \text{TEMP}_T]_T$; (iv) means that $[\text{TEMP}_T\ S]_T$ is interpreted as the set of all points in the lifespans of the event particulars that enter into f($\genfrac{}{}{0pt}{}{S}{T}$). Intuitively, $[\text{TEMP}_T\ S]_T$ is interpreted as the set of all times that the sentence S is true.

Clause (iv) is used to interpret English temporal noun phrases like

(1) time(s) that John kissed Mary

in contexts like

(2a) Every time that John kissed Mary

(2b) The time that John kissed Mary

(2c) Most times that John kissed Mary

(2d) Many times that John kissed Mary

(2e) Almost all times that John kissed Mary

(2f) No time that John kissed Mary

(2g) Some time that John kissed Mary

(2h) A few times that John kissed Mary

Such reference to time provides a certain kind of <u>indirect</u> quantification on sentences by directly quantifying on temporal noun phrases like (1) rather than on sentences.[94]

The Temporal Morpheme PERIOD

The sentence

(1) John always ran on Monday

has at least the two meanings in English:

(2) John ran only (at some time) on Monday

or

(3) John ran (at some time) on every Monday

We can express (2) and (3) by their more colloquial paraphrases, which replace "always" by "whenever." The intended paraphrases of (2) and (3) are (2.1) and (3.1), respectively:

(2.1) Whenever John ran, it was (at some time on) Monday

(3.1) Whenever it was Monday, John ran (at some time) on that Monday)

Note 94. However, a <u>direct</u> quantification on sentences is also possible in our treatment although it does not occur in English (in the sense that this construction in SYN_L^{TR} does not have analogues in English). If direct quantification on sentences did occur in English, they would have forms like the following, exclusive of the parenthesized inserts, which have been included to exhibit possible canonical alternates:

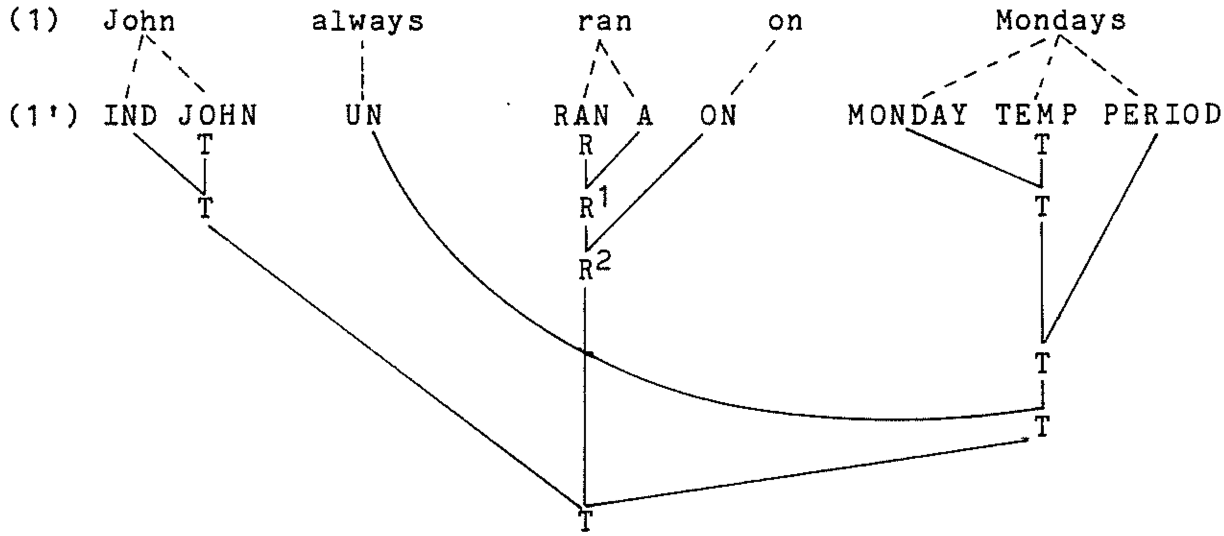
(3a) Every (case of) John kissed (having kissed) Mary
(3b) The (case of) John kissed (having kissed) Mary
(3c) Most (cases of) John kissed (having kissed) Mary
(3d) Many (cases of) John kissed (having kissed) Mary
(3e) Almost all (cases of) John kissed (having kissed) Mary
(3f) No (case of) John kissed (having kissed) Mary
(3g) Some (case of) John kissed (having kissed) Mary
(3h) A few (cases of) John kissed (having kissed) Mary

Nonetheless, such forms may be possible in some TR-languages. The semantic axioms governing TEMP and the semantic axioms governing those logical representational morphemes of which the above natural language determiners are analogues are such that the corresponding pairs of sentences among (2a)-(2h), (3a)-(3h) above have equivalent normal readings.

The intended meaning of (1) appears best approximated by (2) or (2.1) if "Monday" (in (1)) is stressed, and appears to be best approximated by (3) or (3.1) if "always" is stressed in (1). The parenthesized word-string "at some time" in (2), (3), (2.1), and (3.1) carries information that is understood (though less so than in (2) and (2.1)), and would usually be absent in ordinary speech.

In the following discussion I attempt to provide readings for (1) which captures the distinction between (2) and (2.1) on the one hand and (3) and (3.1) on the other, and which are such that, under those readings, (2) and (2.1) inter-entail each other, and (3) and (3.1.) inter-entail each other.

We introduce these considerations by first noting that, if we understand (1) in the sense of (3), then (1) means, not that John ran at some time on every Monday, but that John ran at some time on every Monday in a class of relevant Mondays, e.g., a class of Mondays each of which has its lifespan fall within a relevant interval of the moment of utterance, e.g., at least within the span of John's life and probably considerably less. Thus, the phrase "always on Monday" in (1), taken in the sense of (3), means "at some time in each of a class of relevant Mondays." Formally, we represent "Monday" as a modifier on $\underset{T}{\text{TEMP}}$ which, in application to $\underset{T}{\text{TEMP}}$, yields a set of subsets of time points, i.e., elements of D_T: namely the set of all "Monday times. Where a given "Monday time" is the set of all time points in a given Monday. Thus to say that John always ran on Monday in this sense is to say that for each of a given class of relevant Monday times B, John ran at at least one time point $m \in B$.. This sense of (1) is represented in $SYN^{TR}_{English}$ by the syntactic representation:



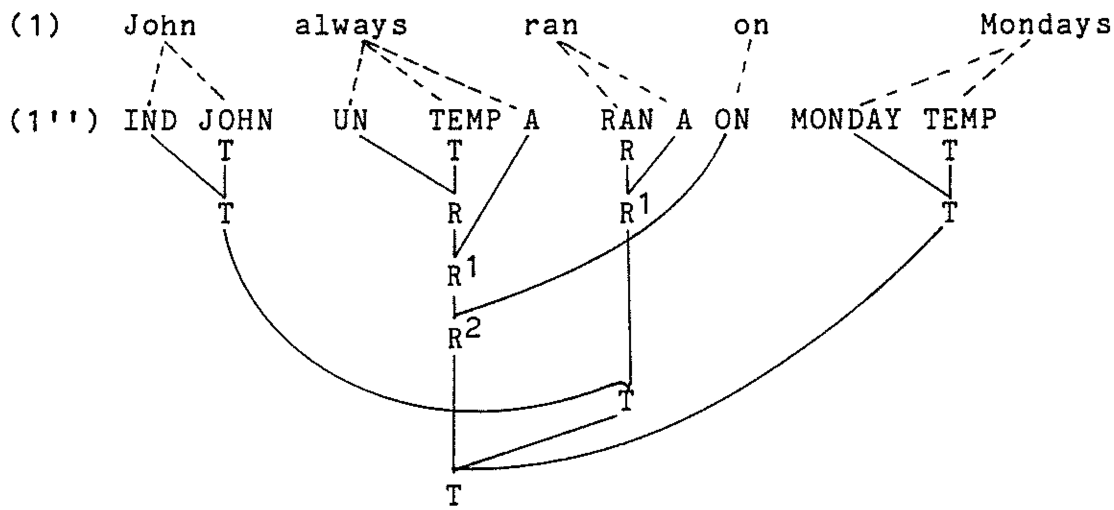
To accommodate this meaning, the morpheme PERIOD is semantically interpreted as a function which assigns to every set A of sets of time points a set B comprised of a set of subsets of elements of A. Intuitively, B would consist of just those subsets of elements of A all of whose member time points belong to a given interval. We cannot specify the interval precisely via the semantics of course, but rather allow PERIOD to be a member set-wise subset function, as expressed in the logical semantic axiom:

L.S.A.(26). For all $A \subseteq P(R^{+})$, $(f(PERIOD))(A) \subseteq A$.

L.S.A.(27). If e is a sentence of SYN^{TR}_{L}, then

$$f(e\ TEMP_{T})_{T} = P\ sp(f(e))$$

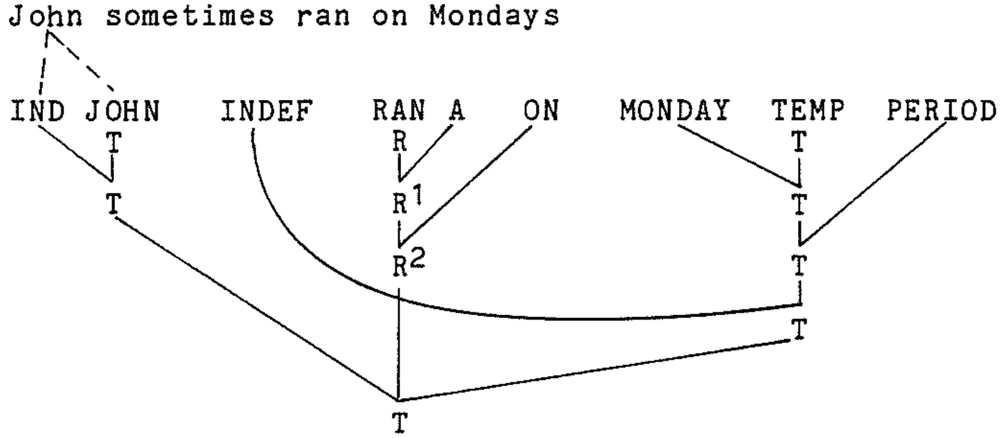
Let us now turn to the sense (2) of (1), namely that sense of (1) whereby (1) means "At all times in a given class of times, John ran on Mondays." This sense of (1) is represented in $SYN^{TR}_{English}$ by:



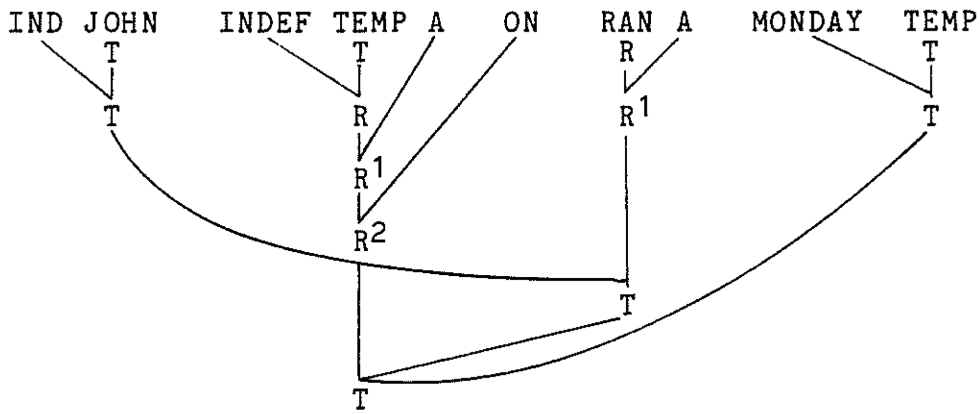
which means (by our semantic paradigm), that for all time points of a given set of time points, the set of time points at which John ran on Mondays is included in that set. Colloquially we can express this as: "For a given time period, John ran only on Mondays".

We can compare the representations (1') and (1'') of (1) with the representations (4') and (4'') of (4) below, which are, respectively, the syntactic representations of the dominant normal readings of (5) and (6), respectively, to follow.

(4) John sometimes ran on Mondays

(4')


(4) John sometimes ran on Mondays

(4'')


(5) John ran on some Mondays

(6) John ran sometimes on Monday

(5) and (6) can be more illuminatingly paraphrased by (7) and (8) respectively:

(7) Sometimes when John ran, it was Monday

(8) Sometimes when it was Monday, John ran

Under L.S.A.(28), below, we can prove that the dominant normal readings of (5) and (6), whose respective syntactic representations appear here as (4') and (4'') above, are equivalent, whereas (2) and (3) have inequivalent dominant normal readings whose respective syntactic representations appeared earlier as (1') and (1'').

L.S.A.(28). Let a be a monadic determiner of SYN_L^{TR}. Then

f(a TEMP A ON) = {<e',e''>:e',e'' are such that there are
T
R
R^1
R^2

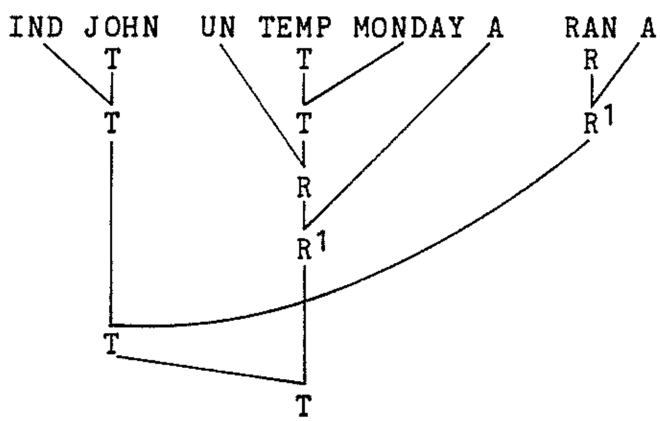
modifiers e_1,e_2 of SYN_L^{TR} such that $f(e_1)[f(T)]$ = e', and $f(e_2)[f(T)]$ = e'' and, for some x such that x ∈ f(a e_1 TEMP), x is
T
T
T

such that x ⊆ f(e_2 TEMP)}
T
T

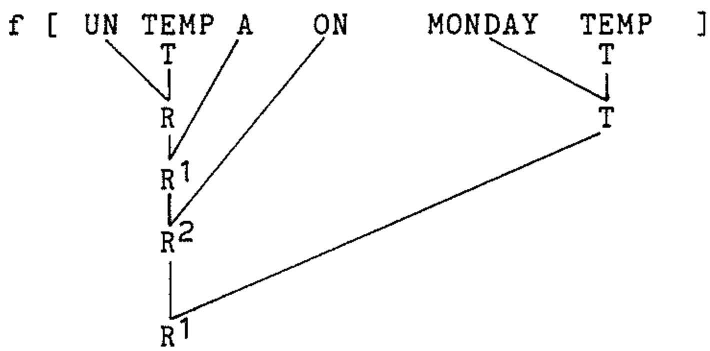
Let us consider another alternative syntactic representation (1''') of (1), which represents the discontinuous phrase "always ... on Monday" as a 1-place relation.

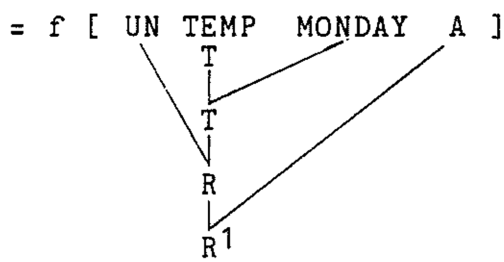
(1) John always ran on Monday

(1''')

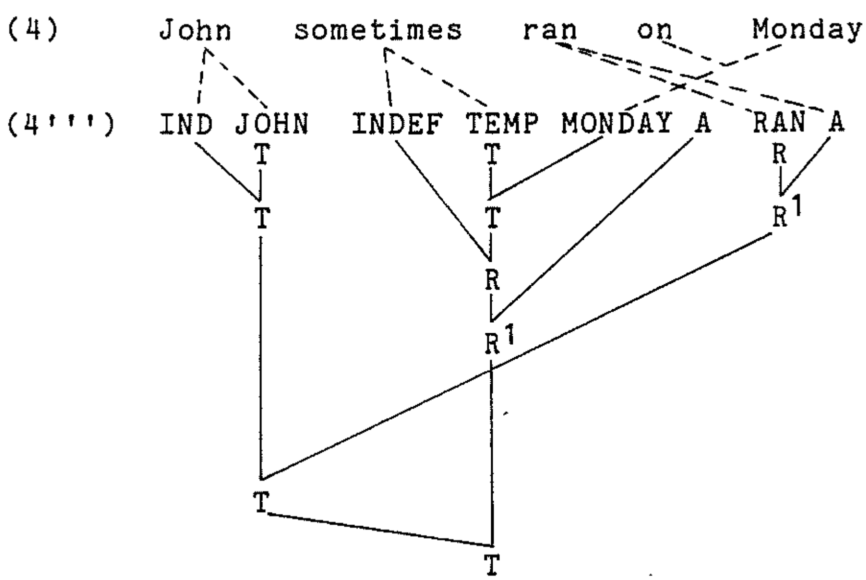


We can compare representations (1''') and (1') by noting that the following identity holds:



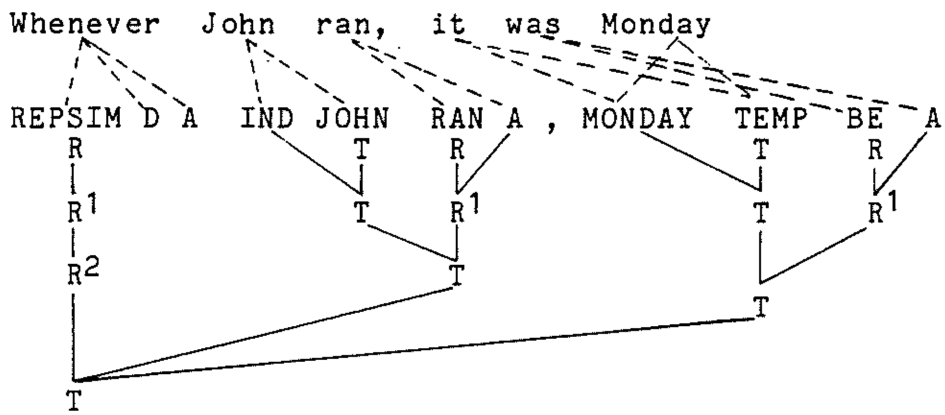


We can also compare (1''') with the following representation (4''') of (4):



Using the semantic clauses below (page 315) which interpret REPSIM, whose English analogue is "whenever," it can be shown that (1) and (2) are equivalent under the reading (1'') of (1) and (2°) of (2)

(2) / (2°)



External and Internal Representations

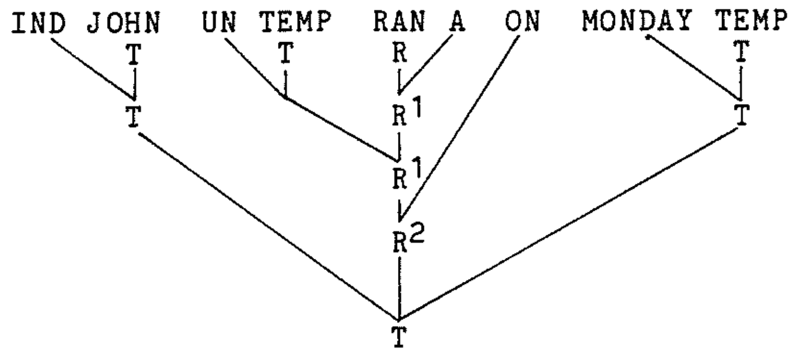
In representations (1'), (1''), and (1''') of (1), the temporal adverb "always" is represented as applying to some word-string other than the verb ("ran"). We say that (1) is externally temporized in each of these representations, since in them, the representation of "always" (indicated by the expressions into which the dotted lines issuing from "always" terminate) is an expression external to the major relation expression of the representation of (1).

We next consider an alternative representation of (1) which is not external in this sense, that is, a representation of (1) in which the representation of the temporal adverb "always" is a subexpression of the main relation-expression of that representation of (1). We call such a representation of (1) a representation of (1) in which (1) is internally temporized.

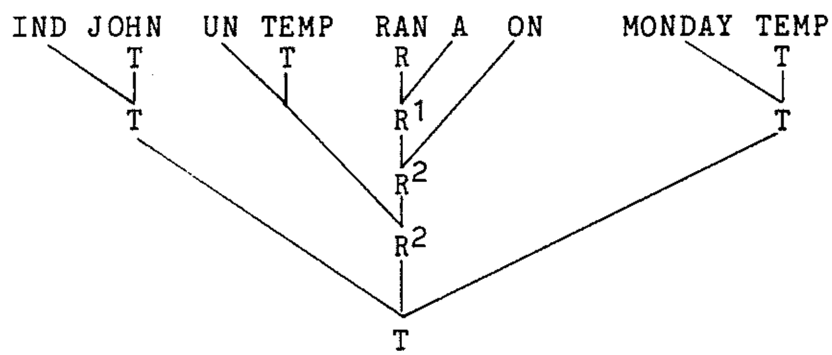
More generally, we call a representation e' of a sentence e of L an externally temporized representation if the temporal morphemes modifying the main verb of e are represented in e' by expressions which are not immediate subexpressions of the major relation of e' and as an internally temporized representation if those temporal morphemes are immediate subexpressions of the major relation of e'.

We exhibit below two internally temporized representations of (1), and then give the semantic axiom that interprets that internal temporization.

(1) John always ran on Monday

(1'''') 

(1) John always ran on Monday

(1''''') 

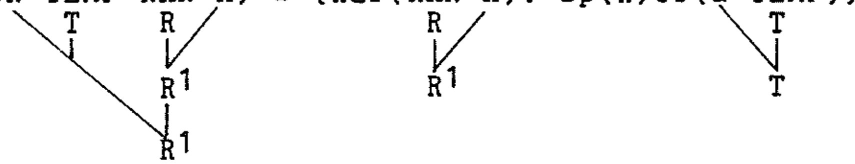
The following logical semantic axiom determines the interpretations of the major relation of an internally temporized representation:

L.S.A.29 If a is a monadic determiner, and b is a modifier, then

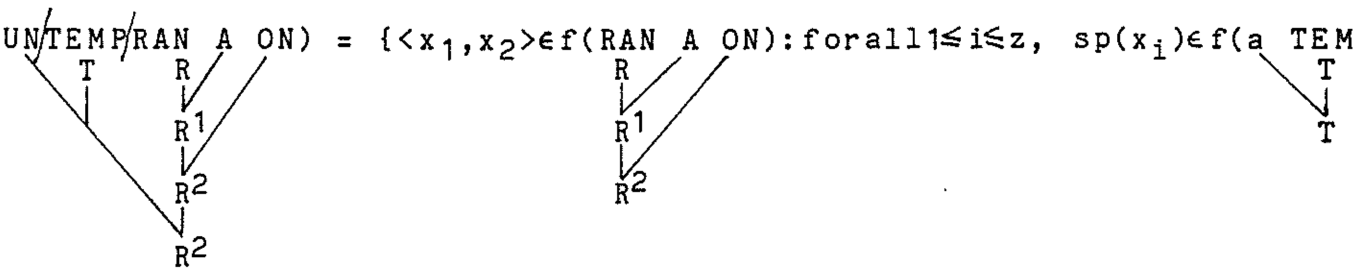
$$f((a\ \mathrm{TEMP}_{T})_{T}\ b_{R^n})_{R^n} = \{\langle x_1,\ldots,x_n\rangle \in f(b_{R^n}) : \text{for all } 1 \leq i \leq n,\ sp(x_i) \in f(a\ \mathrm{TEMP}_{T})_{T}\}$$

For example, the interpretations of the major relation-expressions in (1'''') and (1''''') would be, respectively:

f(UN TEMP RAN A) = {x∈f(RAN A): sp(x)∈f(a TEMP)}



f(UN TEMP RAN A ON) = {<x_1,x_2>∈f(RAN A ON):forall1≤i≤z, sp(x_i)∈f(a TEMP

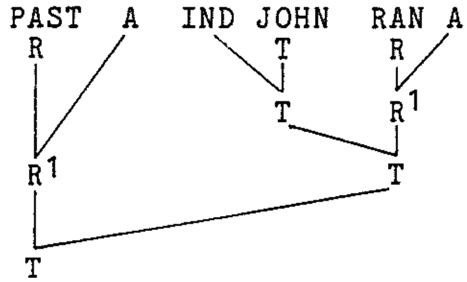


An interesting special case of the distinction between external and internal representation is that involving the special temporal morphemes we call <u>tense morphemes</u>.

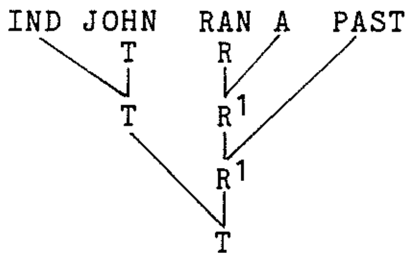
Consider the English sentence:

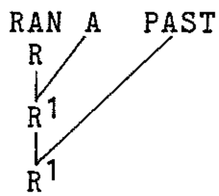
(5) John ran

We can represent the past tense morpheme of (5) externally in several (equivalent) ways, one of which is (5'):

(5') 

or internally as (5''):

(5'') 

The interpretation of 

in (5'') is given by L.S.A.(29) above; the interpretation of 

in (5') is given by the following:

L.S.A.(30) If e is a sentence, then

f(PAST e) [f(e)[f(T_n)]] =
(PAST: R → R^1)

= {x ∈ [f(e)[f(T_n)]]: sp(x) ∈ ⋃f(PAST TEMP)}
(PAST TEMP: T → T)

i.e., f(PAST A) is the set of all event particulars whose
(PAST A: R → R^1)

lifespan is in the past.

Analogous statements obviously hold for the tense morphemes PRESENT and FUTURE.

It can be easily shown that representations (1'') and (1''''') are equivalent, as are (5') and (5''). However, externally and internally temporized readings are not always equivalent, as can be seen in the following example:

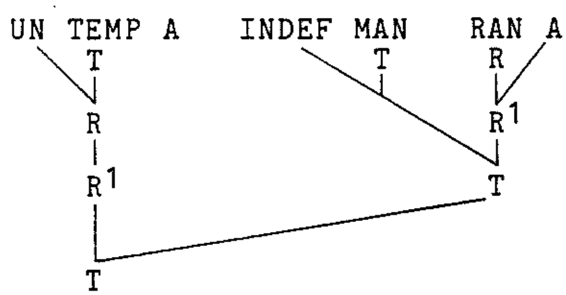
(6) Some man always ran

The dominant normal reading of (6) is an internally temporized reading (6') below. However, the externally temporized reading of (6), (6'') below, is also possible, which is the dominant normal reading of (7):

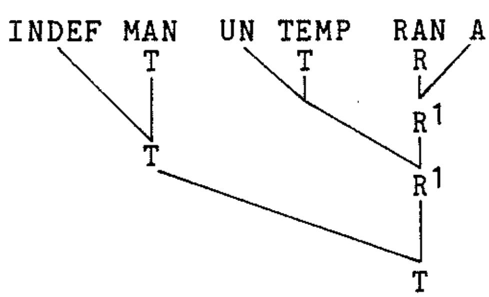
(7) There was always some man who ran

It is clear that (6) and (7) are intuitively inequivalent. Formally, this is reflected in the fact that representations (6') and (6'') below are inequivalent.

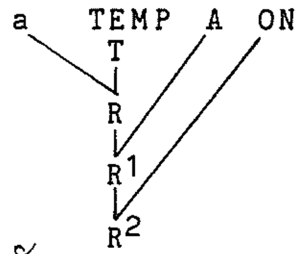
(6')



(6'')



An interpretation for binary temporal morphemes of the form



was given by L.S.A.(27). Binary temporal morphemes of SYN_L^{TR} were used in representations (1'), (4''), (1''''), and (1''''') above. In this subsection we formulate a logical semantic axiom to interpret representations of a number of binary temporal (representational) morphemes of SYN_L^{TR}, whose respective English analogues are: (i) "when" and "as"; (ii) "as soon as"; (iii) "during" and "while"; (iv) "until"; (v) "whenever"; (vi) "at"; (vii) "before" and "earlier than"; (viii) "after" and "then"; (ix) "henceforth" and "hereafter"; (x) "heretofore" and "up till now."

Temporal Morphemes

In this section we provide a semantic axiom that interprets eight binary temporal relations (i)-(viii) on arbitrary things, and two monadic temporal relations (ix) and (x) on arbitrary things.

L.S.A.(31)

(i) $f(\mathrm{OVLPSIM}_{R \to R^1 \to R^2}\ A\ D) = \{\langle x,y\rangle : x,y \subseteq PD$ and there are $x' \in x$, $y' \in y$

R
R^1
R^2

and closed intervals $u \subseteq sp(x')$, $v \subseteq sp(y')$ such that $u' \cap v' \neq \phi\}$

(ii) $f(\mathrm{INTSIM}_{R \to R^1 \to R^2}\ A\ D) = \{\langle x,y\rangle : x,y \subseteq PD$ and there are $x' \in x$, $y' \in y$

R
R^1
R^2

and closed intervals $u \subseteq sp(x')$, $v \subseteq sp(y')$ such that u, v have the same initial point and $u \cap v$ is a closed interval.}

(iii) $f\mathrm{INTVSIM}_{R \to R^1 \to R^2}\ A\ D$

R
R^1
R^2

(iv) $f(\mathrm{EXCLSIM}_{R \to R^1 \to R^2}\ A\ D) = \{\langle x,y\rangle : x,y \subseteq PD$ and there are $x' \in x$,

R
R^1
R^2

$y' \in y$ and closed intervals $u \subseteq sp(x')$, $v \subseteq sp(y')$ such that the terminal point of u is identical with the initial point of v.}

(v) f(REPSIM A D) = $\{\langle x,y\rangle : x,y \subseteq PD$ and there are $x' \in x_1$,

R

R^1

R^2

$y' \in y$ such that for every interval $u \subseteq sp(x')$, there is an interval $v \subseteq sp(y')$ such that $u \cap v \neq \phi\}$

(vi) f(POINTSIM A D) = $\{\langle x,y\rangle : x,y \subseteq PD$

R

R^1

R^2

and there are $x' \in x, y' \in y$ and such that $sp(x') \cap sp(y') \neq \phi\}$.

(vii) f(PREDEC A D) = $\{\langle x,y\rangle : x,y \subseteq P\ D$ and there are $x' \in x$, $y' \in y$

R

R^1

R^2

and closed intervals $u \subseteq sp(x')$, $v \subseteq sp(y')$ such that for all $r_1 \in u$, $r_2 \in v$, $r_1 < r_2$.}

(viii) f(SUCC A D) = $\{\langle x,y\rangle : x,y \subseteq PD$ and there are $x' \in x_1$ $y' \in y$

R

R^1

R^2

and closed intervals $u \subseteq sp(x'), v \subseteq sp(y')$ such that for all $r_1 \in u$, $r_2 \in v$, $r_1 > r_2$.}

(ix) f(FULFUT A D) = {x:x⊆PD and for all j such that

R
R^1
R^2

j∈ ⋃f(FUTURE TEMP), there is an x'∈x such that sp(x') = j}

T
T

(x) f(FULPAST A D) = {x:x⊆PD and for all j such that

R
R^1
R^2

j∈ ⋃f(PAST TEMP), there is an x'∈x such that sp (x') = j}

If we consider the <u>restriction</u> of the above relations to <u>events</u>, the following formulations can be given:

(i') f(OVLPSIM^{O} A D) = {<x,y>: x,y are events and there are

R
R^1
R^2

x',y' such that x' is an event particular of x and y' is an event particular of y, and the base y'' of some interval instance of y' overlaps the base x'' of some interval instance of x', i.e., the initial point of y'' is less than or equal to the initial point of x''}

(ii') f(INTSIM^{O} A D = {(x,y): x,y are events and there are x', y'

R
R^1
R^2

such that x' is an event particular of x, and y' is an event particular of y, and the base y'' of some interval instance of y' overlaps the base x'' of some interval instance of x' such that the initial point of y'' is equal to the initial point of x''} (iii') f(EXCLSIM° A D) = {<x,y>: x,y are events and there are

R
R¹
R²

x',y' such that x' is an event particular of x and y' is an event particular of y, and the base y'' of some interval instance of y and the base x'' of some interval instance of x are such that the terminal point of x'' is identical with the initial point of y''.}

(iv') f(REPSIM° A D) = {<x,y> x,y: are events and there are

R
R¹
R²

x', y' such that x' is an event particular of x and y' is an event particular of y, and the base of every interval instance of y' overlaps the base of some interval instance of x'.}

(v') f(POINTSIM° A D) = {<x,y>: x, y are events and there are

R
R¹
R²

x',y' such that x' is an event particular of x and y' is an event particular of y, and the base of some point instance of x' belongs to the base of some interval instance of y'.}

(vi) $f(PREDEC^{o}\ A\ D)$ = {<x,y>: x,y are events and there are

R

R^1

R^2

x',y' such that x' is an event particular of x and y' is an event particular of y, and the base of some interval instance of x' wholly precedes the base of some interval instance of y'.}

(vii') $f(SUCC^{o}\ A\ D)$ = {<x,y>: x,y are events and there are

R

R^1

R^2

x',y' such that x' is an event particular of x and y' is an event particular of y, and the base of some interval instance of y' wholly precedes the base of some interval instance of x'.}

This axiom, together with the preceding axioms of this chapter, has the following consequences:

Consider the following sentences:

(1) John left

(2) Mary arrived

(3) John left whenever Mary arrived

(4) John left when Mary arrived

(5) John left after Mary arrived

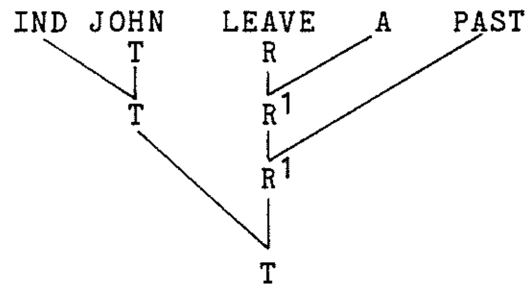
(6) Mary arrived before John left

(7) Mary arrived at 5 o'clock

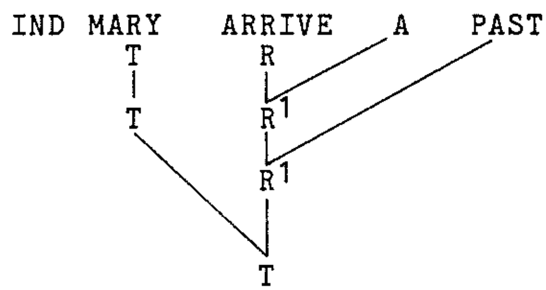
(8) John left after 5 o'clock

Then there are normal readings of (1)-(8) under which: each of (1) and (2) are entailed by each of (4), (5), and (6); (5) and (6) are equivalent; (6) and (7) together entail (8); (3) entails (4), but (4) does not entail (3). The syntactic representation components of these respective readings are as follows:

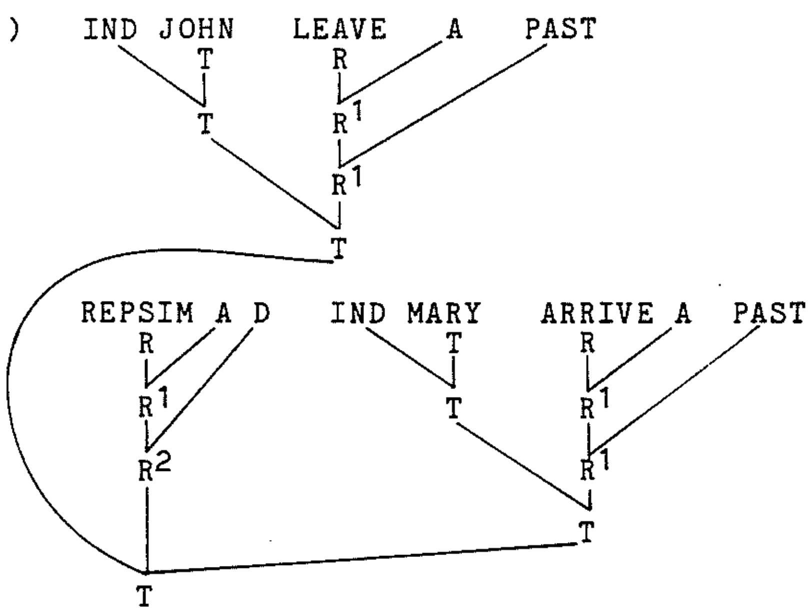
(1')



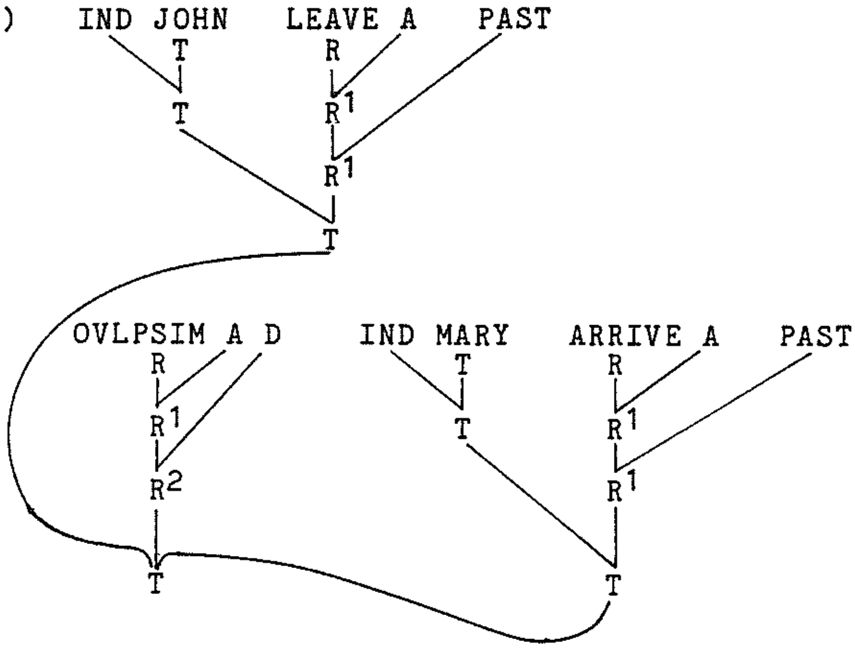
(2')



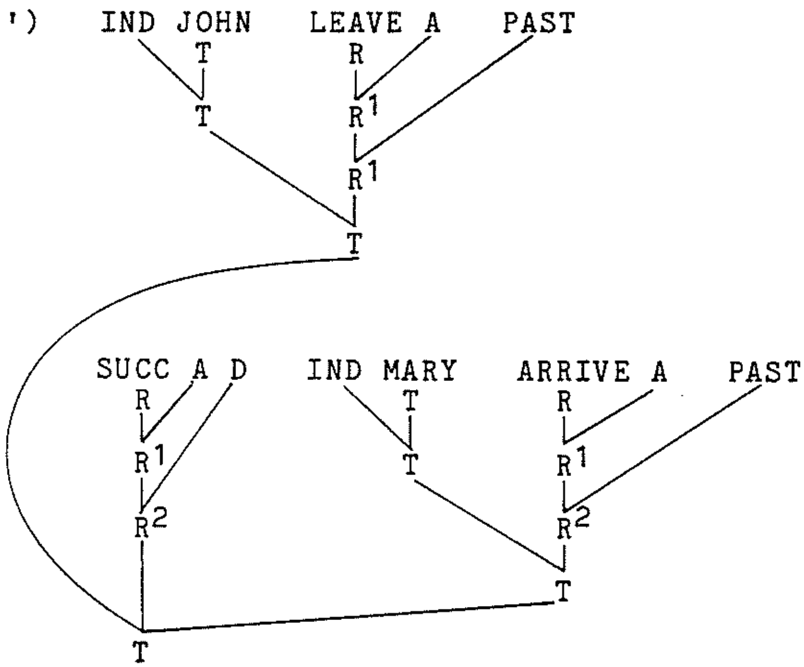
(3')



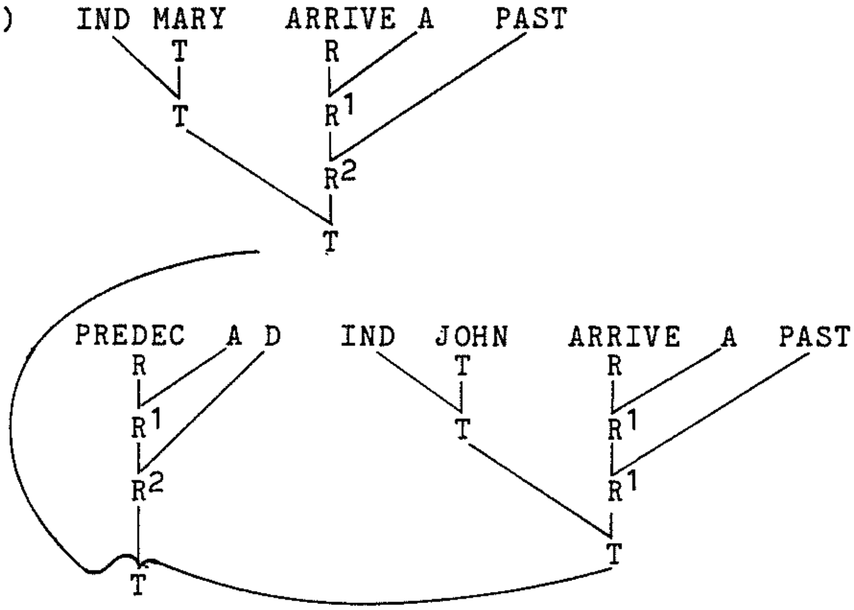
(4')


IND JOHN
LEAVE A
PAST
OVLPSIM A D
IND MARY
ARRIVE A
PAST

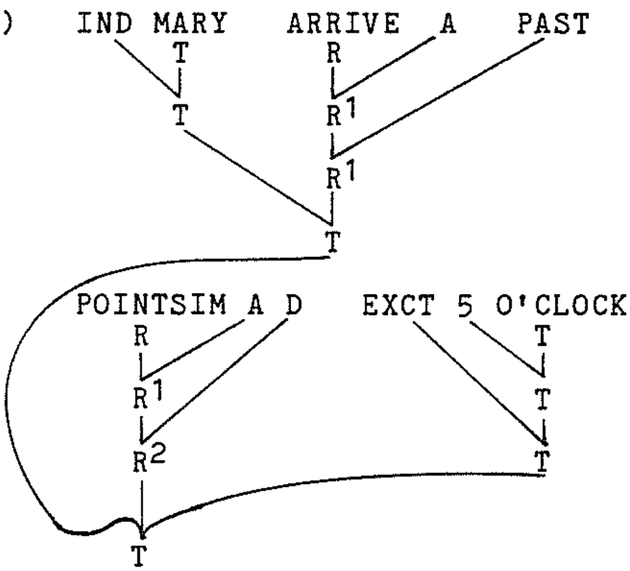
(5')


IND JOHN
LEAVE A
PAST
SUCC A D
IND MARY
ARRIVE A
PAST

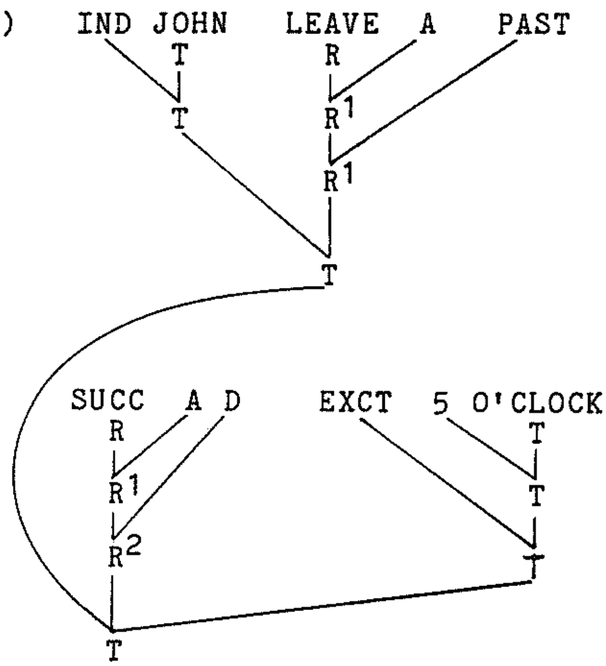
(6')


IND MARY ARRIVE A PAST
PREDEC A D IND JOHN ARRIVE A PAST

(7')


IND MARY ARRIVE A PAST
POINTSIM A D EXCT 5 O'CLOCK

(8')


IND JOHN
T
T
LEAVE A PAST
R
R1
R1
T
SUCC A D
R
R1
R2
EXCT 5 O'CLOCK
T
T
T
T

2.3.3 Pragmatics for L. In this section we briefly sketch some pragmatic notions as they would be treated within a theory of readings.

2.3.3.1 The Set M_L^{TR} of Reading Functions for L 95

Let L be a TR-language. Let q(<w,C#'>) be a relation among readings of a word-string of L produced in a context C# with interpretation C#', and which is such that, for all readings $r_1(w)$, $r_2(w)$ of w, $\langle r_1(w), r_2(w)\rangle \in q(\langle w, C\#'\rangle)$ if and only if $r_1(w)$ has an equal or greater degree of normality with respect to C#' than does $r_2(w)$. M_L^{TR} is a set of reading functions h for L, that is, a set of functions h, each of which assigns, to each word-string token w# and to each interpretation C#' of a context C# in which w# has been produced, a partially ordered set h(<w#,C#'>) of readings r(w) of w, ordered by the relation q(<w,C#'>) on readings r(w) of w. The reading rules for L relative to M_L^{TR}, when suitably specified, would provide practical procedures for

---

Note 95. As remarked in Section 1.7.2 of Chapter 1, we distinguish between a context C as a physical entity and as an interpreted physical entity. When so doing, we use the symbol "C#" to designate the context C as a physical entity, and use the symbol "C#'" to designate the context C as an interpreted physical entity. When we are not discussing matters which are directly affected by this distinction we use the simpler symbol "C" to designate an interpreted physical context. Accordingly, although the production of a particular token w# of a word-string w is a once-only matter, we nonetheless need to consider a multiplicity of possible contexts for w#, depending on the way that the context is interpreted.

approximating the values of the reading functions when applied to particular word-string tokens w#, with respect to particular interpreted contexts-of-utterance C#' at least to the extent of identifying the set Up[h(<w#,C#'>)] of upper bounds, (i.e., normal readings of highest degree), for each partially ordered set h(<w#,C#'>) of readings r(w) of w with respect to C# and, for those h(<w#,C#'>) which have a unique upper bound u∈Up[h(<w#,C#'>)] (i.e., the <u>dominant</u> normal reading of w with respect to C#'), then at least to the extent of identifying that unique upper bound. It is hypothesized that language users employ such reading rules in order to assign suitable readings to word-strings.

The reading rules for a given natural language are specific to that language (since the word-string to which those rules are applied are specific to that language) although similarities across languages inevitably exist.[96] In this section I would like to indicate some aspects of the nature of reading rules, using English as the language for illustration.

Consider the sentences:

(1) John does not love any woman

(2) John does not love every woman

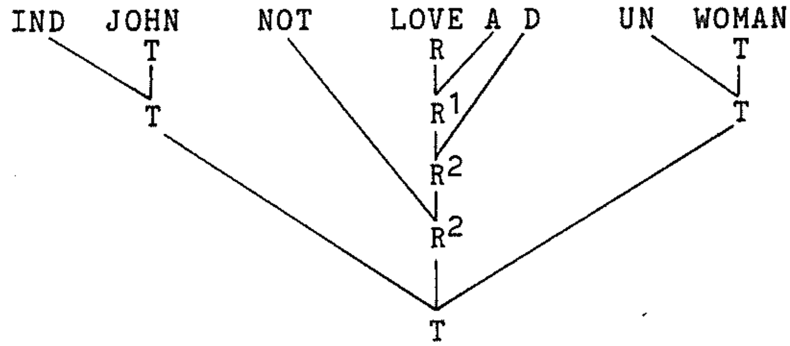
(3) It is false that John loves any woman

(4) It is false that John loves every woman

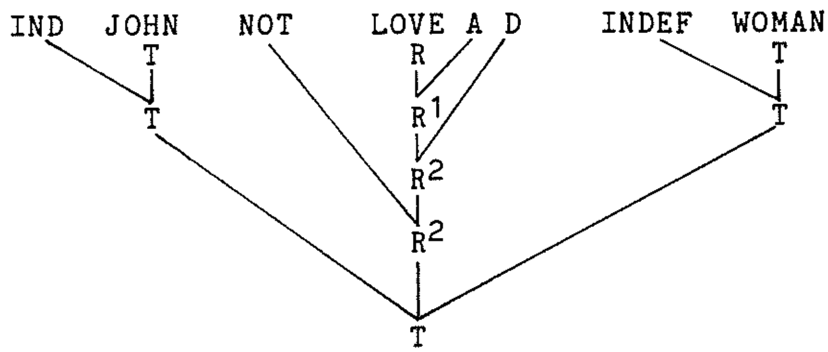
and the readings:

Note 96. I would not consider similarities of reading rules across different languages to be language universals, though there appears to be a tendency in the linguistic literature to so consider them.

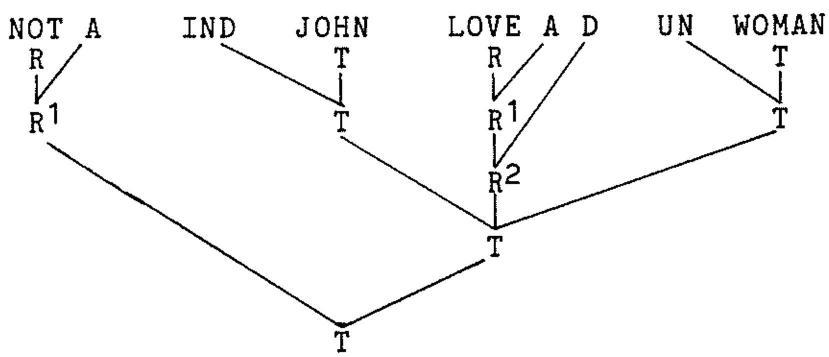
(a)


IND JOHN NOT LOVE A D UN WOMAN
T R T
T R^1 T
R^2
R^2
T

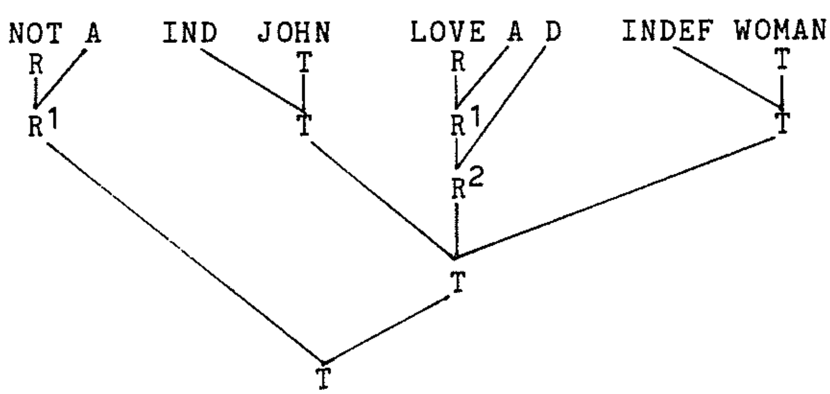
(b)


IND JOHN NOT LOVE A D INDEF WOMAN
T R T
T R^1 T
R^2
R^2
T

(c)


NOT A IND JOHN LOVE A D UN WOMAN
R T R T
R^1 T R^1 T
R^2
T
T

(d)


NOT A IND JOHN LOVE A D INDEF WOMAN
R T R T
R^1 T R^1 T
R^2
T
T

Each of the sentences (1)-(4) has one or more of the readings (a)-(d) as a normal reading; in the case where a sentence has more than one of (a)-(d) as a normal reading, these readings may be of different degrees of normality for that sentence, depending on the interpretation C#' of C# of the context-of-utterance C.[96.1]

Note 96.1. There are many further grammatical constructions besides negation that can influence the interpretation of "any". In particular, we might note the way that the interpretation of "any" is influenced by the circumstance of its occurring in the antecedent or the consequent of a conditional assertion. While cancellable (as will be noted shortly) the tendency is to interpret an occurrence of "any" in the antecedent of a conditional as "some", and an occurrence of "any" in the consequent as "all". For example:

(i) John is attracted to any woman

(ii) If John is attracted to any woman,

Mary will leave him.

(iii) If Mary leaves him, then John will be attracted to any woman.

In the dominant normal reading of (i) and of (ii), "any" would be represented by UN; however, in the dominant normal reading of (ii), "any" would be represented by INDEF. We can combine both cases in:

(iv) If any woman is attracted to John, then John will be attracted to any woman

wherein the occurrence of "any" in the antecedent would, in the dominant normal reading of (iv), be represented by INDEF, and the second occurrence by UN.

We do not of course suggest that these various interpretations of "any" are <u>wholly</u> determined by the circumstance of the occurrence of "any" when the antecedent or consequent of a conditional, for there are situations in which such interpretations are not the dominant ones, such as, say, where they would force one to accept a palpably false statement as true, as in:

(v) If any man loves any woman, then any man loves any woman

where, under the dominant normal reading of (V), <u>all</u> occurrences of "any" would be represented by INDEF (though there is a marginally normal reading where all occurrences of "any" in the antecedent would be represented by INDEF and all those in the consequent would be represented by UN). Rather, what we are suggesting is that the interpretation of "any" is strongly influenced by a variety of constructions within the immediate contextual environment in which "any" occurs. Those can be generated quite easily by compounding further constructions with the above such as the use of restrictive relative clauses occurring within conditional assertions, as in:

(vi) If any woman who is attracted to any man is attracted to John, then John will be attracted to any woman who is attracted to any man.

Here, in the dominant normal reading of (vi), both occurrences of "any" in the antecedent would be represented by INDEF, and the first occurrence of "any" would be represented by UN; however, in the representation of the second occurrence of "any" in the consequent, i.e., that occurrence within the relative clause in the consequent, "any" would be represented by INDEF, (though there are also possible normal readings where it would be represented by UN).

in the consequent

We first consider the normality of readings for each of (1)-(4) relative to unspecified context (under (A) below), that is, relative to the context comprised simply by that sentence.[97] We consider also some partially specified contexts very briefly (under (B) below).

Note 97. As remarked earlier, by a word-string w being _normal relative to unspecified context_ I mean that no specific indication of context is given beyond that suggested by the word-string itself, so that those aspects of context may be assumed which are common to most (or many) usual contexts-of-utterance. Unspecified contexts arise only in the course of analyses, that is, in discussions of the present kind, and not in the course of the actual production of a word-string in ordinary verbal communication, for in practice, any given occurrence of w would inevitably be embedded within a particular context which would indeed carry information beyond the mere fact of the production of w. Rather, by w being normal relative to unspecified context, I mean that w is considered independently of any information that would be specific to any given context of utterance. Context considered in this sense excludes also any signals carried by relative stress of sub-strings within the sentence which, as we indicated above, can plausibly be regarded as comprising an element of context. A reading of w would, then, be normal relative to unspecified context if the entailment relations which that reading induced were consistent with the intuitions of language users who were to consider w independently of any information that was specific to any given context-of-utterance, (though not necessarily independent of other word-strings that were also being simultaneously considered independent of any information that was specific to any given context-of-utterance, and, indeed, within which w# might be embedded.

(A) Relative to Unspecified Context

(i) Readings (b) and (c) are the dominant normal readings for both sentences (2) and (4). Thus in conjunction with internal negation, "every" signals universal quantification and, in conjunction with internal negation, signals existential-quantification.

(ii) Readings (a) and (d) are the dominant normal readings for (3). Thus in conjunction with external negation, "any" signals either universal-quantification and, in conjunction with internal negation, "any" signals existential-quantification.

(iii) Readings (a) and (d) are the dominant readings for sentence (1), and readings (b) and (d) are only slightly less dominant for (1).

(B) Relative to partially specified Contexts[98]

(iv) If either of (1) or (3) were either immediately preceded by "Bill says that John loves every woman. I disagree," or else were immediately followed by "For example, John doesn't love Mary," then it would, like (2)

Note 98. Partially specified contexts, like unspecified contexts, arise only in the course of analyses and would not arise in practice. Partially specified contexts provide only selected contextual information so that only those aspects of context may be assumed which are common to most (or many) usual contexts-of-utterance.

and (4), have reading (c) as its dominant normal reading[99]

(v) For each of (1)-(4), if it were immediately preceded by "Bill says that John loves some woman. I disagree," then it would have readings (a) and (d) as its dominant normal reading.[99]

(vi) If "not" were to receive higher stress than "any" in (1), then (1) would still have readings (a) and (d) as its dominant readings, as it did when "not" was unstressed. This is possibly because of the relatively greater focus rendered onto "not" by the relative order; specifically, it occurs before "any" in (1).

(vii) If "any" were to receive higher stress than "not" in (1), then (1) would have (b) and (c) as more dominant normal readings that either (a) or (d).

(viii) If "any" were to receive higher stress in (3), the result would sound anomalous, precisely because stressing this occurrence of "any" prompts one to understand the negation signalled by the initial "it is false that" as internal negation, which is in conflict with the dominant normal reading of such an initial "it is false that" as external negation.

Note 99. On the other hand, if either (2) or (4) were immediately preceded by "Bill says that John loves some woman. I disagree," it would sound anomalous.

(ix) The effect of stressing either "it is false that", "not" or "every" in (2) or (4) would not change the relative dominance of normal readings of these sentences.

For simplicity of exposition, in the following examples I take the context of utterance of a given word-string simply as context in the narrow sense, i.e., as the immediate verbal context--in each case a sentence--in which that word-string occurs, no further recognition of the wider verbal or non-verbal context is taken.

If a given word string w has only one token in its context of utterance, that token is designated by w#; if it has more than one token in a given context of utterance, they are designated by w#1, w#2, etc., in order of their occurrence.

Let us consider some examples with respect to a normal reading function h on the English sentences (1)-(4) above with respect to unspecified context. Let s_o be the minimal semantic theory on which the semantic axioms L.S.A.(1)-L.S.A.(31) of this chapter hold.

<u>Format</u>:[100] Up[<h (w#, C(W#)] = {<syntactic representation of w in SYN_L^{TR}; semantic theory s_o>}

(1.1) Up[h(<John#, (1)>] = {<IND JOHN , s_o>}

T

T

(1.2) Up[h(<does love#, (1)>)] = {<LOVE A D , s_o>}

R

R^1

R^2

(1.3) Up[h(<not#, (1)>)] = {<NOT , s_o>}

(1.4) Up[h(<any#, (1)>)] = {<UN , s_o>}

(1.5) Up[h(<woman#, (1)>)] = {<WOMAN , s_o>}
T

(1.6) Up[h(<does not love#, (1)>)] = {<NOT LOVE A D , s_o>}
R
R^1
R^2
R^2

(1.7) Up[h(<any woman#, (1)>)] = {<UN WOMAN , s_o>}
T
T

(1.8) Up[h(<1)#, (1)>)] = {<(a) , s_o>}

Note 100. In each of the examples to follow, the set Up[h(<w#, C#')] contains only the one normal reading indicate d, which is the dominant normal reading of w.

(2.1)	Up[h(<John#, (2)>)]	=	{h(<John #, (1)>)}
(2.2)	Up[h(<does love#, (2)>)]	=	{h(<does love#, (1)>}
(2.3)	Up[h(<note#, (2)>)]	=	{h(<note #, (1)>)}
(2.4)	Up[h(<every#, (2)>)]	=	{h(<any# , (1)>)}
(2.5)	Up[h(<woman#, (2)>]	=	{h(<woman#, (1)>)}
(2.6)	Up[h(<does not love#, (2)>]	=	{h(<doesn't love#, (1)>}
(2.7)	Up[h(<every woman#, (2)>]	=	{(<any woman #, (1)>}
(2.8)	Up[h(<(2)#, (2)>)]	=	{<(c), s_o>}
(3.1)	Up[h (<it is false that#,(3)>=		{<NOT A , s_o>} R / R^1
(3.2)	Up[h (<John# , (3)>)]	=	{h(<John# (1)>}
(3.3)	Up[h (<loves# , (3)>]	=	{h(<loves# (1)>)}
(3.4)	Up[h (<any# , (3)>)]	=	{<INDEF) , s_o>}
(3.5)	Up[h (<woman# , (3)>]	=	{h(<woman, (1)>)}
(3.6)	Up[h (<any woman#, (3)>]	=	{<INDEF WOMAN , s_o>} T / T
(3.7)	Up[h(<(3)# , (3)>]	=	{<(d) , s_o>}
(4.1)	Up[h(<it is false that#, (4)>)]	=	{h(<it is false that# , (3)>) }
(4.2)	Up[h(<John#, (4)>)]	=	{h(<John#, (3)>}
(4.3)	Up[h(<loves#, (4)>]	=	{h(<loves#, (3)>)}
(4.4)	Up[h(<every#, (4)>]	=	{<any# , (1)>}
(4.5)	Up[h(<woman#, (4)>)]	=	{<woman#, (3)>}
(4.6)	Up[h(<any woman#, (4)>]	=	{h(<any woman#, (1)>)}
(4.7)	Up[h(<(4)#, (4)>)]	=	{<(c), s_o>}

On the Notion of Well-Formedness for Natural Language Word-Strings

A word-string of a natural language L is a _logically well-formed word-string of L_ just in case it has a logically normal reading, and is a _lexically well-formed word-string of L_ just in case it has a lexically normal reading. Let me urge the reasonableness of this definition as follows: Recall that a word-string w of L has a logically (lexically) normal reading just in case there is some non-trivial set K of sentences of L in which a token w# of w occurs such that there is a logically (lexically) normal reading assignment A on K, that is, a reading assignment A on K that induces an entailment relation on K which is consistent with the logical (lexical) intuitions of L users concerning entailment. Thus, a word-string of L is well-formed just in case it can enter into intuitively correct logical (lexical) entailment relationships among word-strings of L. From an intuitive point of view, the logical or lexical well-formedness of a word-string is constituted wholly by the capacity of that word-string to signal suitable logical or lexical meanings. Within our treatment these meanings are, of course, suitable logically or lexically normal readings. Accordingly, we reflect our intuitions regarding the logical or lexical well-formedness of a word-string within our approach by having the logical or lexical well-formedness of word-strings be defined by their being assignable logically or lexically normal readings. By Assumption A of Section 1.1, the logical or lexical well-formedness of word-strings is defined by their having readings that induce intuitively correct entailment relations among them.

The above only characterizes the meaning of well-formedness, and does not identify the characteristics of a word-string by virtue of which it is well-formed, that is, by virtue of which suitable normal readings can be assigned to it. These characteristics would be yielded by any adequate specification of reading-rules. Linguistic research over the past fifteen years provides a great deal of language-specific data that would apply directly toward the identification of such characteristics. I would consider the results deriving from such linguistic research activity to be directly complementary to the general approach of this paper.

Recall that in Section 1.6 we had described inverses of reading-rules, i.e., generation rules, as rules that assigned word-strings to readings (i.e., expressions of SYN_L^{TR} together with semantic interpretations of them) relative to which those word-strings were normal.

The hearer of a word-string is assumed to apply the reading rules of L to obtain a suitable normal reading <w',i> of w, where w is a syntactic representation of w in SYN_L^{TR} and i is a semantic interpretation of w' in INT_L^{TR}. The hearer must have a knowledge of those reading rules in order to understand w. That is, the reading rules of L enable him to assign to w a representation w' of w, and to understand w' as interpreted by i.

The speaker, on the other hand, is assumed to initially conceptualize that which he ultimately intends to express by a word-string of L as a reading $\langle w',i\rangle \in SYN_L^{TR} \times INT_L^{TR}$, and to then apply the *generation rules* of L to $\langle w',i\rangle$ to obtain a suitable word-string w of L of which $\langle w',i\rangle$ would be a normal reading relative to the context of utterance in which w would be produced.

2.3.3.2 *Assimilating Pragmatics within Semantics*

In this section we attempt to describe how *pragmatics might be at least partially assimilated within semantics*. More exactly, we attempt to describe how the potentially verbalizable part PVC of the context-of-utterance C of a word-string w might be expressed within SYN_L^{TR} and assimilated within the semantic component of the readings that interpret the syntactic representations of w.

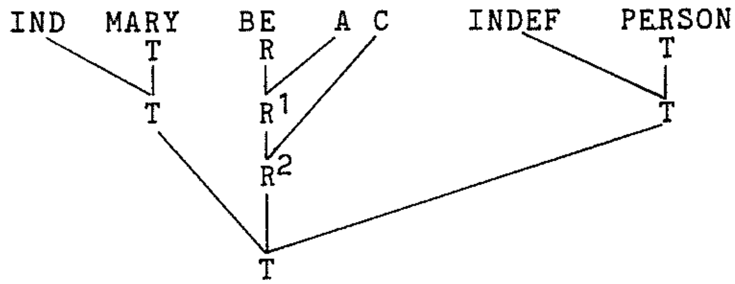
A *lexical refinement* $s' = \langle F',V',R\rangle$ of a semantic theory $s = \langle F,V,R\rangle$ is obtained by adding further semantic lexical axioms to the fund of semantic axioms (logical and lexical) that define the set F of interpretations in s. As remarked earlier, semantic lexical axioms can be specified simply as sentences of SYN_L^{TR}; this has the consequence that for every such semantic lexical axiom e, we have:

$V(\langle e,\langle D,f\rangle\rangle) =$ truth, for every $\langle D,f\rangle \in F'$.

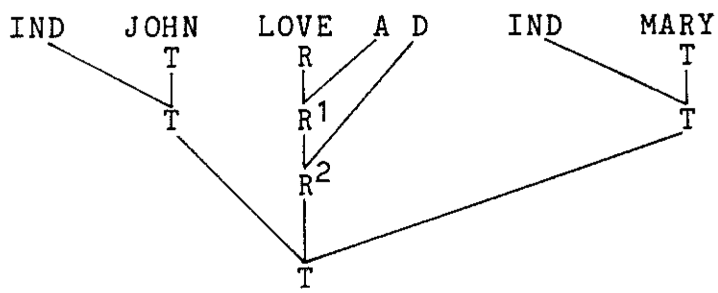
The effect of adding such semantic lexical axioms is to impose further structure on the interpretations $\langle D,f\rangle \in F'$. It is precisely this imposition of additional structure that prompts us to call s' a "refinement" of s.

For example, suppose that s' = <F',V,R> were obtained from s = <F,V,R> by adding the following semantic lexical axiom (1°)

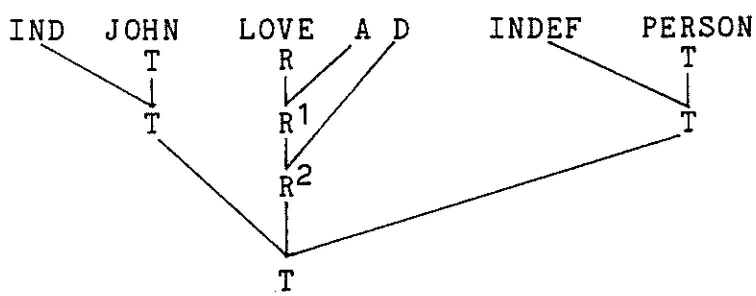
(1°) V(<(1), <D,f>>) = truth, for every <D,f> ∈ F' the specification of F where (1) is as follows below:

(1) 

Then, given the SYN_L^{TR} sentences:

(2) 

and

(3) 

we would have: for some <D,f>∈F, V(<(2),<D,f>>) = truth, while V(<(3),<D,f>>) = falsehood, whereas, for _all_ <D,f>∈F', if V(<(2),<D,f>>) = truth, then V(<(3),<D,f>>) = truth as well, owing to the additional structure imposed on every <D,f>∈F' by

(1^{o}). This latter means, in effect, that (1^{o}) functions as an "assumption" on the structure of every <D,f>∈F'; in particular, every <D,f>∈F' must fulfill the following structural constraint: The sets which are elements of f(IND MARY_{T})$_{T}$ and the sets which are elements of f(INDEF MARY_{T})$_{T}$ must be inter-related as follows: there is an x∈f(IND MARY_{T})$_{T}$ such that, for all y ∈ x, there is a z ∈ f(INDEF PERSON_{T})$_{T}$ such that, for all w ∈ z, y = w. This establishes a definite set-theoretic relationship between the denotations of "IND JOHN_{T}"$_{T}$ and "INDEF PERSON_{T}"$_{T}$ that now defines now defines additional structure for the interpretations of F' as compared to the interpretations of F. These inter-related structures will, in general, hold for some interpretations <D,f> ∈ F but will fail for others. For those for which they fail, (2) may hold while (3) may fail to hold, whereas, for <u>all</u> interpretations <D,f> ∈ F' on which (2) holds, (3) will also hold. The detailed basis for this fact can be argued as follows:

Assume, from (2), that

(i) There is an $x \in f(\text{IND JOHN})$ such that for all $y \in x$ there is

T
T

$z \in f(\text{IND MARY})$ such that for all $w \in z$, $\langle y,w \rangle \in f(\text{LOVE A D})$.

T
T

R
R^1
R^2

Now, from (1), we have

(ii) There is an $x_1 \in f(\text{IND MARY})$ such that for all $y_1 \in x_1$ there

T
T

is a $z_1 \in f(\text{INDEF PERSON})$ such that for all $w_1 \in z_1$, $\langle y_1, w_1 \rangle \in f(\text{BE A C})$,

T
T

R
R^1
R^2

i.e., $y_1 = w_1$.

Let us instantiate x in (i) to x^o, so that we obtain

(i') $x^o \in f(\text{IND JOHN})$ such that for all $y \in x^o$ there is a

T
T

$z \in f(\text{IND MARY})$ such that for all $w \in z$, $\langle y,w \rangle \in f(\text{LOVE A D})$

T
T

R
R^1
R^2

In order to show that (3) is true in $\langle D,f \rangle$ it suffices to show, for x^o as described in (i') that:

(iii) for all $y \in x^{o}$ there is a $z \in f(\text{INDEF PERSON})$ such that for

T

T

all $w \in z$, $\langle y,w \rangle \in f(\text{LOVE A D})$

R

R^{1}

R^{2}

Accordingly, letting $y^{o} \in x^{o}$, we need to show that:

(iv) There is a $z \in f(\text{INDEF PERSON})$ such that for all $w \in z$,

T

T

$\langle y^{o},w \rangle \in f(\text{LOVE A D})$

R

R^{1}

R^{2}

Now by (i') we have that

(v) There is a $z \in f(\text{IND MARY})$ such that

T

T

for all $w \in z$, $\langle y^{o},w \rangle \in f(\text{LOVE A D})$

R

R^{1}

R^{2}

Let us instantiate z in (v) to z^{o}, so that we have

(v') $z^{o} \in f(\text{IND MARY})$ such that for all $w \in z^{o}$,

T

T

$\langle y^{o},w \rangle \in f(\text{LOVE A D})$

R

R^{1}

R^{2}

Now $x_1 \cap z^o \in f$(IND MARY) and

T
T

(vi) for all $y \in x_1 \cap z^o$ there is a $z \in f$(INDEF PERSON)

T
T

such that for all $w \in z, y = w$.

Recall, from (v'), that for all $w \in z^o$, that $\langle y^o, w \rangle \in f$(LOVE A D),

R
R^1
R^2

hence that,

(vii) for all $y \in x_1 \cap z^o, \langle y^o, y \rangle \in f$(LOVE A D)

R
R^1
R^2

Therefore, for all $y \in x_1 \cap z^o$ there is a $z \in f$(INDEF PERSON) such

T
T

that for all $w \in z, y = w$ and, by (vii), $\langle y^o, y \rangle \in f$(LOVE A D).

R
R^1
R^2

Hence for all $y \in x_1 \cap z^o$ there is a $z \in f$(INDEF PERSON) (i.e., {y})

T
T

such that for all $w \in z$ (i.e., y), $\langle y^o, w \rangle \in f$(LOVE A D).

R
R^1
R^2

Hence there is a $z \in f'(\text{INDEF PERSON})$ such that for all
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad T$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad T$

$w \in z, \langle y^{o}, w \rangle \in f(\text{LOVE A D})$, which is just the desired condition (iv),
$\qquad\qquad\quad R$
$\qquad\qquad\quad R^{1}$
$\qquad\qquad\quad R^{2}$

and completes the argument.

There are also, of course, <u>logical refinements</u> as well as lexical refinements of semantic theories. Indeed, the fund of semantic logical axioms forwarded in this paper is in no way to be considered as complete, nor is the fund of logical representational morphemes whose structure they define. As further semantic logical axioms are added to those already specified, we obtain logical refinements of the system to which they are added.

Given a set J of L-sentences generated in a context-of-utterance C, we will indicate how a family of possible lexical refinements of any semantic theory s = ⟨F,V,R⟩ of INT_{L}^{TR} can be obtained from J and C. We first describe how such a family can be obtained for the case where J is a singleton set consisting of a single L-sentence e. Accordingly, Let K be a set of L-sentences, let A be a reading assignment on K, and let B be a reading of e, where both A and B are normal with respect to C, and where 2A = 2B = s = ⟨F,V,R⟩. Let A* be obtained from K, A, e, and B as follows:

We first obtain a lexical refinement s' = ⟨F',V,R⟩ of s = ⟨F,V,R⟩ from J = {e} by requiring that the interpretations

$\langle D,f\rangle \in F'$ be those interpretations of F that also satisfy the semantic axiom:

(e^{o}) $V(\langle e,\langle D,f\rangle\rangle) = \text{truth}$

In particular, the lexical refinement of $s = \langle F,V,R\rangle$ involving examples (1), (2), and (3) above, was obtained by adding the lexical semantic axiom (1^{o}) above to the semantic axioms defining F. Then we define the reading assignment A^* such that, for all $k \in K$, $A^*(k) = \langle 1A(k), s'\rangle$ and $A^*(e) = \langle 1B(e), s'\rangle$; that is, the syntactic representations of the sentences in K under A^* are exactly those under A, but the semantic theory that interprets them is now the lexical refinement s' of s instead of s itself, obtained by imposing the additional lexical structure defined by (A) onto the interpretations $\langle D,f\rangle \in F$, to form the subset $F' \subseteq F$ of interpretations fulfilling that structure. We call A^{o} *the lexical refinement of A determined by K, {e}, and B*. More generally, we can define *the lexical refinement of a reading assignment A determined by sets K, J, of L-sentences and by a reading assignment B of J* such that the domain of A is K and the domain of B is J, and $2A = 2B$ as that reading assignment A^* which has the domain $K \cup J$, and which is such that, for all $k \in K$, $A^*(k) = \langle 1A(k), s'\rangle$, where for all $j \in J$, $A^*(j) = \langle 1B(j), s'\rangle$, and s' is a semantic theory which is an element of INT_L^{TR} and is obtained from $2A(K \cup J) = \langle F,V,R\rangle$ by defining $s' = \langle F',V,R\rangle$, where F' is that subset of F defined as follows: $\langle D,f\rangle \in F'$ if and only if $\langle D,f\rangle \in F$ and, for all $j \in J$, $V(\langle j,\langle D,f\rangle\rangle) = \text{truth}$.

The above definition has the following semantic consequence (a) and pragmatic consequences (b) and (c) relative to K, J, A, B as defined above:

(a) The lexical refinement A* of A determined by K, J, and B induces an entailment relation on the sentences of L relative to which every sentence $j \in J$ is entailed by every set K' of sentences of L under every reading assignment A' on K' such that 2A' = 2A*.

(b) If, furthermore, A was a normal reading assignment on K with respect to a context-of-utterance C, then A* would also be a normal reading assignment on $K \cup J$ with respect to C, provided that a language user would regard all the sentences of J as true of C under those ways of understanding them that are formalized by the reading assingment B on J;

(c) Assuming, still, that A is a normal reading assignment of K with respect to C, and if we take the potentially verbalizable part PVC of C and let J be the set of those L-sentences that are intuitively true of C, and if B is a reading assignment of J that is normal[101] with respect to C, then the lexical refinement of A determined by J, K, and B would also be normal with respect to C, for B determines only the pattern of inter-entailments on J, which assures that B is logically normal, which is all one needs to assure that the consequences of J will also be true of C. This is of particular importance in the case where J is a small subset of PVC--as is inevitably the case in applications--so that it is not assured that J is closed under logical consequences, as would of course be the case where J = PVC, since logical consequences of sentences that are true of the context would also be true of the context.

The above situation involves several sorts of oversimplifications of a pragmatic character: The first sort of oversimplification arises from the fact that we have not allowed for the fact that the sentences of K would not, strictly speaking, be produced simultaneously within a single context-of-utterance C so that, for precise delineation of context, the context-of-utterance C should be stratified to yield a different context C_k for each sentence (token) $k \in K$. Indeed, the sentences of K produced before a given sentence $k \in K$ is produced would comprise part of the context-of-utterance of k itself.

The second sort of oversimplification derives from our not having accounted for the fact that, as successive sentences k of K are produced, not only does the context-of-utterance of successive sentences k change, i.e., not only does the set J and its reading assignment B change, but so does the reading assignment A_k on that subset of K containing all the sentences up to K.

Let us consider an example of this latter phenomenon:

Let K = {(4),(5)}:

(4) John saw Henry looking with a telescope

(5) Henry was blind

Thus, if (4) and (5) are produced successively, the context-of-utterance C_5 of (5) differs from the context-of-utterance C_4 of (4) insofar as C_5 includes also the hearer's belief that the speaker wishes to affirm the truth of (5) as well as of (4).

Note 101. By this I mean that all of the SYN_L^{TR}-sentences 1B(j) are true in every interpretation of 2B.

Thus the hearer might well update his choice of syntactic representation in A_4 or (4) after hearing (5).

If the hearer, moreover, accepts (4) (or (5)) as *true*, then the reading assignment itself becomes semantically updated, that is, by forming a lexical refinement of A, within which the reading of (4) was chosen, and in which (4) was true. Thus we have two sorts of updates--one a *syntactic update* wherein the syntactic representation is changed; the other is a *semantic update* wherein the semantic theory itself is lexically refined. In the remainder of this section we will describe several sorts of semantic updates, namely: *lexical updates*, of which the preceding was an example, and *valuation updates*.

When a person asserts a sentence e, i.e., produces a sentence token e#, he usually intends to be understood as affirming the truth of e, that is, as intending a reading $\langle e',s\rangle$ of e under which, letting $s = \langle F,V,R\rangle$, $V(\langle e',\langle D,f\rangle\rangle)$ = truth for all $\langle D,f\rangle \in F$, and not simply a reading that induces some expected type of inter-entailments. On the other hand, the truth of e is in some sense "in contest" between speaker and hearer; that is, while the speaker may intend to be understood as affirming the truth of e, the hearer is under no compulsion to accept e as true, nor does the speaker necessarily hold that the hearer accepts e as true.

Ordinarily, a speaker, upon producing a given sentence S_n, the n^{th} sentence of a sequence of produced sentences, intends that it be regarded as true by the hearer; the hearer, on the other hand, may or may not regard it as true. If he does, then the context-of-utterance C becomes upgraded by having S_n become part of the "given," by a suitable lexical update of the reading assignment in force before the production of S_n, as in the case with example (5), above.

Generally, we may distinguish that which the hearer can reasonably be regarded as doing in a communication situation--namely, identifying how e is to be understood without prejudicing the truth or falsity of e, from that which occurs at a more advanced stage of the communication transaction, namely where e, having been produced, becomes then part of the context-of-utterance for understanding e, as well as for understanding subsequently produced word-strings.

Consider the English sentences:

(6) John loves Mary

(7) John is a person

(8) John is a dog

Suppose that the sentence (6) were produced in a context C_6, where people were being generally discussed, say, mutual acquaintances of the speaker and hearer, and in which the hearer knew Mary, a person, but did not know anything about the entity referred to as "John." The hearer might well assume that the speaker intended that (7) also be regarded as true, since in most contexts-of-utterance like C_6, (i.e., where people are being generally discussed, and where the referrent of "Mary" is already known (by the hearer) to be a person) in which a sentence like (6) is produced (i.e., a sentence whose subject is ordinarily used as the name of a person, and whose verb is ordinarily one used with not only animate but human subjects), the individual name in the subject position is the name of a person. We would say, then, that the hearer, upon hearing (6) in the context C_6, assumes that the speaker intends that the hearer adopt a semantic theory in which, not only (6), but also (7) were true in the sense of holding in all interpretations in that semantic theory. One possible mechanism for obtaining a suitable semantic theory would be to obtain a lexical refinement of that semantic theory in which (6) is true, obtained by adding an additional semantic lexical axiom to that theory to the effect that an appropriate syntactic representation (7') of (7) were true in every interpretation of the refinement. If this mechanism were adopted, we would then say that (7) was a _lexical presupposition of (6) relative to the context_ C_6 _and relative to that refinement_.

As a second example, if (6) were produced in a context-of-utterance C_7 such as the following: The hearer and speaker operate a kennel for dogs and have given names to dogs that are ordinarily appropriate to humans. The hearer knows Mary, a particular dog, but does not know the entity to which "John" refers. Upon hearing (6) in this context, the hearer might well assume that John was a dog, i.e., (8), since in most contexts-of-utterance, like the one in which (6) has been produced, the entity to which "John" refers would be likely to be a dog. We would say, then, that the hearer, upon hearing (6) in the context of C_7, assumes that the speaker intends that the hearer adopt a semantic theory in which, not only (6), but also (8) were true in the sense of holding in all interpretations in that semantic theory. One possible mechanism for obtaining a suitable semantic theory would be to obtain a lexical refinement of that semantic theory in which (6) is true, obtained by adding an additional semantic lexical axiom to that theory to the effect that an appropriate syntactic representation (8') of (8) were true in every interpretation of the refinement. If this mechanism were used to obtain a suitable semantic theory, we would then say that (8) was a _lexical presupposition of (6) relative to the context_ C_7 _and relative to that refinement_.

As an example of another sort, consider

(9) Bill realized that John loves Mary

Upon hearing (9) in a context C_9 where Bill, John, and Mary were known to the hearer, the hearer might well assume not only that the speaker intended that (9) be regarded as true, but that the speaker also intended that

(10) John loves Mary

be regarded as true, since in most contexts-of-utterance in which a speaker uses the word "realize," that speaker intends that the state of affairs which is realized be regarded by the hearer as actually being the case, i.e., that the sentence, like (10), describing that state of affairs, be regarded as true. The mechanism the hearer employs here would be just like the one employed in the earlier two examples. That is, the hearer, upon hearing (9) in given context-of-utterance C_9, assumes that the speaker intends a semantic theory be adopted in which, not only (9), but also (10) is true in the sense of holding in all the interpretations in that semantic theory. One possible mechanism for obtaining a suitable semantic theory would be to obtain a lexical refinement of that semantic theory in which (10) were true, obtained by adding an additional semantic lexical axiom to that theory. If this mechanism were used to obtain a suitable semantic theory, we would then say that (10) were <u>a lexical presupposition of (9) relative to the context C_9</u> and relative to that refinement.

We make the above notion of lexical presupposition explicit in (i) below and extract two grammatical consequences (ii) and (iii) from (i):

(i) An L-sentence b is <u>a lexical presuppositon of an L-sentence a relative to a context-of-utterance-C in which a</u>

is produced and relative to a reading assignment A on the set {a,b} which is normal with respect to C if and only if b is true in A(2), that is, letting A(2) = <F,V,R>, then for all <D,f>∈F, V(<b,<D,f>>) = truth.

(ii) An L-sentence b is a lexical presupposition of an L-sentence a relative to a context-of-utterance C in which a is produced, if and only if for some reading assignment A on {a,b} which is normal with respect to C, b is a lexical presupposition of a relative to C and A.

(iii) An L-sentence b is a lexical presupposition of an L-sentence a, if and only if, for some context-of-utterance C in which a is produced, and for some reading assignment A on {a,b} which is normal with respect to C, b is a lexical presupposition of a relative to C and A.

Let us now alter the context C_6 slightly in the first example 6 to bring up another point: say that, in that example, the speaker had augmented the context C_6 to a context C'_6 which included also his indicating that he intended

(11) John loves a person

to follow from (6). Then that reading r in which (7) were true would be of a "higher" degree of normality with respect to this augmented context, as compared to the normality of r with respect to the original context (i.e., that in which no indication was given the speaker of his intent that (11) follow from (6)). Note that, while still in context C_6, since (6) lexically presupposes (7) relative to C_6, and since (6) and (7) entail (11) under the dominant normal readings of (6), (7), (11), we also have that (6) lexically presupposes (11) relative to C'_6.

There also are cases where more than one mechanism can be used to obtain a suitable semantic theory to accommodate the speaker's apparent intent. Consider:

(12) The man whom John struck is angry

Upon hearing (12) in a context C_{12} where John is known to the hearer, and the hearer has as yet no knowledge of any man whom John might have struck, the hearer might well assume not only that the speaker intended that (12) be regarded as true, but that the speaker also intended that

(13) John struck a man

be regarded as true, since in most contexts-of-utterance in which a speaker employs (i.e., produces) a definite description of the form "The x such that B(x)," that speaker intends that the hearer regard the sentence "There is an x such that B(x)" as true. We would say, then, (as earlier) that the hearer, upon hearing (6) in the context C_9, assumes that the speaker intends that the hearer adopt a semantic theory in which, not only (12), but also (13) were true in the sense of holding in all interpretations of that semantic theory in which (7) were true. There are two possible mechanisms (A) and (B) available to us here: (A) we could, as before, obtain a suitable semantic theory as a lexical refinement of the semantic theory s in which (12) is true, a process we refer to as a lexical updating. More exactly, say s = $\langle F,V,R \rangle$; then we would obtain a semantic theory s' = $\langle F',V,R \rangle$ as a lexical updating of s by adding to the lexical semantic axioms the additional condition for some appropriate syntactic representation (13') of (13):

(*) V(<<(13'),<D,f>>) = truth, for all <D,f>$\in$F,

the effect of which is to restrict the interpretations <D,f> of F to just those satisfying (*), the restricting subset of F being precisely F'. If we chose this mechanism, we would say that (13) were a lexical presupposition of (12) with respect to the context C_9; (B) we could, instead, obtain a suitable semantic theory from s by adding to the *valuational rules* for s, a process we refer to as a *valuational updating*. More exactly, say s = <F,V,R>; then we would obtain a semantic theory s' = <F,V',R> as a valuational updating of s by adding to the truth clause of V the following additional condition : (**) For all definite thing-expressions a occurring in sentence e of SYN_L^{TR}, $f(a)\neq\{\phi\}$. That is, for all sentences e of SYN_L^{TR}, and <D,f>$\in$F, V'(<e,<D,f>>) = truth, just in case V(<e,<D,f>>) = truth and (**).

If mechanism (B) were adopted, we would also say that (13) were *a valuational presupposition of (12) relative to the context* C_9 *and relative to the valuational refinement s' of s*. The choice of whether to treat (13) as a lexical or valuational presupposition of (12) (with respect to C_9) is one of expeditiousness: The advantage of the latter, that is, of choosing mechanism (B), is that it is more economically specifiable than (A), since if one were to choose to treat all definite expressions (i.e., regardless of context) in this way, it would be more economical to choose mechanism (B), which would state the requisite condition once and for all, whereas choosing (A) would require a lexical refinement for each case.

There is another point with regard to the difference between lexical and valuational presuppositions: The former are "lexical" in the sense of providing specific set-theoretic structure to the syntactic representations of lexical morphemes whereas the latter are "logical" in character, in the sense of providing specific (further) set-theoretic structure to the syntactic representations of logical morphemes, i.e., here, to the English determiner "the." Nonetheless, we distinguish between logical and valuational refinements by designating any refinement as valuational which involves the sole set-theoretic condition that some denotation be different from $\{\phi\}$.

The preceding discussion involving lexical and valuation refinements is yet oversimplified in certain additional respects, which need now also to be identified: First, lexical refinements were obtained for a _single_ sentence (rather than for a set of sentences) at a time; second, only _explicitly asserted_ sentences were considered, rather than including the consideration also of sentences which are not explicitly asserted but which correctly describe some features of the context-of-utterance in which the explicitly asserted sentences were produced; third, no consideration of the action of the readings of subsequently produced sentences on the readings of earlier-produced sentences was made, i.e., no back-up operations for updating the readings of lexical-referrents-in-progress have been provided.

When a speaker produces a sentence e_1 of L within a context-of-utterance C_1, the context C_1 supplants that context C_o which "prevailed" just prior to the production of e_1 and represents an updating of C_o by the production of e_1. C_1 can differ from C_o in various ways, including, besides the hearer's updated estimate of facts to which the specific content of e_1 pertains, also the hearer's updated estimate of the speaker's intent in uttering e_1, his estimate of the speaker's estimate of the hearer's estimate of facts to which the specific content of e_1 pertains, and of the speaker's estimate of the hearer's estimate of the speaker's intent in uttering e_1, and so on. The reading r_1 assigned to e_1 by the hearer is determined by C_1; and the reading r_2 assigned to e_2 is determined[102] by C_2. The reading r_1 of e_1 is updated to $r_{1,2}$ after the production of e_2, where $r_{1,2}$ is a normal reading of e_1 with respect to C_2; the reading $r_{1,2}$ is a normal reading of e_1 with respect to C_2; the reading $r_{1,2}$ of e_1 is then updated to $r_{1,2,3}$ after the production e_2 where $r_{1,2,3}$ is a normal reading of e_2 with respect to C_3, and so on. Thus, we obtain a sequence of produced sentences e_1, e_2,...,e_n each with its fully updated normal reading, that is, each updated up till the production of the last produced sentence e_n:

Note 102. Clearly, the reading r_1 assigned to e_1 by the hearer may also be a function of special characteristics of the hearer, that is, of possibly idiosyncratic ways by which he arrives at estimates of content, intent, etc. as mentioned above. However, here, as elsewhere, we assume a "typical" rather than "idiosyncratic" hearer. This simplifying assumption would be waived when examining "non-normal" readings.

sentence	updated reading
e_1	$r_{1,2,\ldots,n}$
e_2	$r_{2,3,\ldots n}$
e_{n-1}	$r_{n-1,n}$
e_n	r_n

It is understood, throughout, that each of the above fully updated readings has the same semantic theory s, so that we can define a reading assignment A_K on the set $K = \{e_1,\ldots,e_n\}$ such that, for each $1 \leq i \leq n$, $A_K(e_1) = \langle 1r_{i,i+1,\ldots,n}, s\rangle$

We next attempt to provide a model of where such a reading assignment A_k might come from:

Let PVC_i be the potentially verbalizable part of the context-of-utterance C_i, described in L by the set $(PVC_i)^*$ of sentences of L, which is that existing just before the production of sentence e_i, and let $r_{i,i+1}$ be obtained from r_i by lexically enriching r_i by some reading assignment B_i of $(PVC_i)^*$ which is normal with respect to C_i. The sentences of $(PVC_i)^*$ give verbal expression within L to all the information that is so expressible concerning C_i and which the typical hearer might reasonably be expected to know and apply to his determining a reading of the sentence e_i that is normal with respect to C_i. Then the semantic theory s of A_K is simply the semantic theory of the reading assignment B_{n+1}. It is convenient to stratify lexical presuppositions as follows: First level lexical presuppositions of a sentence a of L with respect to the context-of-utterance in which a is produced are lexical presuppositions that involve no reference to the speaker or hearer. _Second level lexical presuppositions_ of a with respect to C are those lexical presuppositions of a that _do_ involve such reference to the speaker or hearer. We have seen some examples of first level lexical presuppositions. As an example of a second level lexical presupposition of the sentence (6) with respect to the context C_6 above (page 349) we can consider

(14) The speaker believes that John loves Mary

Let us assume that the context C_9 is like C_6 except that, instead of sentence (6) being produced (and thus being considered as part of that context) the following sentence (15) was instead produced:

(15) It is John that loves Mary

While sentences (6) and (15) are equivalent under their dominant normal readings, they do not have the same lexical presuppositions with respect to their respective contexts-of-utterance. In particular (15) has a second level lexical presupposition with respect to C_9 which (6) does not have with respect to C_6, namely:

(16) The speaker believes that the hearer already knows that something (or someone) loves Mary

Thus the speaker would produce the sentence (15) instead of (6) if he wanted the hearer to adopt a semantic theory in which (16) were true, and would produce the sentence (6) instead of (15) if he wanted the hearer to adopt a semantic theory in which (16) were not true. In ways such as this, the speaker guides the hearer's choice of a reading assignment by means of which the hearer comes to understand the speaker's utterances as the speaker intends them to be understood. Pragmatics is concerned precisely with such matters, that is, with the way that the structure of a word-string signals its various normal readings in the communications transaction between speaker and hearer. The fragmentary remarks of this section are intended only to indicate how such considerations, already forwarded in various ways in the general literature on presuppositions, might be accommodated within the framework of a theory of readings. Since the literature on presuppositions deals in part with the notion of "focus," we will attempt to relate the above discussion to that notion as well: The distinction between (6) and (15) can be described as a distinction in focus. That is to say, under the dominant normal readings of (6) and (15) with respect to their contexts-of-utterance, (15) distinguishes old information (that someone loves Mary) from new information (that John is that someone), whereas (6) does not so distinguish these items, or does so only to such a faint degree--as imposed by the fact that the occurrence of "John" in (6) precedes that of "Mary"--that it is easily over-ridden by other factors in the communication situation.[103]

We end this discussion by briefly noting some cases where lexical presuppositions are influenced by focal differences in

---

Note 103. As remarked earlier, in a fuller specification of a reading, one would formalize, beyond the entailment aspects of understanding, also the "focal aspects" of understanding, by which I mean the notational enrichment of a reading as thus far developed in this paper to include the focal inter-relationships among word-string parts and how they relate to the context-of-utterance.

discourse structure (in (17),(18) below) rather than in sentence structure, brought about by a difference in order of occurrence of the constituent sentences of that discourse. Consider the following:

(17) John sat down. The stool turned over.

(18) The stool turned over. John sat down.

(19) John had tried to sit on the stool.

(20) John had not tried to sit on the stool.

(21) John's sitting down was the cause of the stool's turning over.

(22) The speaker intends that the hearer believe that John's sitting down was the cause of the stool's turning over.

While (17) and (18) have the same entailments, (17) lexically presupposes (19), (21), and (22) but not (20). On the other hand, (18) lexically presupposes (20) but neither (19), (21) nor (22).

CHAPTER 3

Some Classes of Distinguished Readings

This chapter deals with some special types of readings of natural language word-strings, each corresponding to some common notional characteristic pertaining to word-strings, specifically, to the notional characteristics of (being an instance of) sentence, clause, discourse, anaphora, ellipsis; adjective, adverb, ergativity, intensionality, and branching quantification. Accordingly, in the following sections, we discuss their corresponding types of readings, called, respectively, sentential readings, clausal readings, discourse readings, anaphoric readings, elliptical readings, adjectival readings, adverbial readings, ergative readings, intensional readings, and branching quantifier readings.

Our intent is not to treat any of the above types of readings in a thorough way, but rather to illustrate the application of the theory of readings to an account of some standard notional characteristics of word-strings and, by such illustration, to indicate how the discussion of standard grammatical notions might be conducted within a theory of readings.

Throughout this chapter we let s_0 be the minimal semantic theory satisfying the logical semantic axioms L.S.A.(1)-L.S.A.(31) of Chapter 2, that is, we let s_0 be the intersection of all semantic theories satisfying those axioms; moreover, we adopt the convention that, when a reading of a given word-string is identified without explicit mention of its semantic theory, (that is, when a reading of a word-string is identified simply by giving the syntactic representation of that word-string), then s_o is understood to be the semantic theory comprising the second component of the reading rather than, say, some special refinement of s_o.

3.1. Sentential Readings

The sentence unit appears fundamental in treatments of natural language.

The notional significance of a sentence is that it describes an event or circumstance, allowing that we take ellipsis into account whereby fragments of sentences are understood as complete sentences by virtue of having implicitly realized components.

A sentential reading of a word-string of a natural language L is a reading of that word-string whose syntactic component is a sentence of SYN_L^{TR}, (as described in Section 2.3.1.1.2.3 of Chapter 2).

Any natural language word-string can be accorded a sentential reading (if we allow ellipsis), but not all such sentential readings would be normal.

By a sentence of L, then, would be meant any word-string of L that has a normal sentential reading.

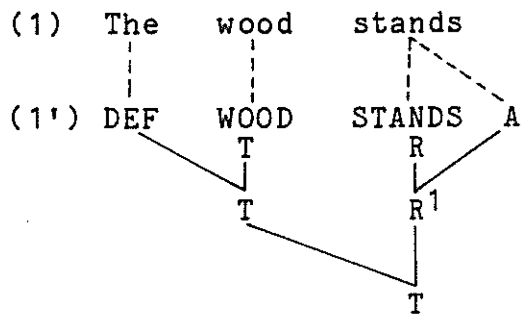
The notion of a natural language sentence, then, is therefore essentially a semantic rather than a purely syntactic notion. Let us consider some examples:

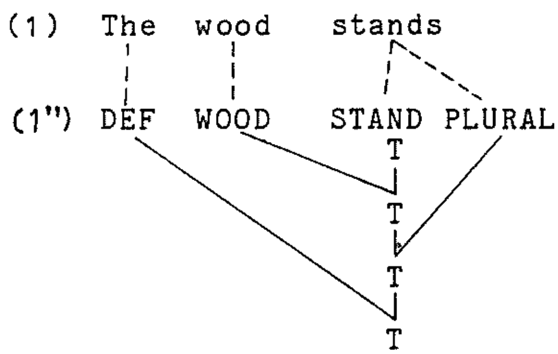
The English word-string

(1) The wood stands

has, among its normal readings, at least one sentential reading and at least one non-sentential reading, whose respective syntactic components in $SYN_{English}^{TR}$ are[104]:

Note 104. Recall that, by our above convention, when a reading is identified solely by giving its syntactic component as we are here doing, the semantic component of that reading is understood as the minimal semantic theory s_0 satisfying the logical semantic axioms L.S.A.1 - L.S.A.31.





Thus, whether we regard (1) (or any other natural language word-string) as a sentence is wholly dependent on the reading we intend to accord it.

Let us look at an example of a sentential reading that is, at best, marginally normal. The English word-string

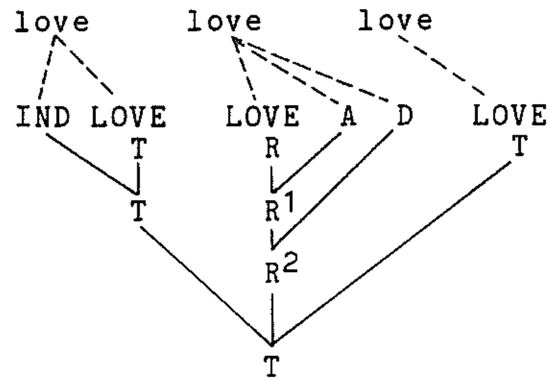
(2) love love love

probably does not have any strong normal readings since it is unlikely that an English speaker, under common conditions of use, would intuitively regard any word-string as being entailed by or entailing (2), at least in any compelling way. One possible (though weak) intuitive entailment involving (2) is that whereby (2) entails (3):

(3) love love

This intuitive entailment can be obtained by assigning the following respective readings to (2) and (3):

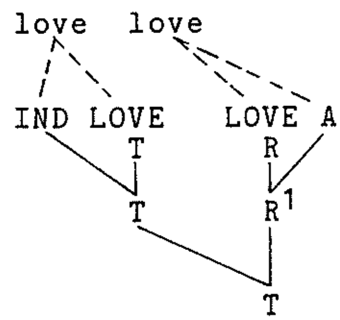
(2)
(2')



which could be paraphrased as: "Love loves itself"

and

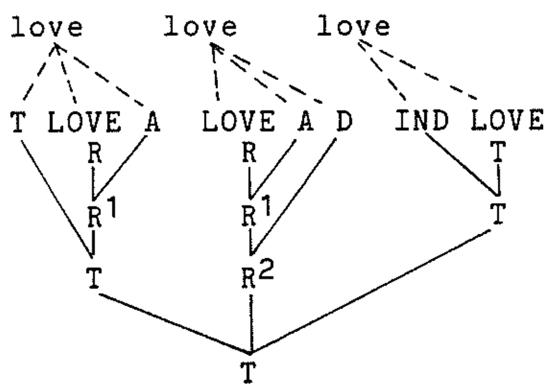
(3)



which would be paraphrased as: "love loves."

Somewhat stronger readings of (2) and (3) would be the following:

(2')
(2'')

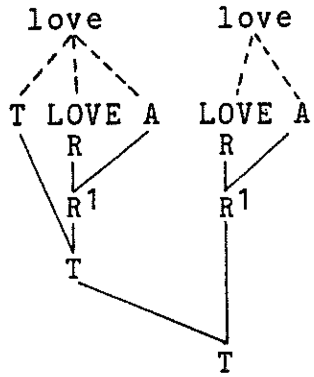


which could be paraphrased as: "Everything that loves, loves love"

and

(3)

(3'')



which could be paraphrased as: "Everything that loves loves."

Both (2') and (2'') would each comprise stronger normal readings of the (more grammatical) English word-string:

(2^{o}) love loves love

and both (3') and (3'') would each comprise stronger normal readings of the (more grammatical) word-string

(3^{o}) love loves.

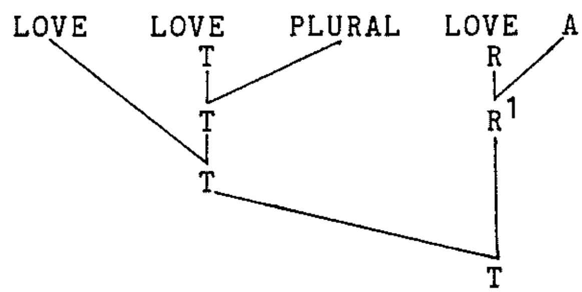
The reason that (2') and (2'') would be stronger normal sentential readings of "love loves love" than of (2) is that "love loves love", unlike (2), is sufficiently marked to signal the reading of (2') or (2'') as possible (normal) readings of (2). The markings themselves include the occurrence of the terminal "s" on "loves", and the order of occurrence of "loves" relative to the two occurrences of "love," which themselves do not have a terminal "s". These possible normal readings would of course have to be further filtered through the context-of-utterance in which (2) is produced, in order that one of them survive as that reading of "love loves love" that has the highest degree of normality with respect to that context. By this latter is meant, of course, that, if other sentences $k_1,\ldots,k_n$, (such as "love loves") were to be produced within that same context-of-utterance, that some reading assignment that assigned readings (2') or (2'') to "love loves love" would induce an entailment relation on the set {"love loves love", $k_1, .., k_n$} that was more consistent with English speakers' intuitions regarding entailment with respect to that context than any other entailment relation induced on that same set by a reading assignment on the sentences of that set which, in particular, assigned a reading to "love loves love" other than (2') or (2'').[105]

Generally, any word-string of a natural language can be assigned a sentential reading, but cannot necessarily be assigned a <u>normal</u> sentential reading for, ultimately, the normality of a reading depends on whether users of that language would tend to agree with the entailments induced on sets of sentences containing that word-string as element. Thus a word-string that is unusual or deviant in its markings cannot be expected to enjoy a consensus among language users regarding the intuitive entailments into which it enters. In this sense, a normal reading of a word-string can be regarded as a reading of that

---

Note 105. Other possible, but very weak, normal sentential readings of "love loves love" could be identified; these would include, for example, the reading:

(2''')



which has the meaning, very roughly, that certain things, namely "love-loves" themselves love.

word-string that is commonly (though not necessarily usually) assigned to it by language users -- at least to a degree to enjoy what we might regard as a consensus.

We had earlier remarked that, because (2) was deviant in the above sense, that the reading (2') would be only a marginally normal reading for the word-string (2). "Marginality" here means that, if A were a reading assignment on a non-trivial set K of sentences that contained (2) as an element, and which was such that $A(2) = \langle (2'), s_0 \rangle$, then it would be unlikely that any common context-of-utterance in which K was produced[106] would be able to carry sufficient auxiliary information which, in conjunction with K, would enable language users to come to any consensus regarding the entailments induced on K by the reading assignment A.

3.2. Discourse Readings

The notion of a discourse is an intuitive semantic one, just as is the notion of a sentence. A discourse is not merely a set of natural language sentences; rather, it is a set of natural language sentences that are inter-related by explicitly or implicitly marked natural language relational morphemes. When explicitly marked in English, these morphemes are variously realized by word-strings such as "and", "but", "so", "therefore", "later", "as soon as", "then", and so on. These morphemes are

---

Note 106. Strictly speaking, as has been pointed out in Section 2.3.3, the context-of-utterance is slightly different for each occurrence (token) of (2). Broadly speaking, however, these contexts are usually so similar as to allow us, in practice, to consider them identical.

implicitly marked by various devices, including the order of production of the sentences, intonation, stress, the pattern of content-interconnections and referential links joining word-string-parts of the sentences entering into the discourse. Indeed, a succession of sentences implicitly marked by various of these devices can sometimes be understood (i.e., "read") as being

successively related by the implicit natural morpheme ordinarily explicitly marked by "and" in English, as is often done in oral speech and represented in normal readings of the representational language by the representational morpheme $\text{CONJ}_{R \to R^1 \to R^2}$ A D.

A discourse reading is a special sort of sentential reading, namely one whose syntactic component is such that each of its major thing-expressions is of the form $\overset{a}{T}$, where a is a sentence. That is, a _discourse reading_ of a word-string is a sentential reading of that word-string whose syntactic representation component is such that each of its major thing-components is a sentence of SYN_L^{TR}.

A word-string is a _discourse_ if and only if its dominant normal reading with respect to most usual contexts-of-utterance is a discourse reading.

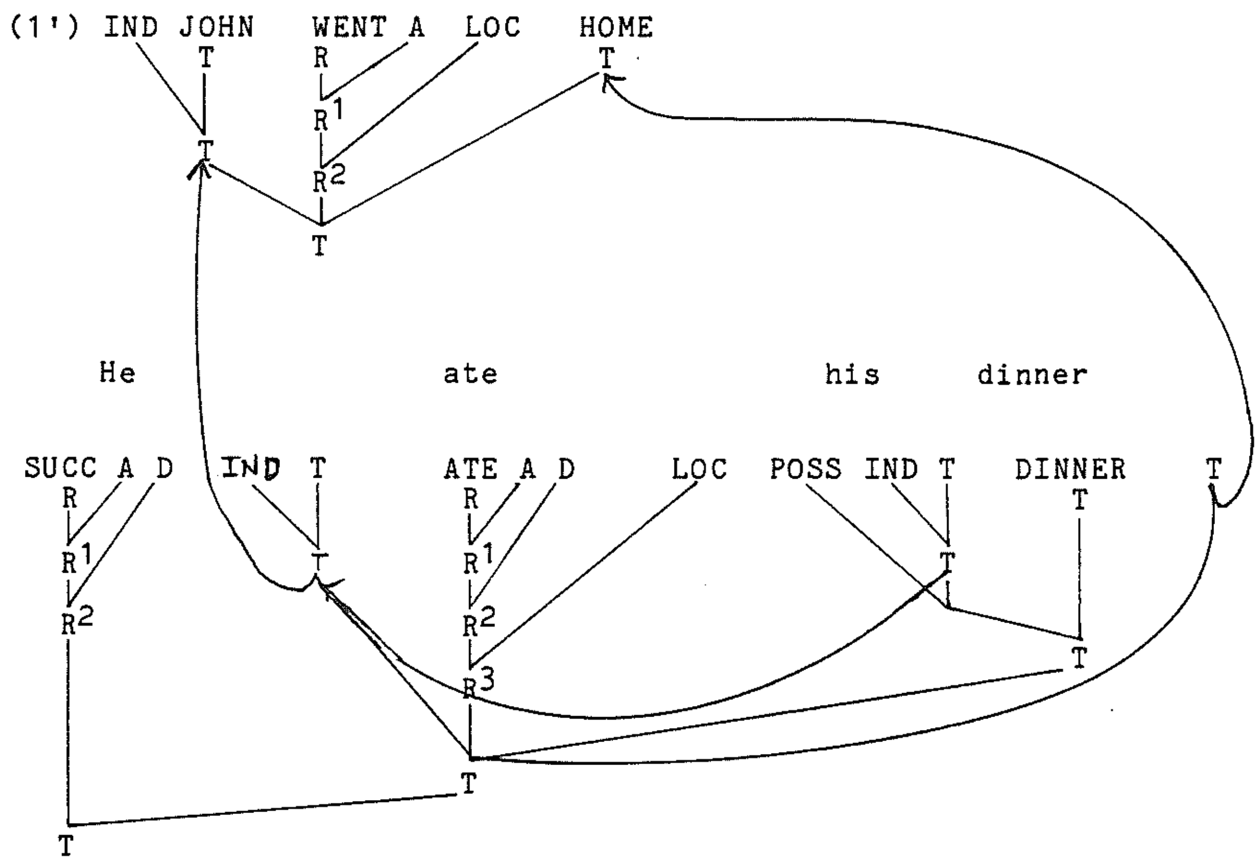
We give some examples of discourse readings; consider this set of English sentences:

(1) (i) John went home

(ii) He ate his dinner

We assign a discourse reading to (1), that is, a reading which connects (1i) and (1ii) within a discourse; namely:

(1) John went home



Roughly speaking, the reading (1') of (1) is that which would be equivalent to the dominant normal reading of

(2) (i) John went home.

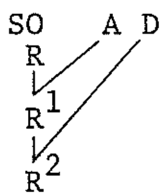
(ii) Then John ate John's dinner at home.

In (2) the (natural language) morpheme "then" is explicit; whereas in (1), it is implicit, indicated essentially by the order in which the sentences (1)(i) and (1)(ii) are exhibited (or produced).

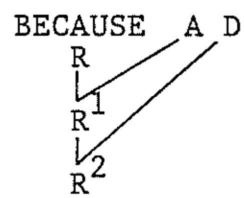
In the following example, all intersentential relations are explicit:

(3) John loves Mary. So John likes Mary. But he dislikes Nancy. So he does not love Nancy.

Below we indicate the dominant normal discourse reading (3') of (3), and the dominant normal non-discourse reading (3'') of (3). It is clear that the discourse reading (3') would have a higher degree of normality than the non-discourse reading (3''), insofar as the intersentential relations in (3) are explicit. In particular, under suitable logical semantic axioms for



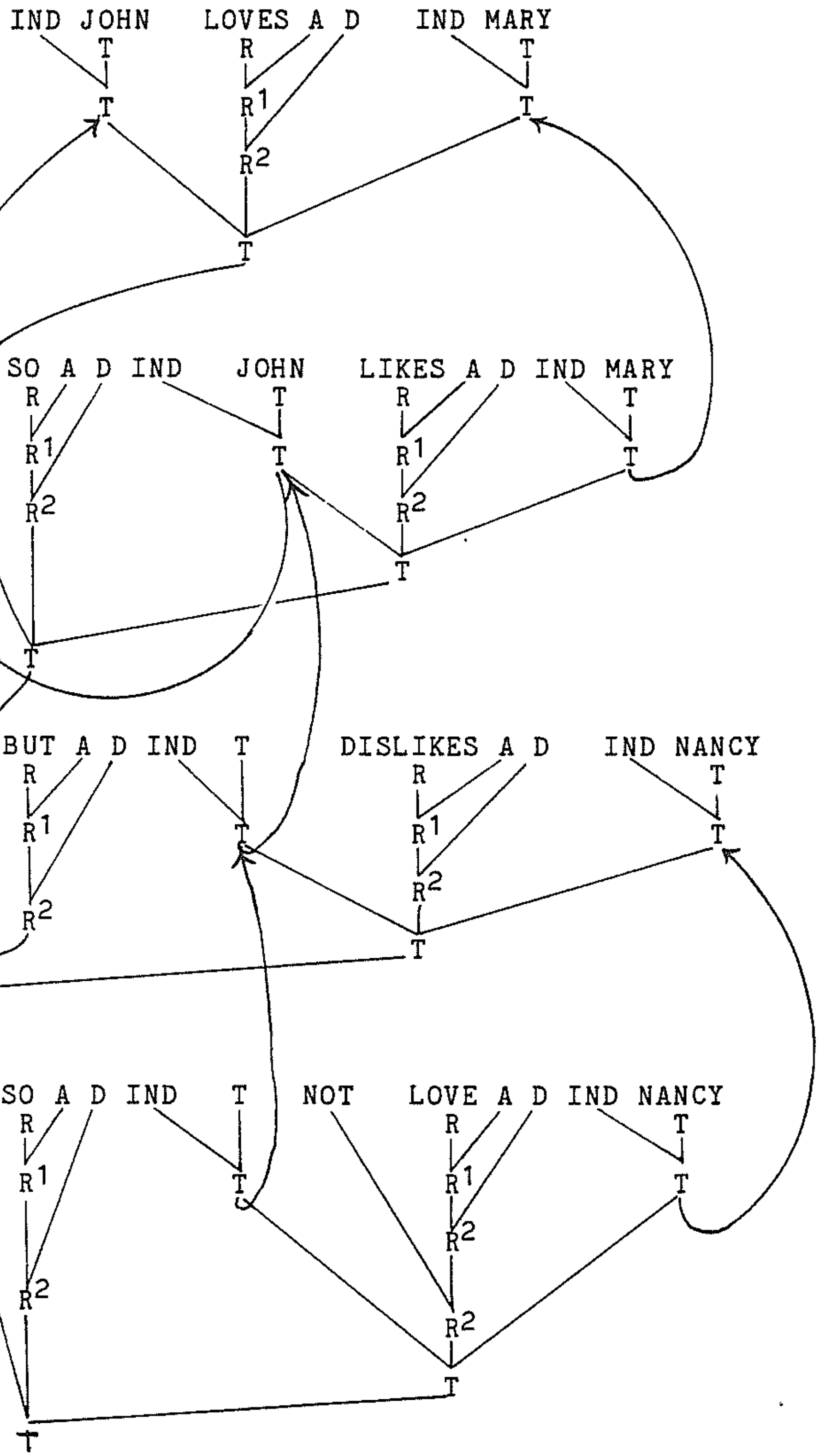
and



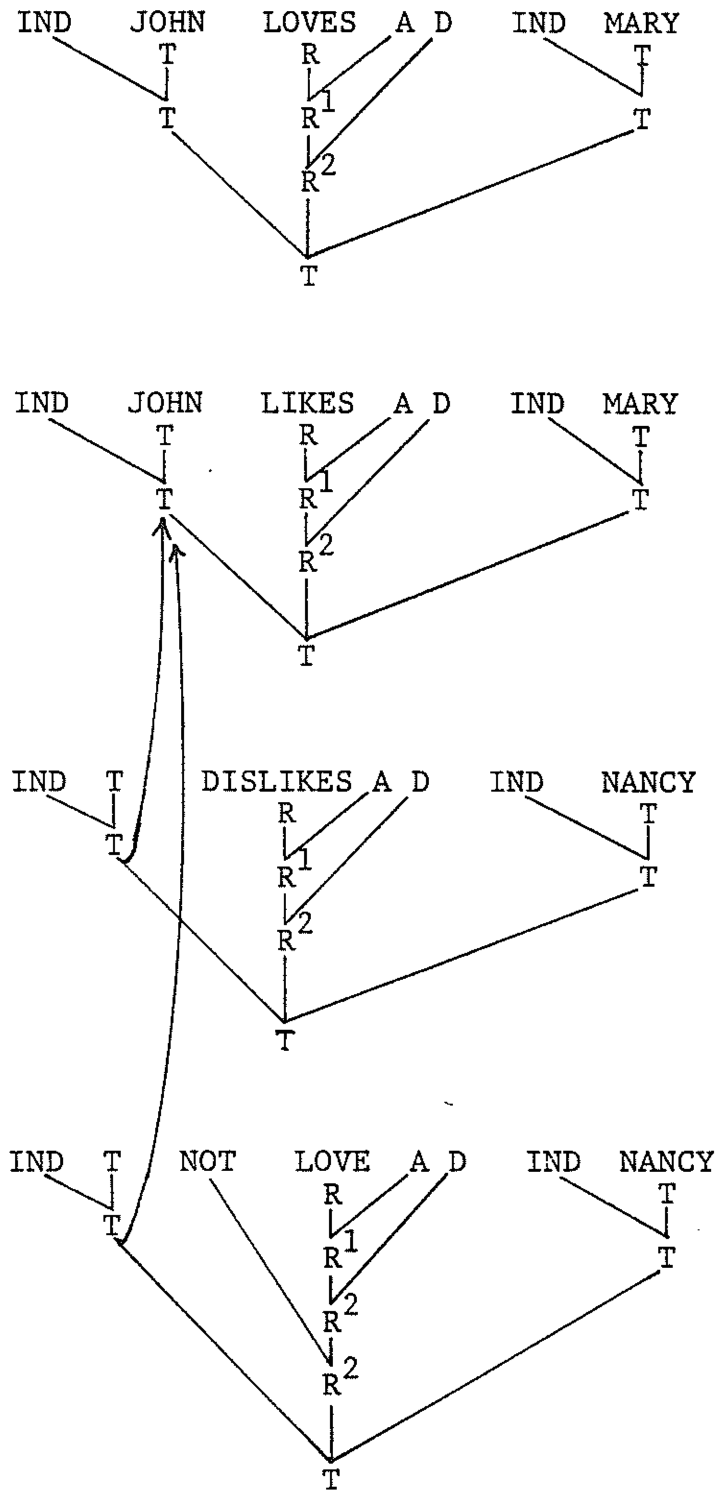
,

and under the discourse reading (3') of (3), (3) would entail "John liked Mary because he loved Mary" under the dominant normal reading of the latter, but under the non-discourse reading (3'') of (3), (3) would not entail it.

(3')


IND JOHN LOVES A D IND MARY
T R T
T R1 T
R2
T
SO A D IND JOHN LIKES A D IND MARY
R T R T
R1 T R1 T
R2 R2
T
T
BUT A D IND T DISLIKES A D IND NANCY
R R T
R1 T R1 T
R2 R2
T
T
SO A D IND T NOT LOVE A D IND NANCY
R R T
R1 T R1 T
R2 R2
R2
T
T

(3'')


IND JOHN LOVES A D IND MARY
T R T
T R1 T
R2
T
IND JOHN LIKES A D IND MARY
T R T
T R1 T
R2
T
IND T DISLIKES A D IND NANCY
R T
T R1 T
R2
T
IND T NOT LOVE A D IND NANCY
R T
T R1 T
R2
R2
T

106.1

Note 106.1. By virtue of L.S.A. 8.1, it clearly makes no semantic difference if we use IND T or T as the referencing
T
expression, insofar as its denotation is determined wholly by the denotation of its referenced expression. In the present series of examples, IND T is sometimes used also to highlight
T
its possible nonreferencing use (i.e., when no arrow issues from it) in normal readings of singular, i.e., individual, pronouns. In the earlier examples of referencing given on pages 192 through 194.7, we had used the simpler thing-expression T as the referencing expression rather than the thing-expression IND T as in the present examples. Generally, the simpler refer-
T
encing expression T is used.

Thus, the difference between a discourse reading and a set of sentential readings is that a discourse reading treats the syntactic representations of those sentences associated with those readings not as elements of the set of their syntactic representations but as sub-expressions of a single syntactic representation. The latter representation interconnects the former representations within a network. The relations interconnecting the sentence representations are variously temporal relations, spatial relations, causality, and so on.

The intuitive entailments induced by a discourse reading are different from those induced by readings on the set of sentences entering into that discourse. For example, the entailments induced by the set of sentences

(4) John went home

(5) John went to sleep

under a non-discourse reading would not include "John went to sleep after he went home," whereas under at least one normal discourse reading, would include this sentence. This derives partly from the displayed order of occurrence of the sentences (4) and (5) which suggests that the time of the event denoted by sentence (5) is later than the time of the event denoted by (4), and the fact that people usually sleep in their homes, which suggests that people usually go to sleep after they go home rather than before. Thus, the sentences (4) and (5) can be read "temporally" or "atemporally."

More generally, intersentence relations involved in discourse readings would include relations with base morphemes of the following special kinds: temporal morphemes ("then," "while," "later," "after," etc.), spatial morphemes ("there," etc.), incompatibility morphemes ("but," "however," etc.), compatibility morphemes ("moreover,") consequenting morphemes ("because," "for," etc.), addition morphemes ("and," "besides," "moreover," etc.).

Compare the preceding sentences (4) and (5) with the following sentences:

(6) John went crazy

(7) John went to sleep

There is a temporal suggestion here (by virtue of the order of the sentences, and the fact that sometimes when people have fits, etc., they go to sleep (as, for example, from exhaustion). But there is no locational suggestion here, because "crazy" is not a place. Thus, while the analogue of the reading (4) of (1) is possible for (6), the analogue of the reading (3) of (1) is not.

We note that a discourse is not simply an interconnected set of sentences: it is a set of sentences interconnected by relations among them. Thus, for example, the presence of a referencing expression in a given sentence which has a referrent expression in a different sentence does not render those sentences a discourse: it only interconnects them. Of course, the presence of such cross-sentence referencing may help signal that some particular relation does hold among those sentences, in which case those sentences could comprise a discourse.

3.3 Elliptic Readings

An elliptic reading of a word-string is one that "completes" the word-string by adding further words to that word-string that are implicit, being signalled by the immediate verbal and nonverbal context in which the word-string is produced.

In this section we introduce the special sort of elliptic readings that are signalled (or signallable) by the immediate verbal context in the sense that the words that are to be added to complete a given word-string occur already within that verbal context, and call it explicit ellipsis.[107]

We treat this sort of ellipsis from the point of view of the multiple grammatical functions that the words to be added can have within a given verbal context.

It is a common phenomenon in natural language that a single occurrence of a contained word-string within a containing word-string can be used to signal multiple grammatical functions within that containing word-string, rather than have each such intended function be signalled by a separate occurrence of that word-string. Within the syntactic representation language SYN_L^{TR}, we indicate such multiple grammatical functions of a given word-string a within a containing word-string by multiple lines descending from the syntactic representation a' of a within the syntactic representation b' of b. We say that a reading of b

---

Note107. This is consistent with our usage of the word "explicit" elsewhere in this paper, where, in general, a signal is explicit if it is comprised dominantly by the actual occurrence of a given word-string or word-string part.

with a syntactic component b' having this characteristic is an explicit <u>elliptic reading</u> of b. That is, an explicit <u>elliptic reading</u> of a word-string b is a reading of b whose syntactic component contains an expression a' with multiple lines descending from a'. For example, if we accord the sentence:

(1) John sang besides Harriet

with that normal reading ordinarily accorded to

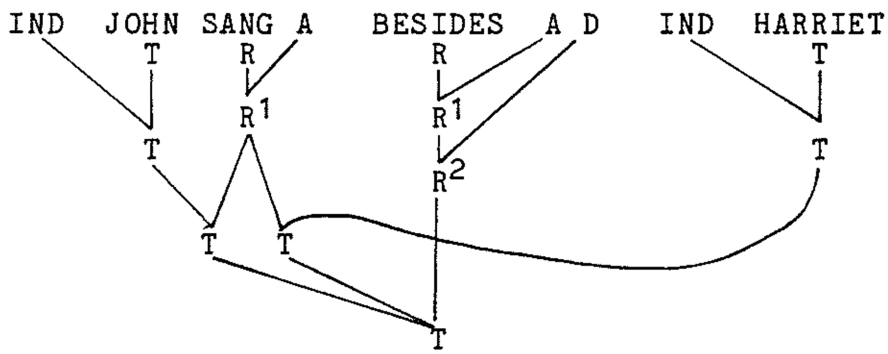
(2) John sang besides Harriet singing

under which (2) would have the sense[108] of

(3) Besides Harriet singing, John sang,

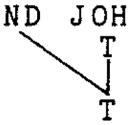
then (1) would have the following sytntactic representation:

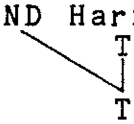
(4)



In (4) we use "SANG A" as the relation expression for both

"IND JOHN" and for "IND Harriet." Notationally, we simply

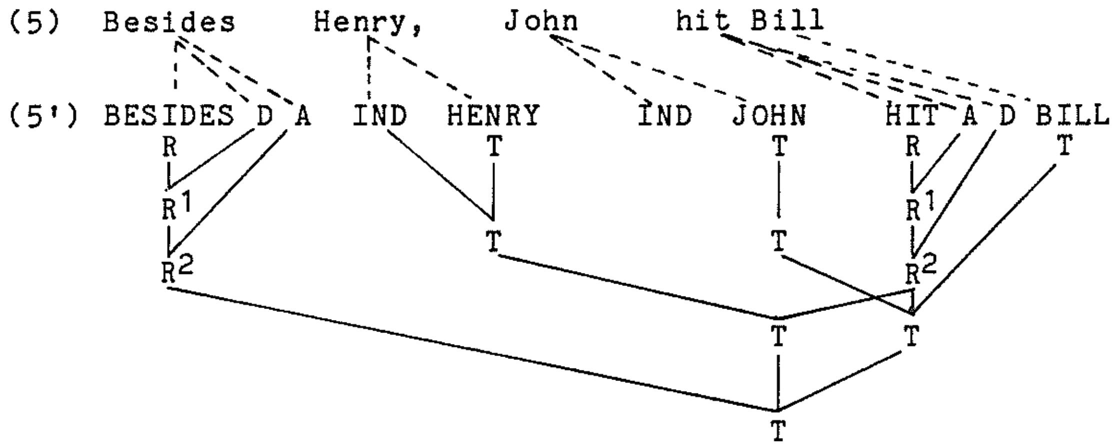




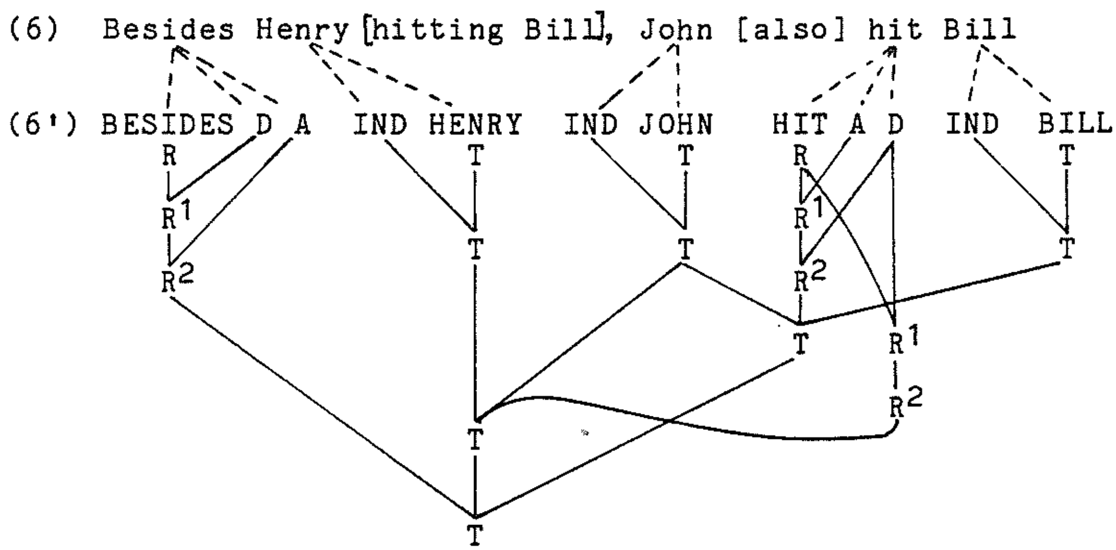
connect expressions as needed. We continue with some further

Note 108. By this I mean of course that the normal reading of (2) in question is that under which (2) would be equivalent to (3), when (3) were accorded its dominant normal reading.

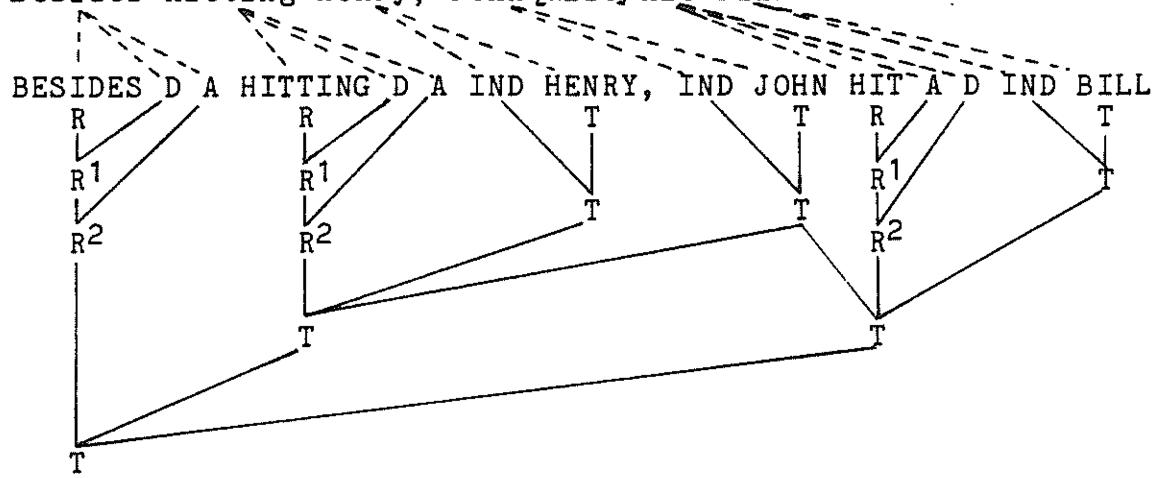
examples: the dominant homologous normal reading of (5) is the elliptic reading (5'):



(5') is equivalent to the dominant homologous normal reading (6') of (6) below, as well as to the dominant homologous normal reading (7') of (7) which follows later; both readings are, as indicated, elliptic.

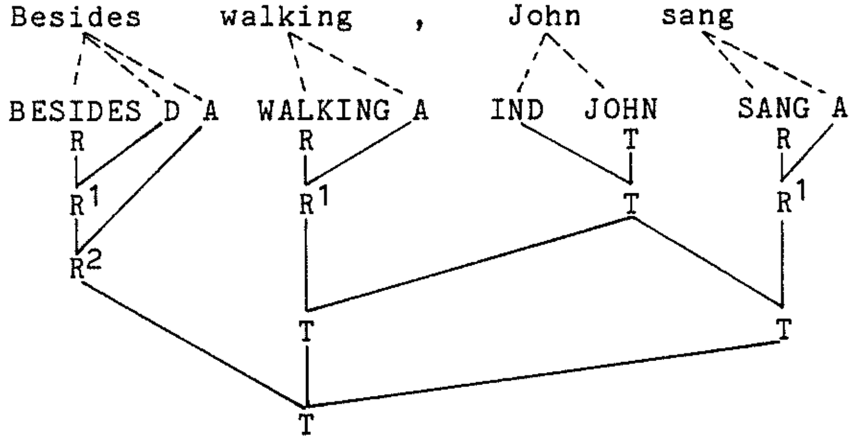


(7) Besides hitting Henry, John [also] hit Bill

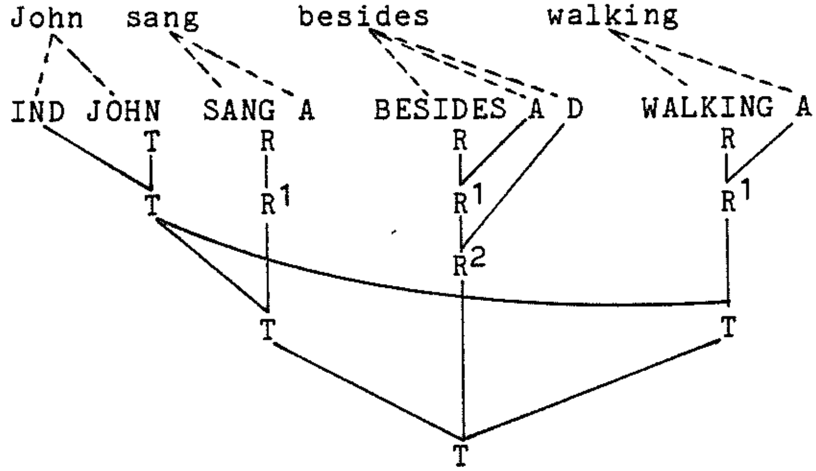
(7') 

Let us consider several other sets of examples of dominant homologous normal readings which are elliptic:

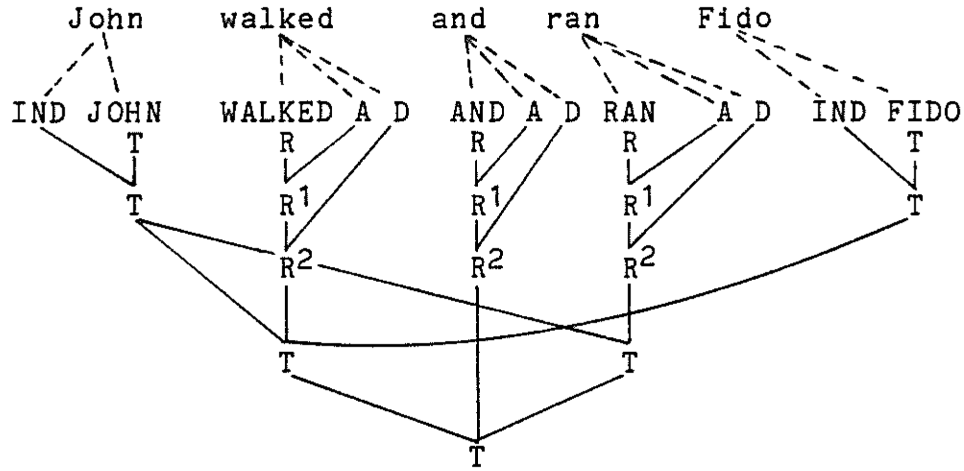
(8) Besides walking , John sang

(8') 

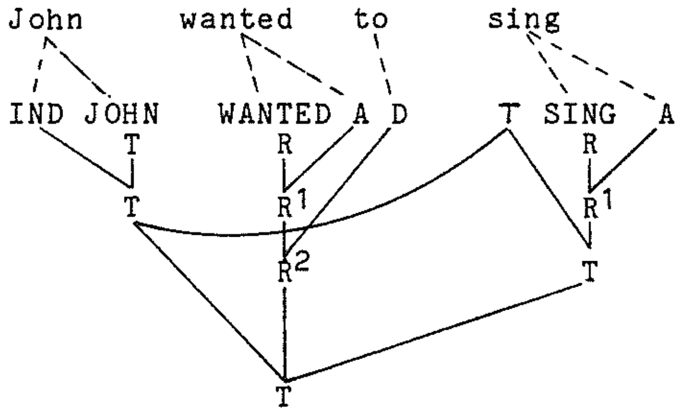
(9) John sang besides walking

(9') 

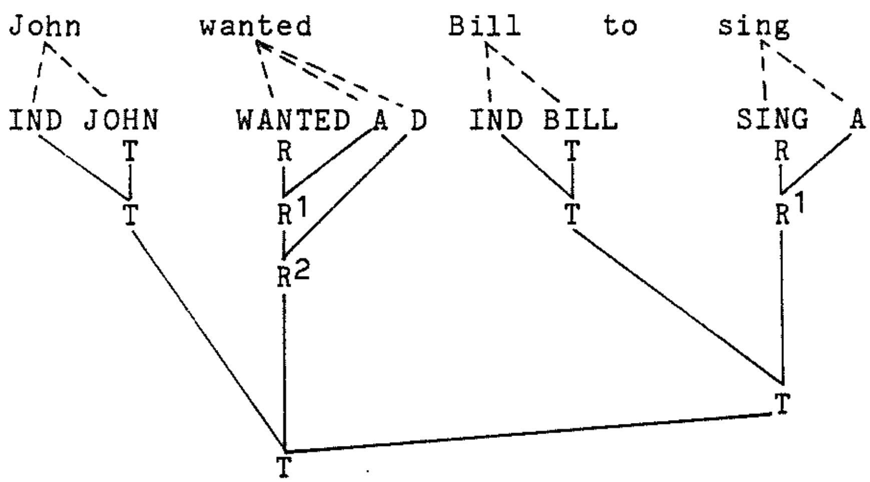
(10)
(10')

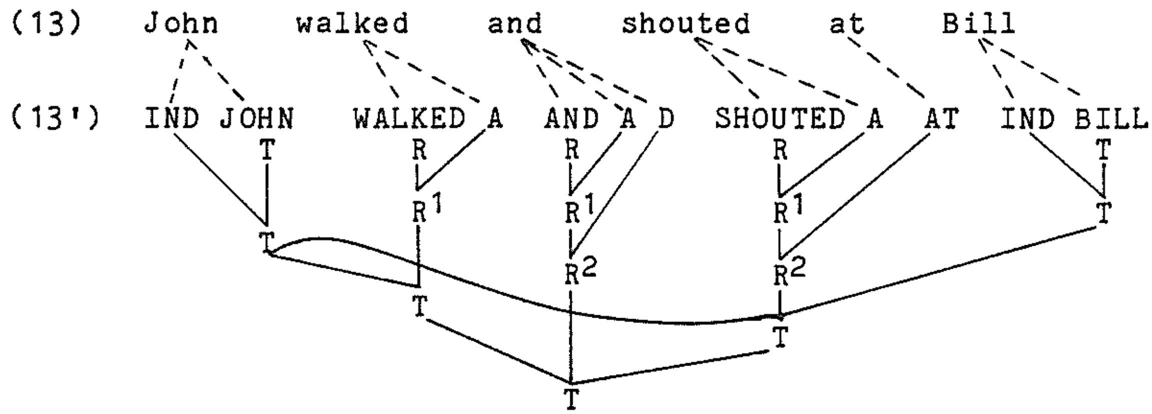


(11)
(11')



(12)
(12')



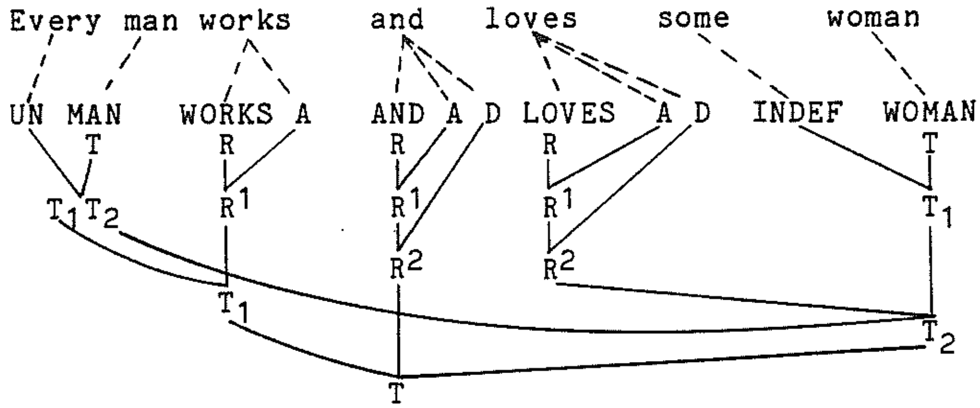


The above examples illustrated the case where a given word-string may have a multiple grammatical function relative to other word-strings in a containing word-string. We now consider the case where a given word-string may also have _multiple roles relative to those functions_, where a "role" is its status as a thing-expression, relation-expression, or modifier, or as its status relative to determiner ordering.

In the examples the multiple grammatical functions of the ellipsized word-string are identical within the containing word string and so can be indicated by a single label. We call this _single-function explicit ellipsis_ and distinguish it from cases of explicit ellipsis where the ellipsized word-string has two or more functions within a containing word-string that are not identical within that containing word-string. We call this latter kind of ellipsis _multiple-function explicit ellipsis_.

We indicate multiple-function explicit ellipsis within a syntactic representation by the use of sequences of labels, each indicating a specific grammatical function, such as T_1T_2, T_2T_1, T_1 R^n, R^n R^m, etc., as discussed earlier in Section 2.3.1.1.3.2. The following reading (14') is a normal compact multiple-function elliptic reading of the English sentence (14):

(14) (14')

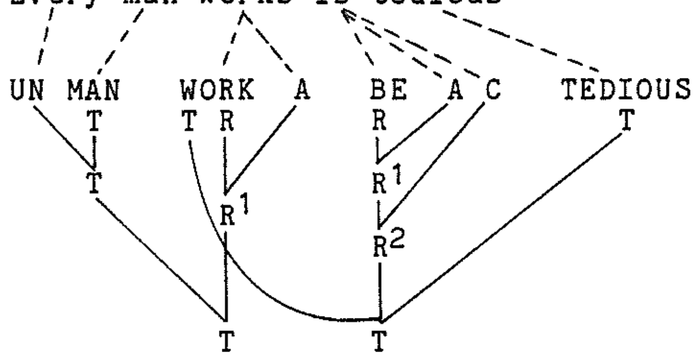


In the above example, "every man," while the agent of both "works" and "loves" has its quantifier "every" taken first (and alone) in conjunction with "works" and taken after the quantifier "some" in "some woman" in conjunction with "loves," which gives the sentence "every man works and loves some woman" the meaning of "Every man works and there is some woman that every man loves," rather than that of "every man works and he loves some woman" or "every man works (understood transitively) and loves some woman."

There are other examples in which the labels are more heterogeneous, in the sense that the "T" and "R" labels both occur within the same sequence. It appears that, in English, this heterogeneous kind of multiple-function ellipsis is most compactly syntactically represented by the non-restrictive relative construction (discussed in Section 3.5 below). A heterogenous multiple-function elliptic reading would be one having a syntactic component like the following.

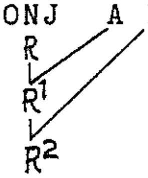
(15) Every man works is tedious

(15')

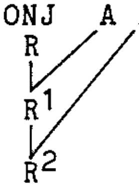


(15) is an ungrammatical sentence of English, though analogous forms may be grammatical in other languages. In English, however, (15) would be avoided in favor of a non-restrictive relative clause construction, which can be expressed in English by (16), and represented by the non-restrictive relative clausal reading (16') of (16) which is a more homologous reading of (16) than (15') is of (15).[108.1]

108.1 When we form a non-restrictive relative clause on a sentence, we render the conjunction explicit by inserting the logical relation morpheme CONJ A D in order to preserve the basic structure of a

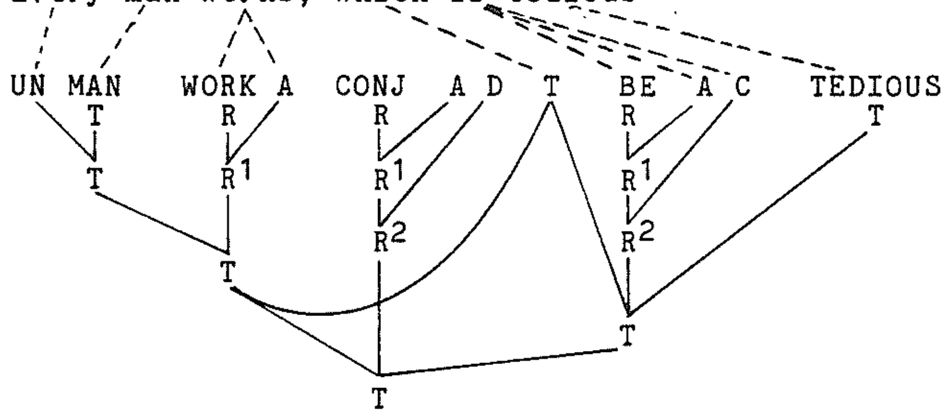


sentence as comprised of m major thing-expressions and an m-place relation expression. This is tantamount to treating non-restrictive relative clauses on sentences as special cases of discourse readings wherein the non-restrictive relative clause is represented as a separate sentence that is anaphorically linked to a preceding sentence, and where the two sentences are related by the CONJ A D relation.



See Section 3.5 for the general treatment of relative constructions, which includes a discussion of restrictive and non-restrictive relative clauses and phrases on thing-expressions, relation-expressions, modifiers, and sentences.

(16) Every man works, which is tedious

(16') 

The elliptic reading (15') of (15) and the non-restrictive relative clausal reading (16') of (16) are equivalent under the logical semantic axioms of Chapter 2. On the other hand, while (16') is a homologous normal reading of (16), (15') is a homologous reading of (15) but is not a normal reading of (16), since (15), being ungrammatical, has no normal readings (in English).

It would appear that there are no multiple-function heterogenous, explicit elliptic homologous readings of grammatical word-strings in English, and that the closest grammatical variant of a word-string that had such a homologous reading would be a word-string which had a non-restrictive relative reading as a homologous (normal) reading. In English the non-restrictive relative clause construction is used within word-strings to signal heterogeneous multiple function explicit elliptic readings. We use the notion of a non-restrictive relative clausal reading that is equivalent to a multiple function explicit elliptic reading in order to provide homologous readings of word-strings that have a non-restrictive clausal surface structure.

3.4. Anaphoric Readings

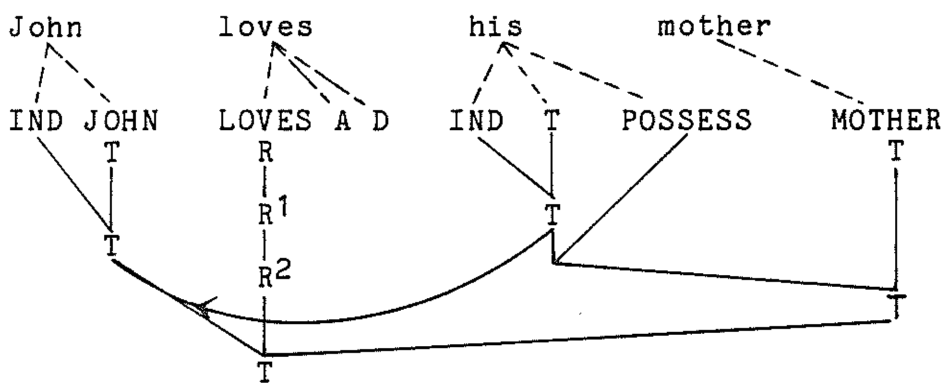
Roughly speaking, anaphora is a relation between two word-string occurrences which holds when those two occurrences have (or are intended to have) the same denotation.[109]

Accordingly, we define an anaphoric reading of a word-string as a reading of that word-string whose syntactic component contains two expressions (which can be thing-expressions, relation-expressions, or modifiers) with a coreferential link between them.

Let us consider some examples. Example (1) is of the simplest sort, involving a link between a pronoun and a preceding referent proper noun within a single sentence.

(1)

(1')



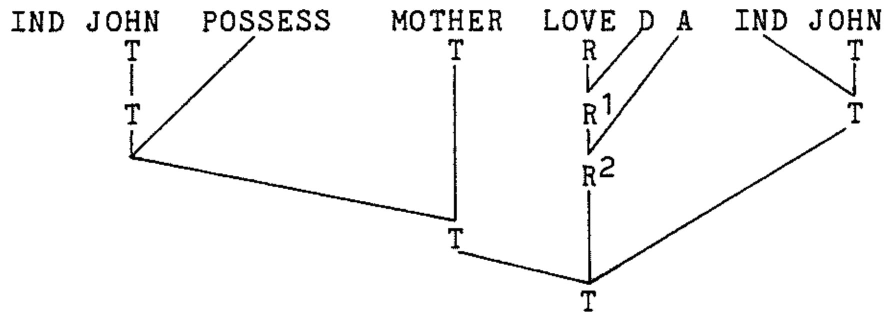
As a sample entailment from (1) under the reading (1') we have

Note 109. In some treatments in the literature, ellipsis is regarded as a special case of anaphora, called zero-anaphora, where an empty word-string has the same denotation as some non-empty word-string. Such an approach is reasonable in characterizing anaphora for natural language word-strings, but is not reasonable for characterizing anaphora for their syntactic representations.

(1.1) John's mother is loved by John

under the reading (1.1') of the latter:

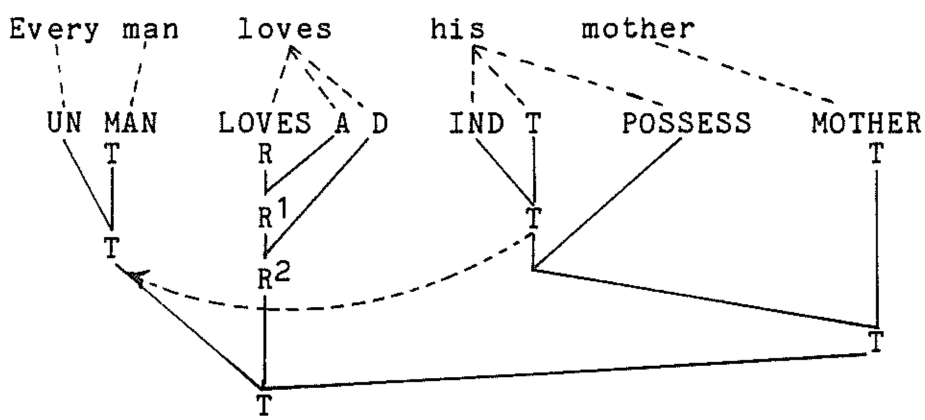
(1.1')



Example (2) below is like example (1), except that example (2) involves an <u>ineliminable pronoun</u> whereas (1) involves an <u>eliminable pronoun</u>, whereby "his" is eliminable by replacing it by "John." At the syntactic representation level, the structure is wholly analogous. The distinguishing difference is in the semantic interpretation.

(2)

(2')

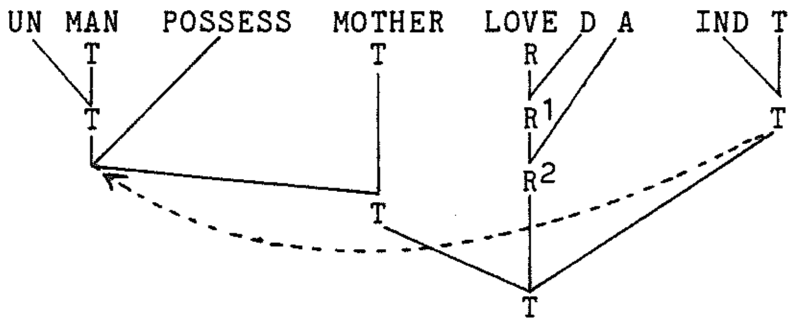


As a sample entailment from (2) under the reading (2') we have

(2.1) Every man's mother is loved by him

under the reading (2.1') of the latter:

(2.1')



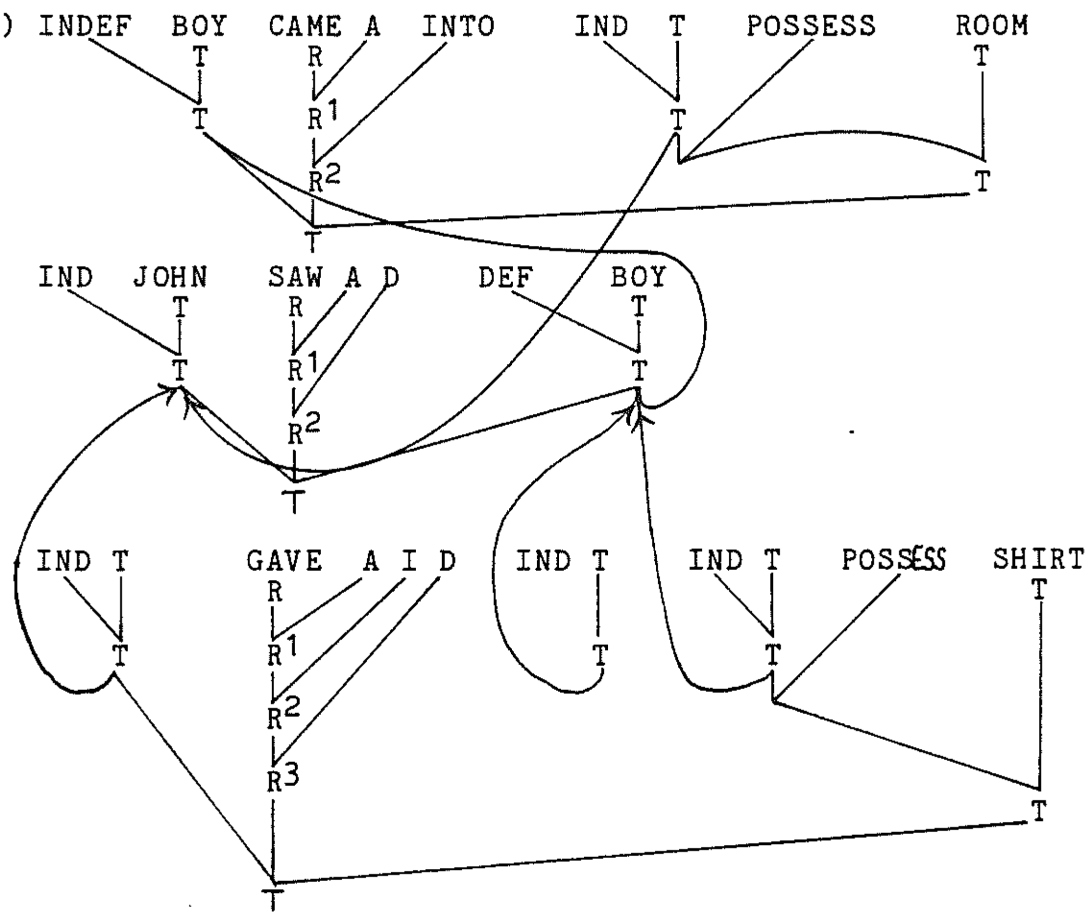
Example (3) below involves anaphora across sentence boundaries, thereby constituting a discourse reading of (3).

(3) A boy came into his room.

John saw the boy.

He gave him his shirt.

(3')

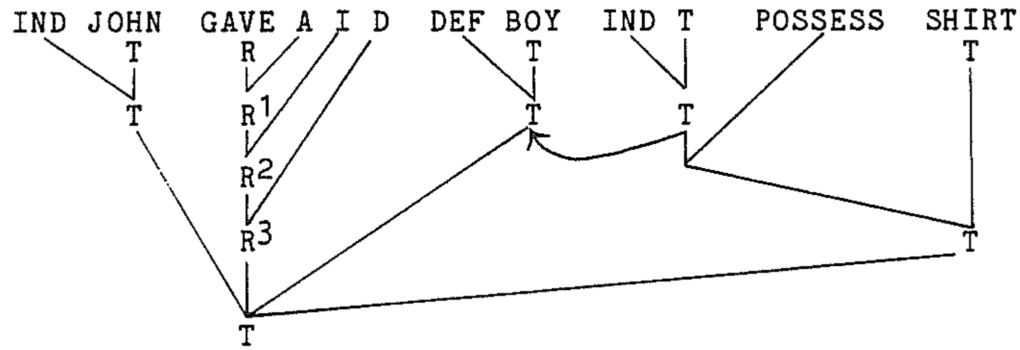


As a sample entailment of (3) under the reading (3') we have

(3.1) John gave the boy his (the boy's) shirt

under the reading of the latter:

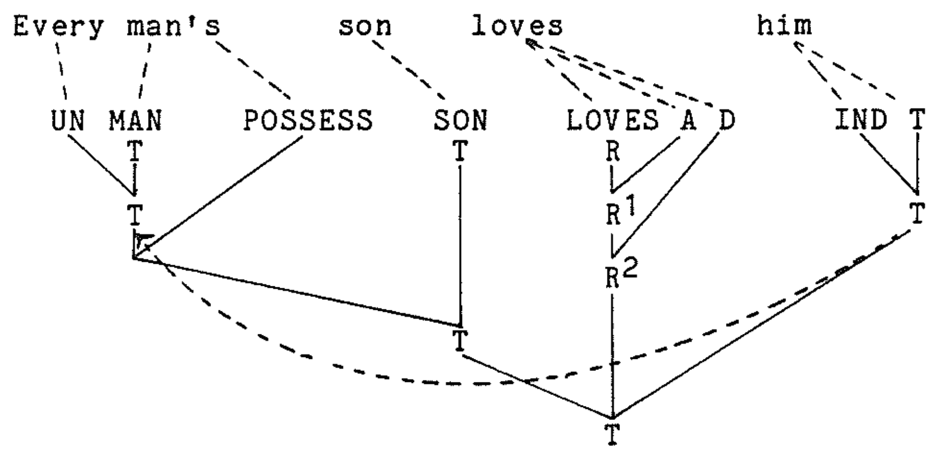
(3.1')



The following example illustrates further case of pronominal anaphora:

(4)

(4')



3.5 Ordinary Restrictive and Non-Restrictive Relative Readings

In Section 2.3.1.1.2.1.2 we introduced the two compound determiners: The differentiated relative determiner and the ordinary restrictive relative determiner. Our purpose there was to distinguish the ordinary differentiated relative readings from ordinary restrictive relative readings. In this section our concern is to distinguish between the ordinary restrictive relative and the ordinary non-restrictive relative readings.

The notional significance of the difference between the ordinary restrictive relative and the ordinary non-restrictive relative is that the former restricts the denotation of the expression to which it is applied whereas the latter does not.

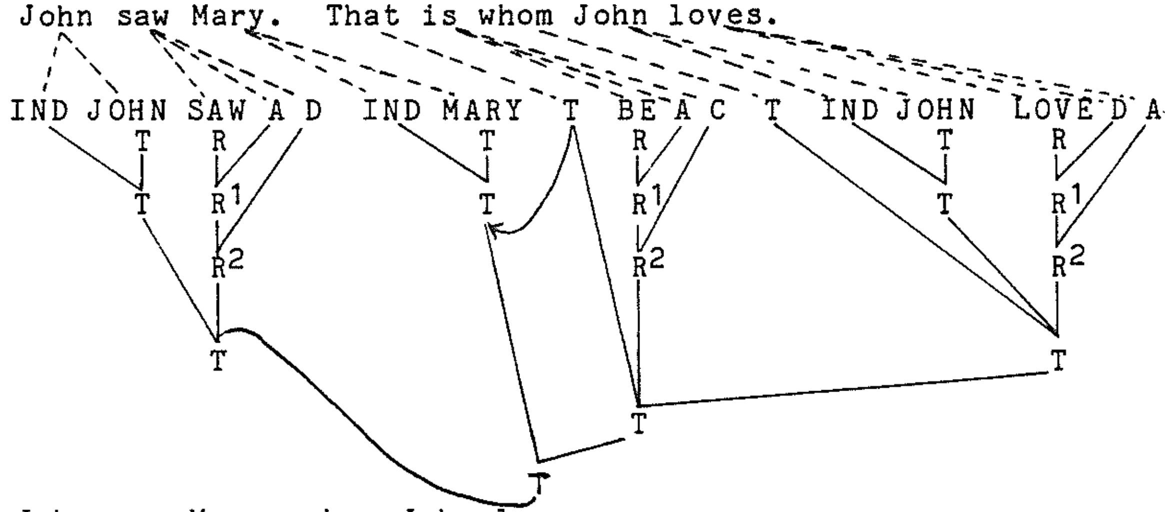
Our interest in this study is to describe those differences between the ordinary restrictive and non-restrictive relative readings of natural language word-strings that induce differences in their entailments. That is to say, our concern is to describe the formal basis for only the entailment-relevant aspects of the difference between these two types of ordinary relative readings, ignoring for the present any discussion of the focal-relevant aspects of the difference between them, which pertain to ways of distinguishing, say, new from old information or, more generally, which pertain to ways of distinguishing information that is more focally relevant from that which is less focally relevant.

Generally speaking, a non-restrictive relative clause is a construction that is used to include, within the boundaries of a given sentence, another sentence which elaborates on some component of that given sentence, and which could have been stated as a separate sentence.

In example (a) below, the potential non-restrictive relative clause occurs as a separate sentence; in (b) it occurs as a non-restrictive relative clause within the boundaries of a single sentence containing the original sentence as a part. (a') and (b') are their respective dominant *homologous* normal readings:

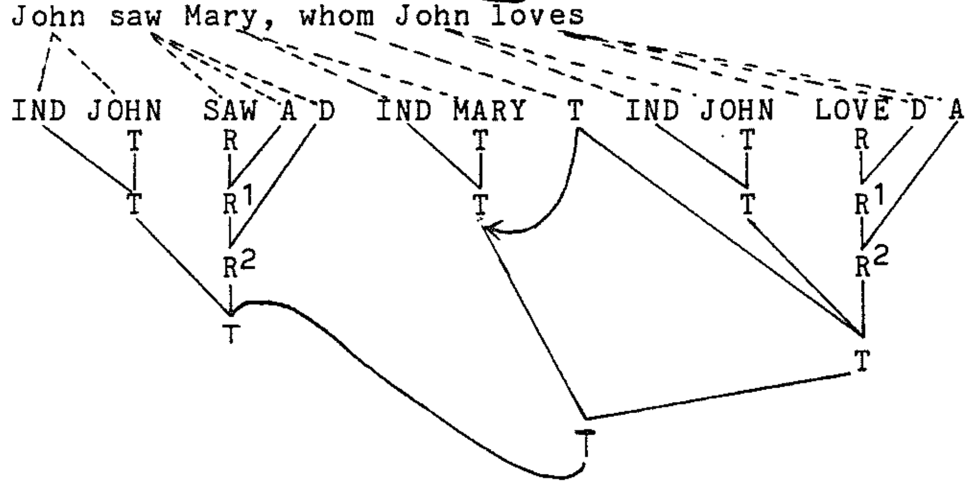
(a) John saw Mary. That is whom John loves.

(a')



(b) John saw Mary, whom John loves

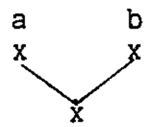
(b')



The non-restrictive relative clause in (b) is equivalent to what could be obtained by splitting off the clause from within the containing sentence and asserting it separately, as in (a). (a) and (b) are intuitively equivalent; this is reflected in the fact that the readings (a') and (b') are semantically equivalent relative to the minimal semantic theory s_o of Chapter 2.

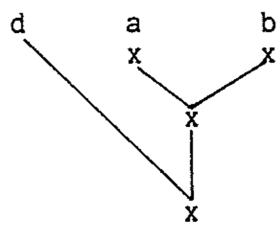
An *ordinary relative reading* of *a word-string e of a natural language L* is a reading of e whose syntactic component is of the form

(A)



for some modifier expressions a, b of SYN_L^{TR} and where x is a thing label, relation label, or the (empty) modifier label (but, where necessary for visual perspicuity, we will use a star * as modifier label in place of the empty label). An ordinary relative reading of e is said to be *restrictive* if the expression (A) occurs immediately within an expression of the form

(B)



where d is a determiner[110], and is said to be *non-restrictive* otherwise. Thus, as can be seen from inspecting (A) and (B) above, the distinction between the ordinary restrictive and non-restrictive relative is a distinction in the *scope* of the expression to which it is applied, that is, in the extent of the

---

Note110. While our examples in this study deal only with determiners on thing-expressions, there is no reason, in principle, why determiners could not be applied to relation expressions as well.

modified expression which the ordinary relative "modifies" relative to the governing determiners of that expression. If some initial governing determiner in the modified expression fails to be included in the scope of the ordinary relative, that ordinary relative is restrictive; if all initial governing determiners (if any) in the modified expression are included in the scope of the ordinary relative, it is non-restrictive. Thus restrictive relative readings are short-scoped ordinary relative readings, while non-restrictive relative readings are long-scoped ordinary relative readings.

Another distinction that is useful to make is the following:

There are two types of ordinary restrictive and non-restrictive relative readings which we distinguish as clausal and phrasal, and which differ in that the former represents the ordinary relative modifier (that is, the expression b in (A) and (B) above) as a clause, whereas the latter does not. The next section illustrates the difference between ordinary clausal restrictive and non-restrictive relative readings of some sample English word-strings, and the section which follows it illustrates the difference between ordinary phrasal restrictive and non-restrictive relative readings of some further sample English word-strings.

Ordinary Clausal Restrictive and Non-Restrictive Relative Readings

Consider the following sentences of English.

(1) John loves the girl who is kind

(2) John hates the girl who is cruel

(3) John loves the girl

(4) John hates the girl

(5) John loves a girl who is kind

(6) John hates a girl who is cruel

(7) John loves a girl

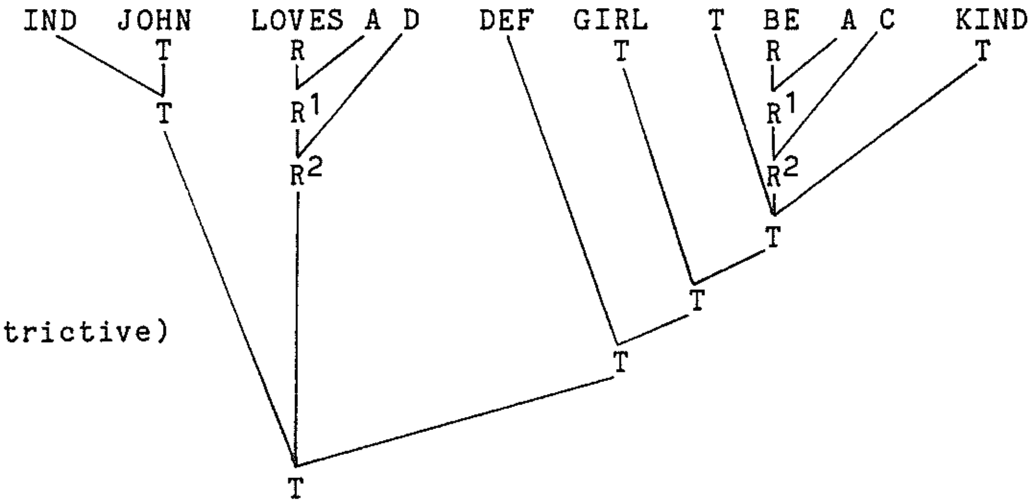
(8) John hates a girl

In each of (1), (2), (5), and (6) the word-string "girl who is ... " can be assigned a (clausal) restrictive or non-restrictive ordinary relative reading. There are normal readings r(1), r'(1) of (1); r(2),r'(2) of (2); r(3) of (3); and r(4) of (4) such that r(1) has, as a subreading, a restrictive relative reading $r^{\vee}(1)$ of "girl who is kind," r'(1) has, as a subreading, a non-restrictive relative reading $r'^{\vee}(1)$ of "girl who is kind," such that r(2) has, as a subreading, a restrictive relative reading $r^{\vee}(2)$ of "girl who is cruel," and r'(2) has, as a subreading, a non-restrictive relative reading $r'^{\vee}(2)$ of "girl who is cruel," which are such that (1) entails (3) under the readings r'(1) of (1) and r(3) of (3), (2) entails (4) under the readings r'(2) of (2) and r(4) of (4), (1) does _not_ entail (3) under the readings r(1) of (1) and r(3) of (3), and (2) does _not_ entail (4) under the readings r(2) of (2) and r(4) of (4).

Also, there are normal readings r(5), r'(5) of (5), and r(6), r'(6) of (6), r(7) of (7), and r(8) of (8) such that r(5) has, as a subreading, the restrictive relative reading $r^{\vee}(1)$ of "girl who is kind," r'(5) has, as a subreading, the non-restrictive relative reading $r'^{\vee}(1)$ of "girl who is kind," r(6) has, as a subreading, the restrictive relative reading $r^{\vee}(6)$ of "girl who is cruel," and r'(6) has, as a subreading, the non-restrictive relative reading $r'^{\vee}(6)$ of "girl who is cruel," which are such that (1) entails (5) under the readings r(1) or r'(1) of (1) and r(5) or r'(5) of (5); (2) entails (6) under the readings r(2) or r'(2) of (2) and r(6) or r'(6) of (6); (5) entails (7) under readings r(5) or r'(5) of (5) and r(7) of (7); (6) entails (8) under readings r(6) or r'(6) of (6) and r(8) of (8); (3) entails (7) under readings r(3) of (3) and r(4) of (7); (4) entails (8) under readings r(4) of (4) and r(8) of (8); the readings r(5) and r'(5) of (5) are equivalent, as are the readings r(6) and r'(6), in the sense that their syntactic components have the same denotation under s_0 (the normal semantic theory satisfying L.S.A.(1)-L.S.A.(31)).

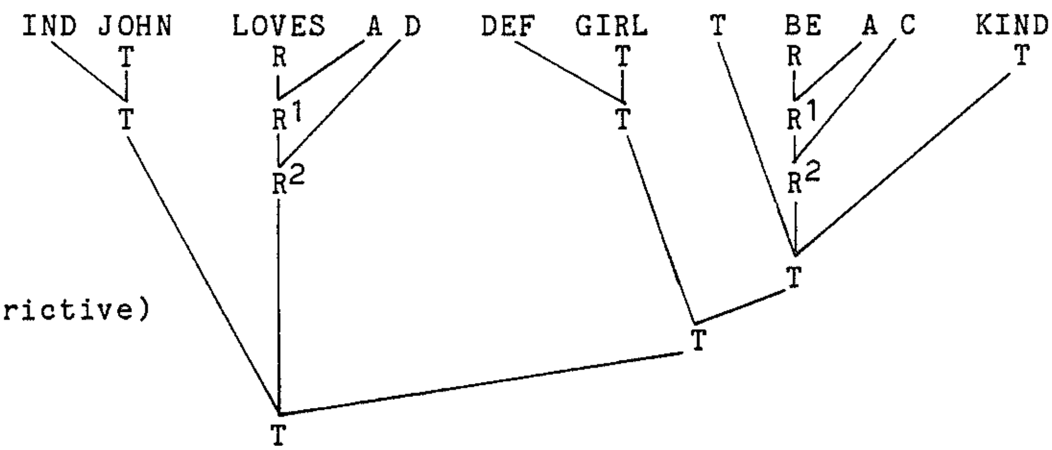
In the following, we list the syntactic components of the various readings alluded to above. Specifically, the syntactic component of r(1) appears as (1') below, that of r'(1) appears as (1''), that of r(2) appears as (2'), that of r'(2) appears as r(2''), that of r(3) appears as (3'), that of r(4) appears as (4'), that of r(5) appears as (5'), that of r'(5) appears as (5''), that of r(6) appears as (6'), that of r'(6) appears as (6''), that of r(7) appears as (7'), and that of r'(7) appears as (7'').

(1')



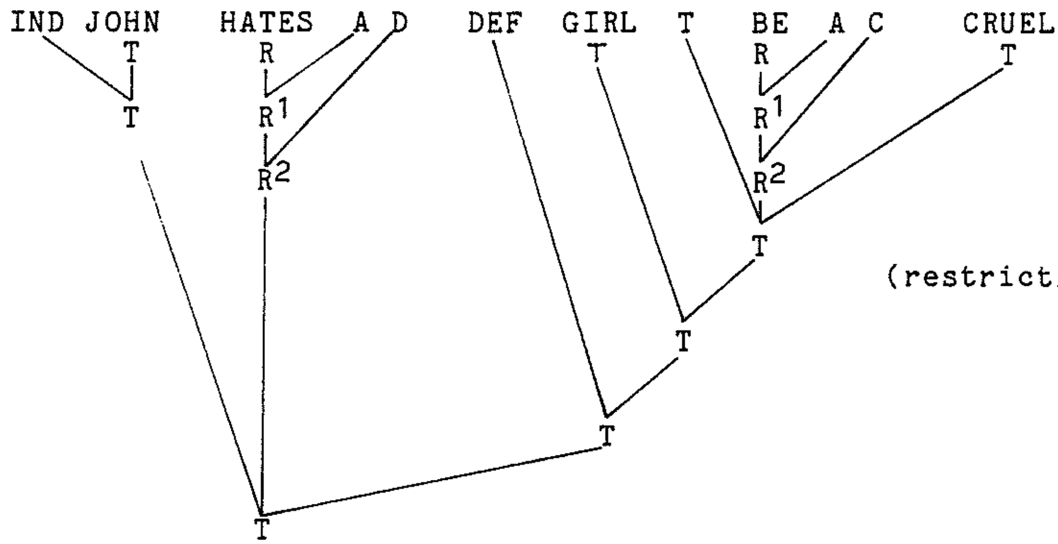
(restrictive)

(1'')



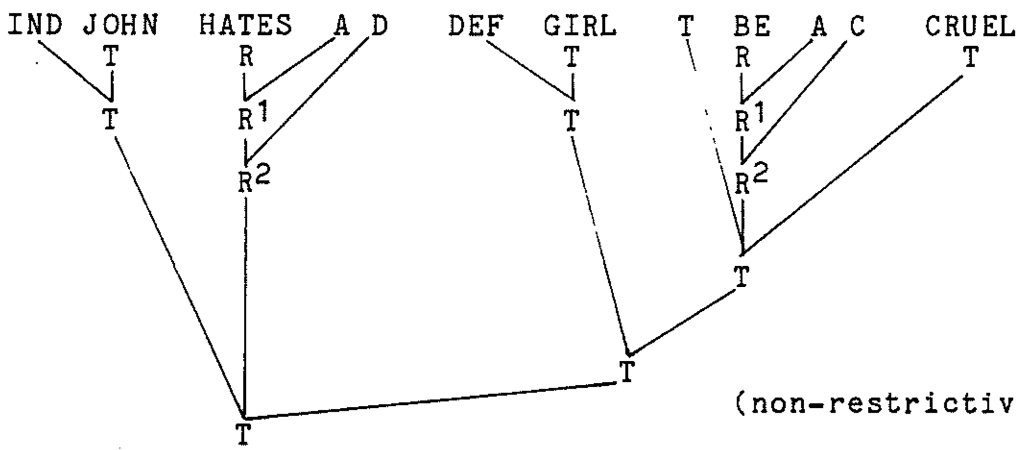
(non-restrictive)

(2')



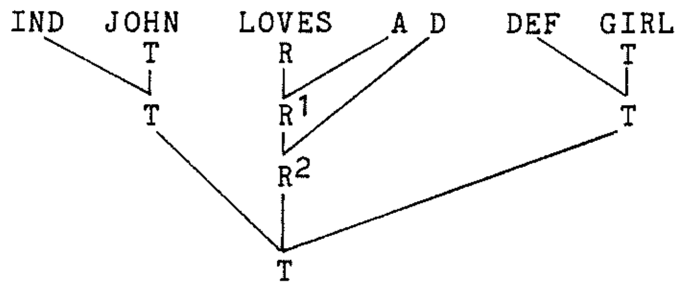
(restrictive)

(2'')

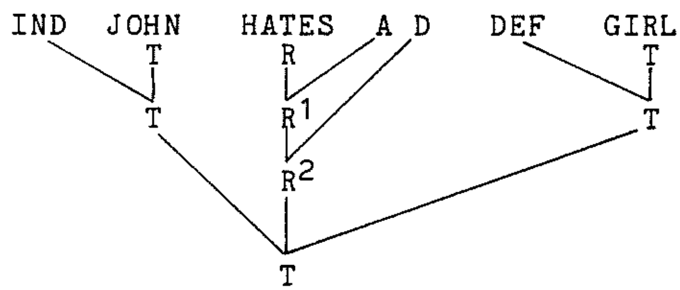


(non-restrictive)

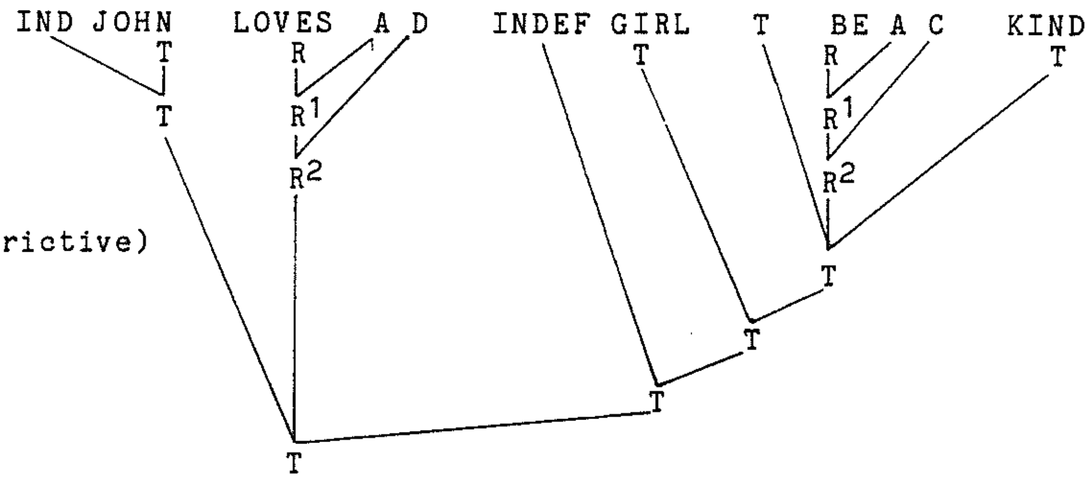
(3')


IND JOHN LOVES A D DEF GIRL
T R T
T R1 T
R2
T

(4')

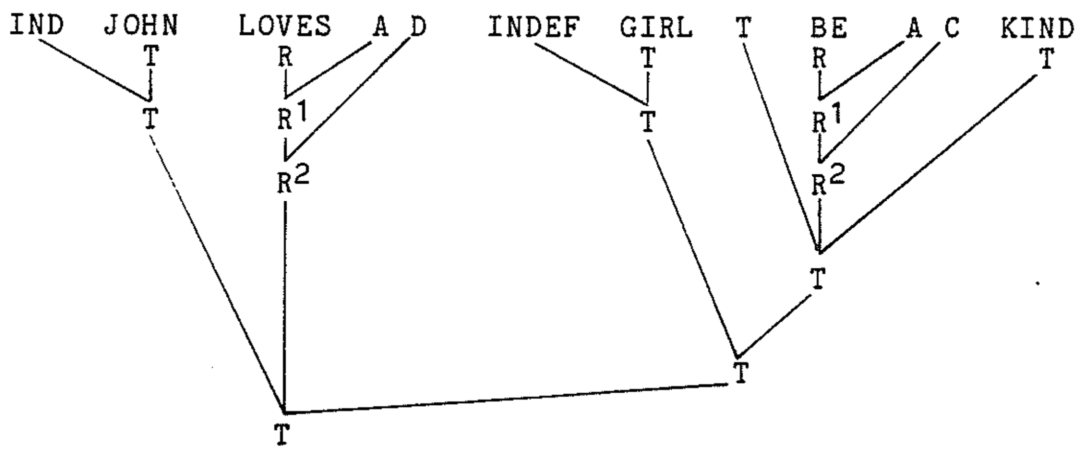

IND JOHN HATES A D DEF GIRL
T R T
T R1 T
R2
T

(5')



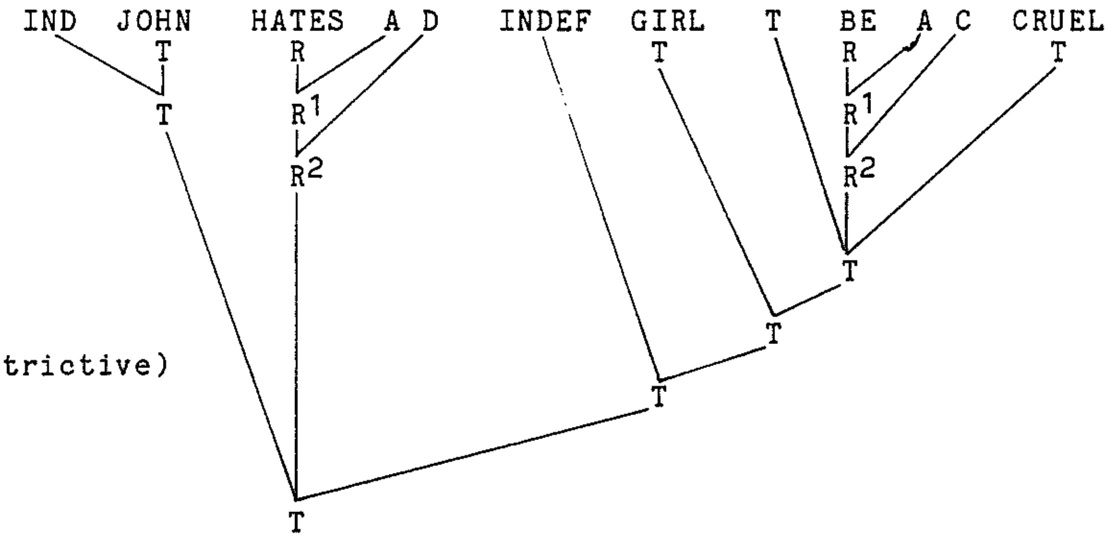
(restrictive)

(5'')



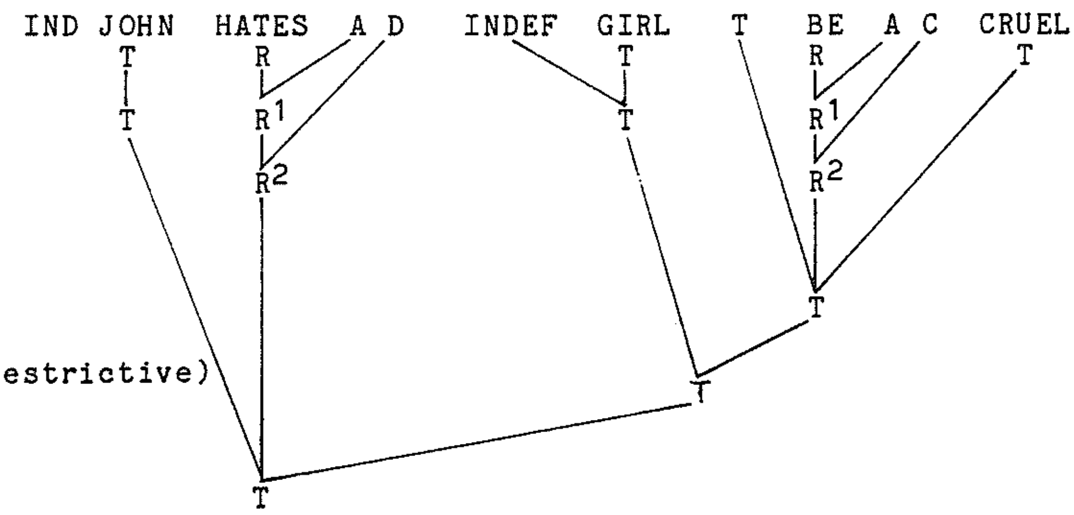
(non-restrictive)

(6')



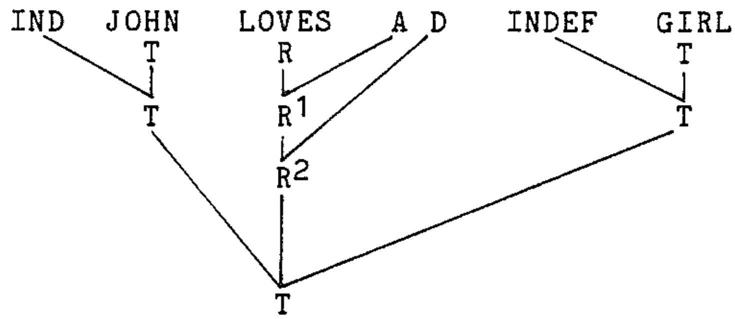
(restrictive)

(6'')

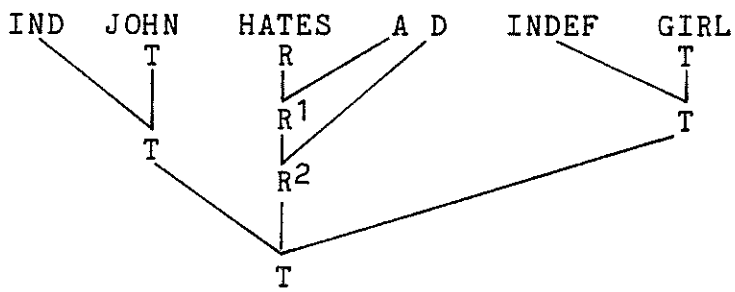


(non-restrictive)

(7')


IND JOHN LOVES A D INDEF GIRL
T R T
T R1 T
R2
T

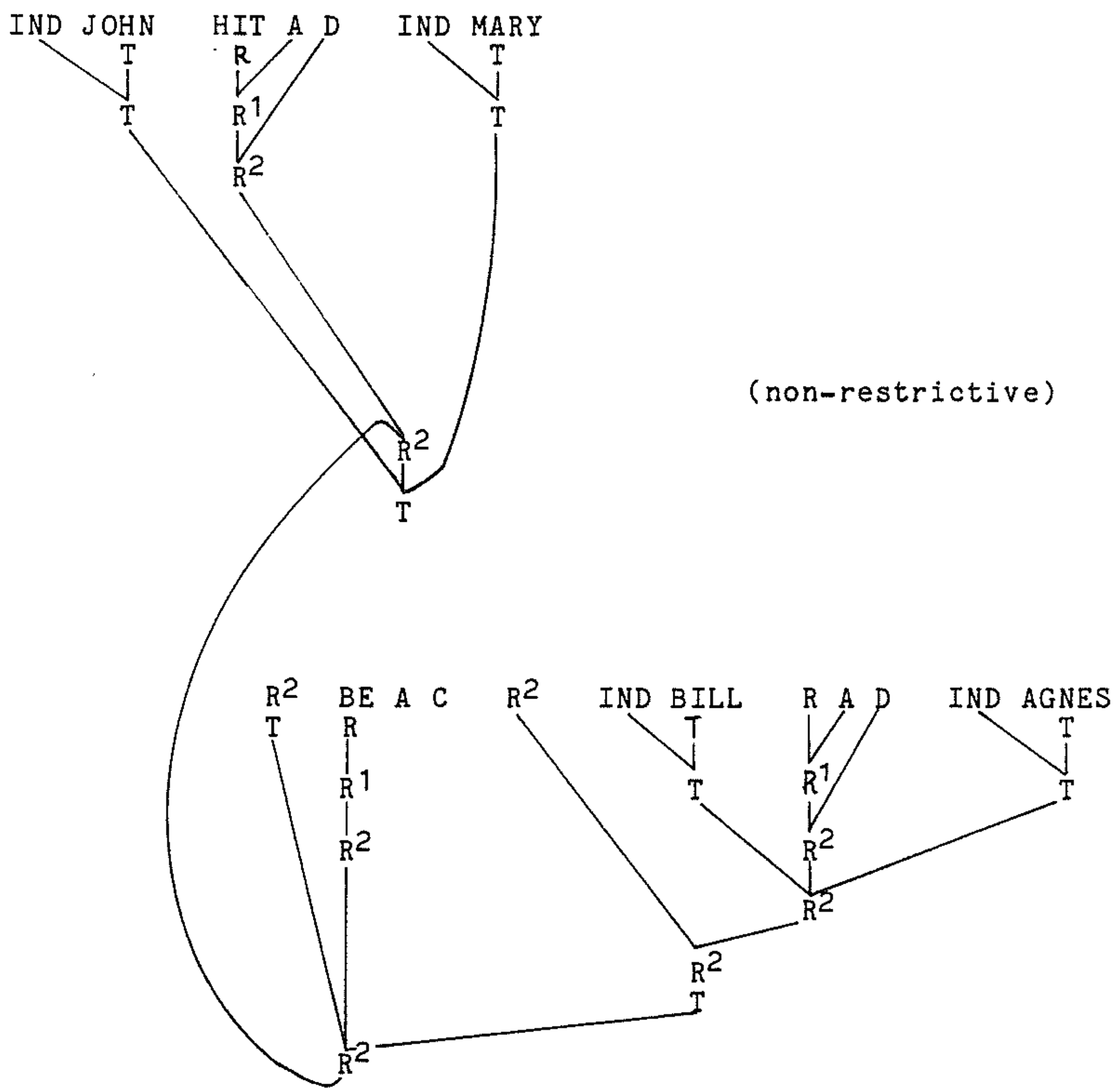
(7'')


IND JOHN HATES A D INDEF GIRL
T R T
T R1 T
R2
T

The above examples of ordinary clausal non-restrictive relative readings were homologous normal readings of the simple and somewhat stilted constructions of (1)-(8). The following examples of ordinary clausal non-restrictive relative readings exhibit homologous normal readings of sentences that are more characteristic of English vernacular, ((9)-(13)):

(9) John hit Mary, which is what Bill did to Agnes

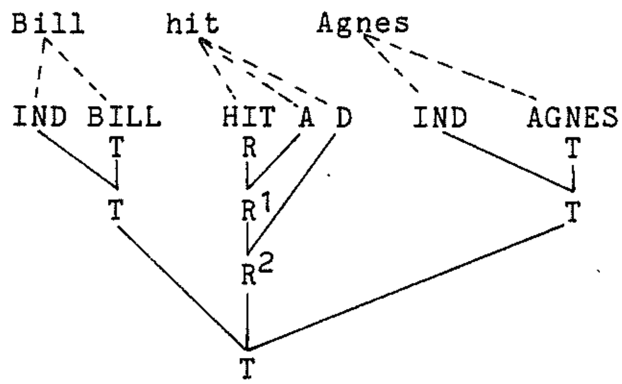
(9')



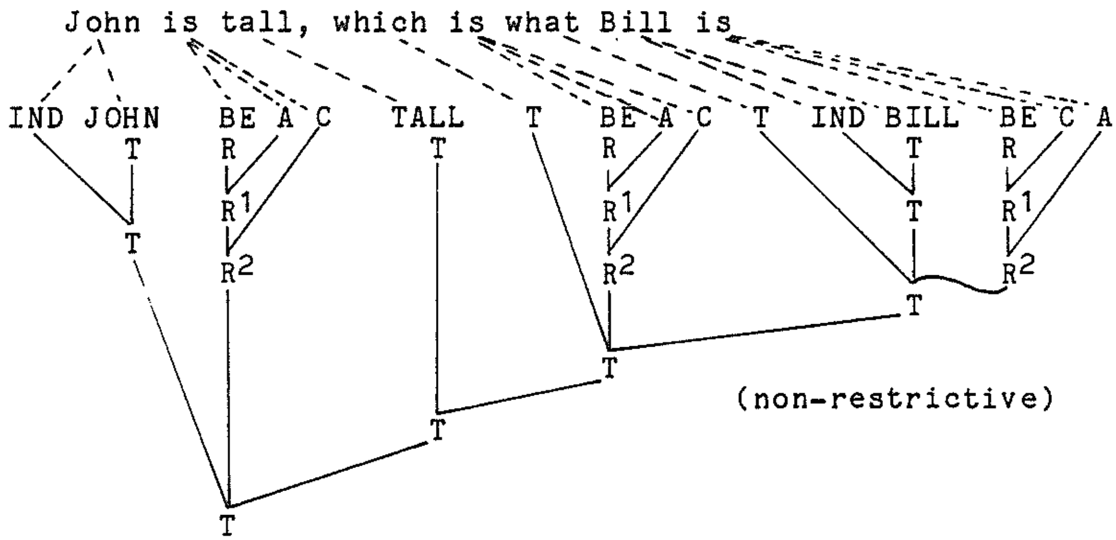
Under the reading (9'), and under the reading (9.1') of (9.1), (9) entails (9.1):

(9.1) Bill hit Agnes

(9.1')

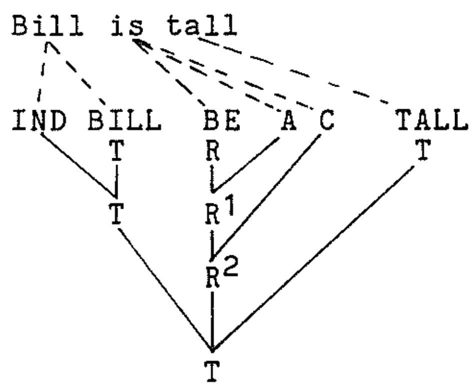


(10)
(10')



Under the reading (10') of (10), and under the reading (10.1') of (10), (10) entails (10.1)

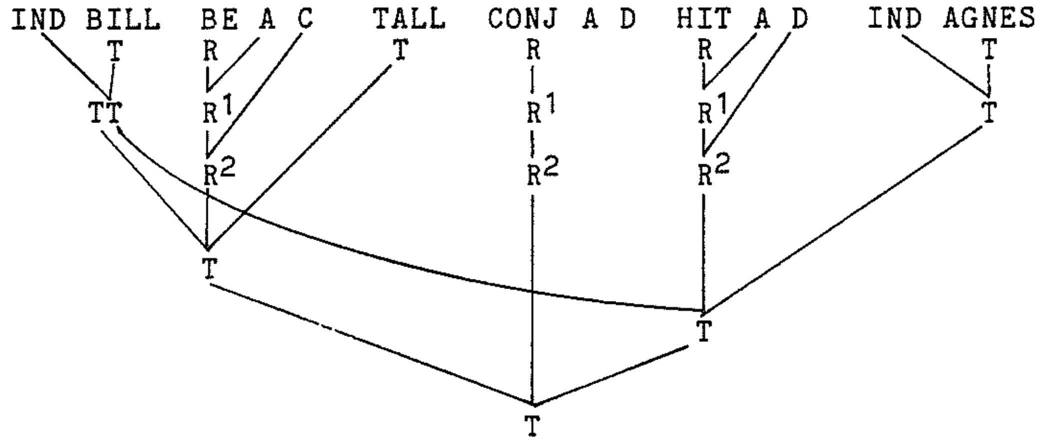
(10.1)
(10.1')



Under the readings (9.1') of (9.1), (10.1') of (10.1), and (10.2') of (10.2), (9.1) and (10.1 entail (10.2) (hence, so do (9) and (10) under the readings (9') and (10'), respectively):

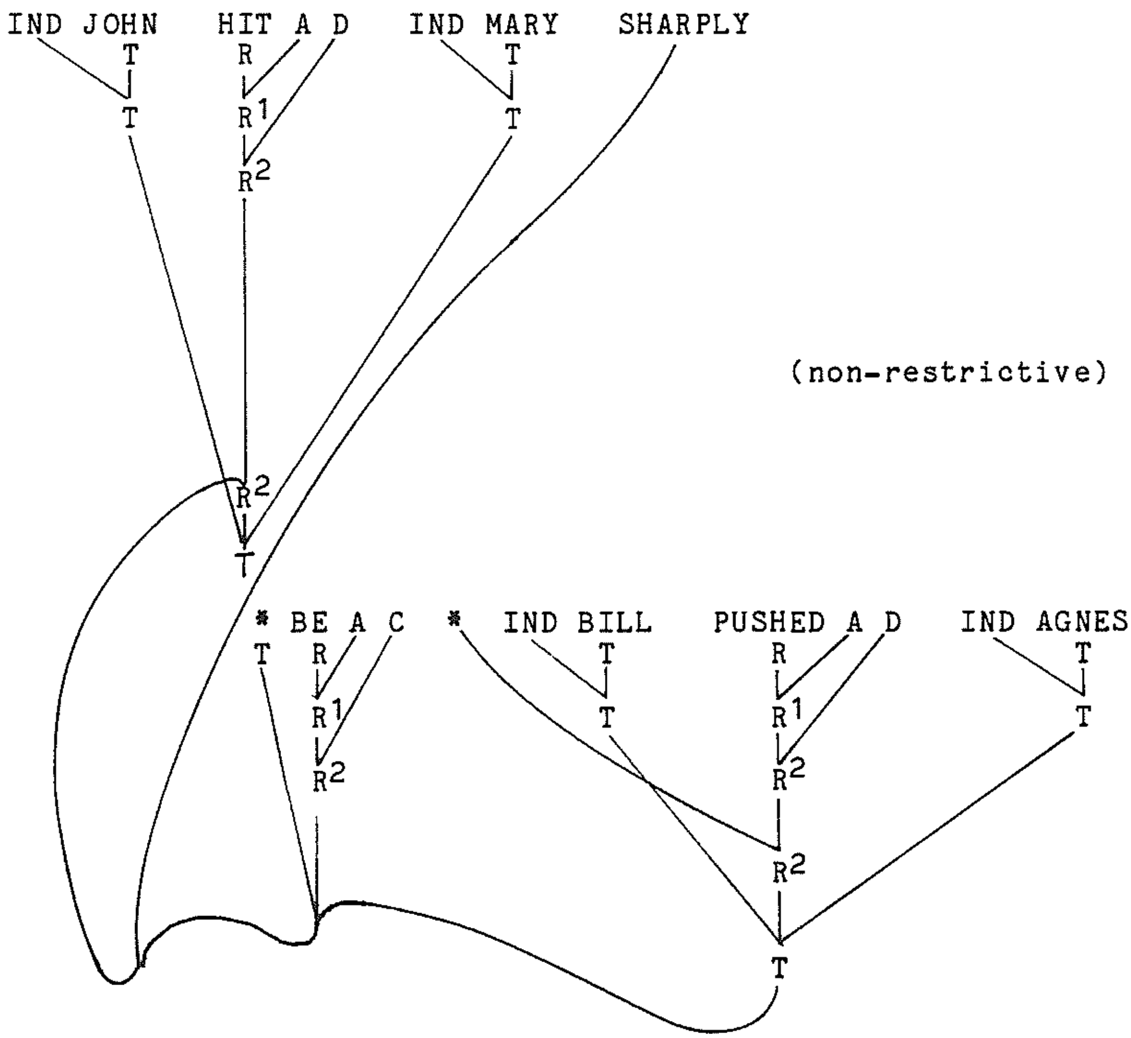
(10.2) Bill is tall and hit Agnes

(10.2')



(11) John hit Mary sharply, which is how Bill pushed Agnes

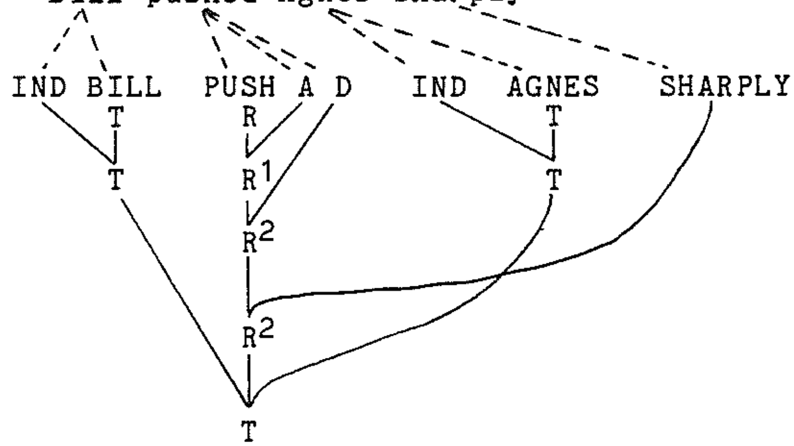
(11')



Under the reading (11') of (11), and under the reading (11.1') of (11.1), (11) entails (11.1):

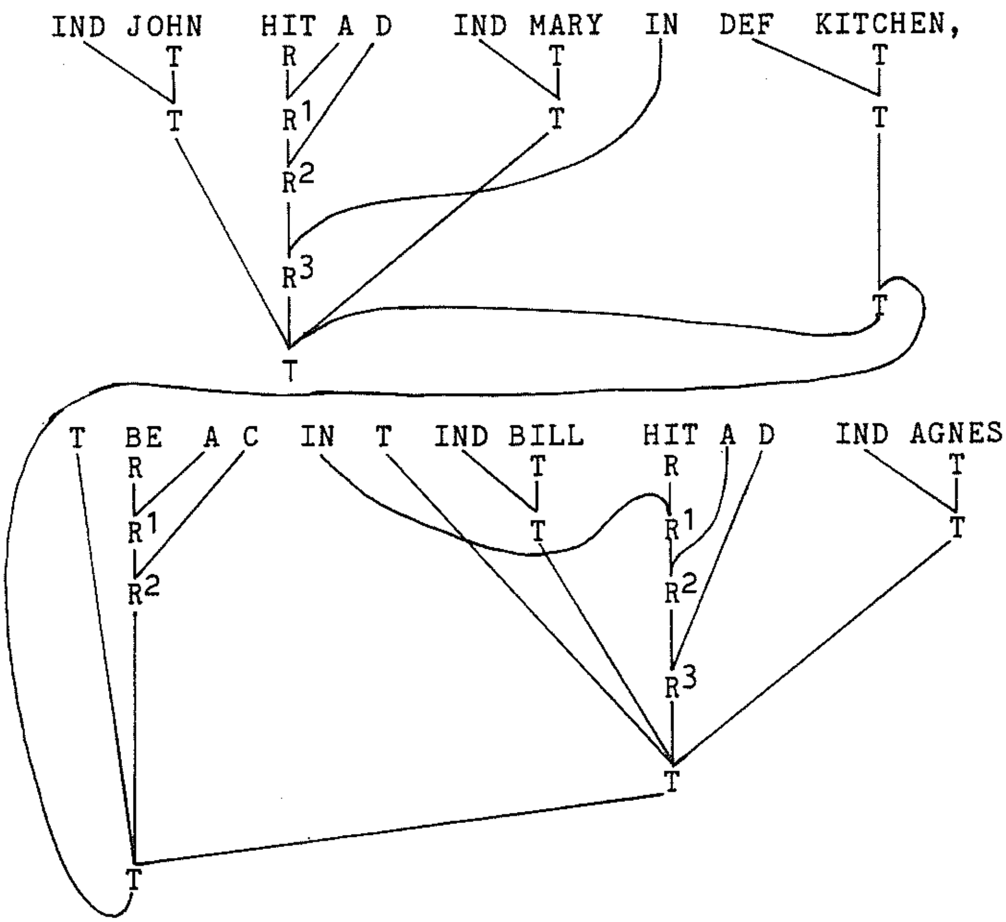
(11.1) Bill pushed Agnes sharply

(11.1')



(12) John hit Mary in the kitchen, which is where Bill hit Agnes

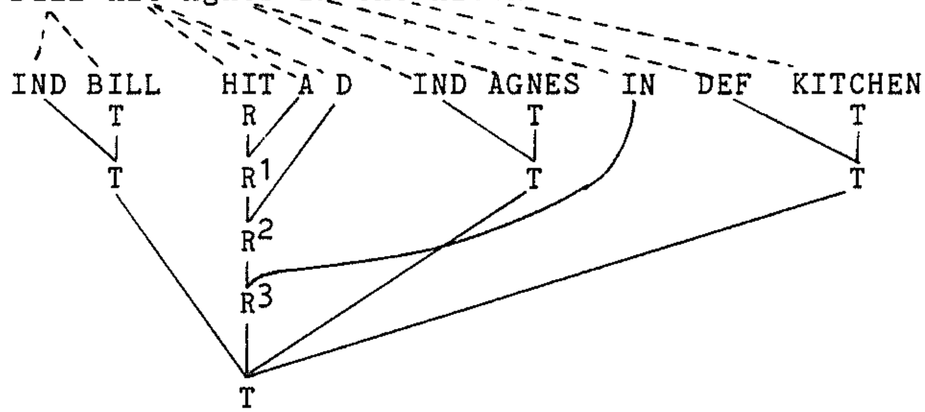
(12')



Under the reading (12') of (12), and under the reading (12.1') of (12.1), (12) entails (12.1):

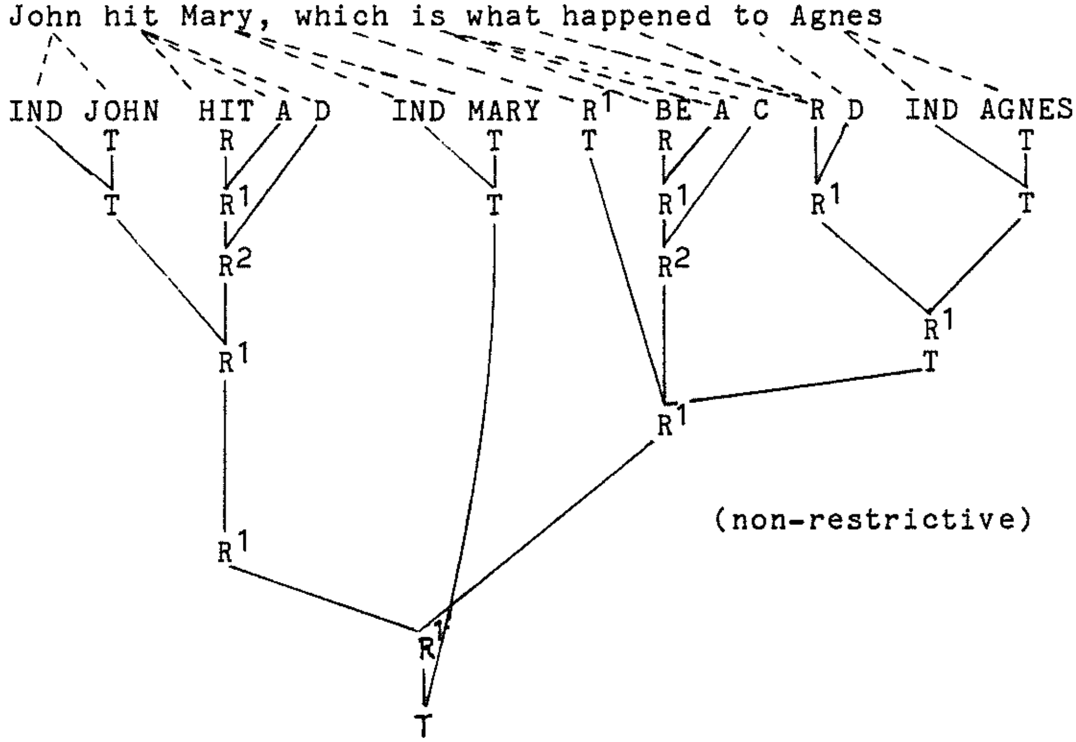
(12.1) Bill hit Agnes in the kitchen

(12.1')



(13) John hit Mary, which is what happened to Agnes

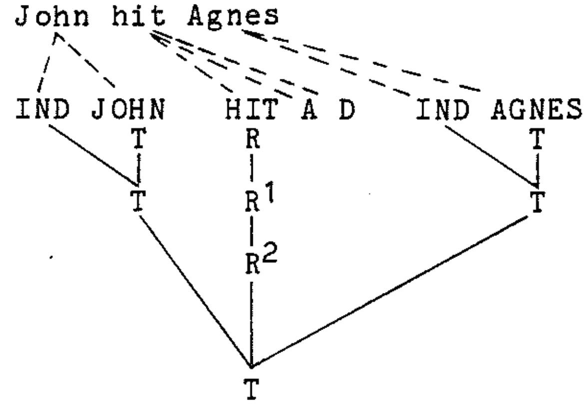
(13')



Under the reading (13') of (13), and under the reading (13.1') of (13.1), (13) entails (13.1):

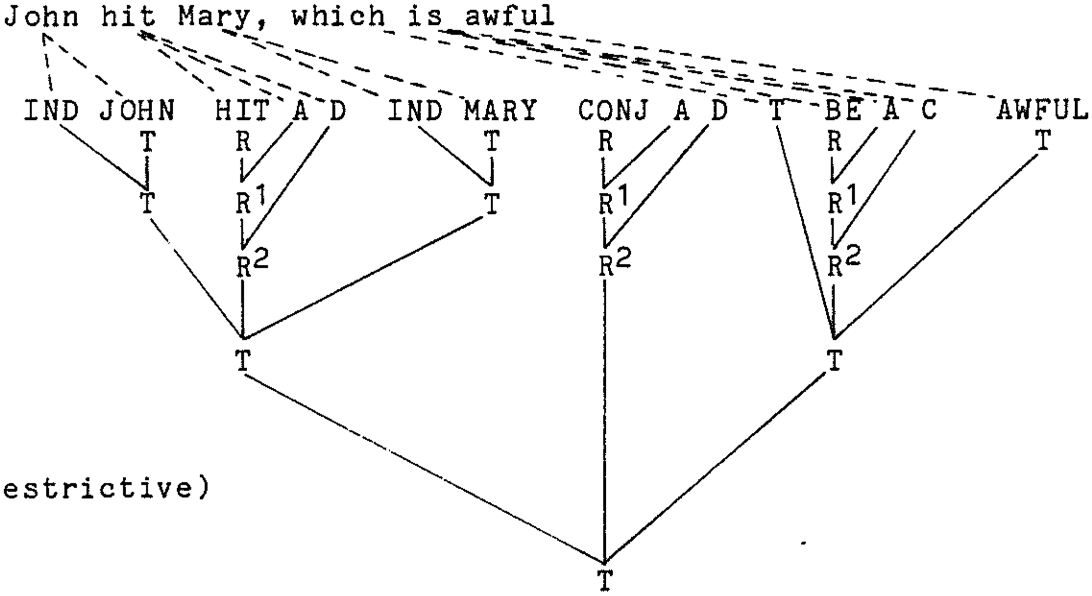
(13.1) John hit Agnes

(13.1')



(14) John hit Mary, which is awful

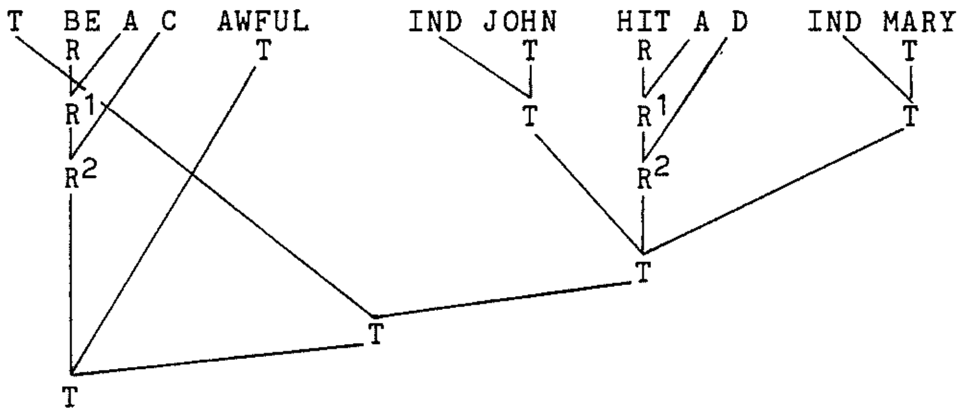
(14')



(non-restrictive)

Under the reading (14') of (14), and under the reading (14.1') of (14.1), (14) entails (14.1):[111]

(14.1) It is awful that John hit Mary

(14.1') 

Note 111. One difference between (14) and (14.1) is a difference in _focus_, wherein the ascription of awfulness to John's hitting Mary is subordinated to John's hitting Mary in (14), whereas, in (14.1), the ascription of awfulness is not subordinated, but is dominant. The subordination is indicated in (14') by the case morphemes A and D, wherein A, signifying agency, governs the main clause, and D, signifying recipient of the action, governs the relative clause. The significance of agency of action and reception of action in this case is somewhat degenerate; but serves to distinguish the given direction, as opposed to what we might try to convey in English by "what is awful is that John hit Mary", which has a dominant normal reading equivalent to the reading (14.1').

We might compare (14) and (14.1) to the following English sentences which have dominant normal readings equivalent to (14') and (14.1).

(14.2) That John hit Mary is awful

(14.3) John hit Mary and that is awful

(14.4) What is awful is that John hit Mary

The differences among (14), (14.1), (14.2), (14.3), and (14.4) are differences in aspects other than entailment, including, in particular, focus.

Ordinary Phrasal Restrictive and Non-Restrictive Relative Readings

Consider the sentences:

(15) John loves the kind girl

(16) John hates the cruel girl

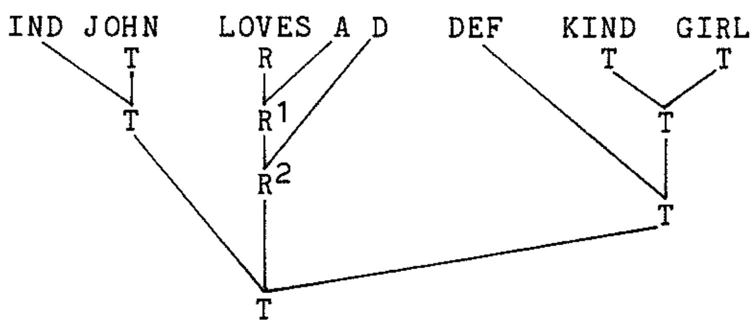
(17) John loves a kind girl

(18) John hates a cruel girl

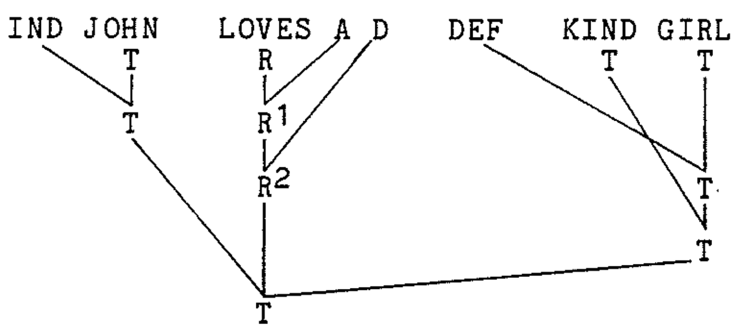
In each of (15), (16), (17), and (18), the word-string "kind girl" or "cruel girl" can be assigned a phrasal restrictive or non-restrictive reading. There are normal readings r(15), r'(15), r(16), r'(16), r(17), r'(17), r(18), and r'(18) such that: (i) r(15), r(16), r(17), and r(18) have, as subreadings, phrasal restrictive relative readings of "kind girl" or "cruel girl," whose respective syntactic components appear below as (15'), (16'), (17'), and (18'); and (ii) r'(15), r'(16), r'(17) and r'(18) have, as subreadings, phrasal restrictive relative readings of "kind girl" or "cruel girl," whose respective syntactic components appear below as (15''), (16''), (17''), and (18'').

Analogous to the case for clausal relative readings, (15) entails (17) under the readings r(15) or r'(15) of (15) and r(17) or r'(17) of (17); (16) entails (18) under the readings r(16) or r'(16) of (16) and r(18) or r'(18) of (18); (17) entails (7) under the readings r(17) or r'(17) of (17) and r(7) of (7); (18) entails (8) under readings r(18) or r'(18) of (18) and r(8) of (8); the readings r(17) and r'(17) are equivalent, as are the readings r(18) and r'(18), in the sense that their syntactic components have the same denotation under s_o; (1) and (15) inter-entail each other under the readings r'(1) of (1) and r'(15) of (15), and similar inter-entailments hold for the pairs (2),(16); (5),(17); and (6),(18).

In the following, we list the syntactic components of the various readings alluded to above, Specifically, the syntactic component of r(15) appears as (15') below, that of r'(15) appears as (15''), that of r(16) appears as (16'), that of r'(16) appears as (16''), that of r(17) appears as (17'), that of r'(17) appears as (17''), that of r(18) appears as (18'), and that of r'(18) appears as (18'').

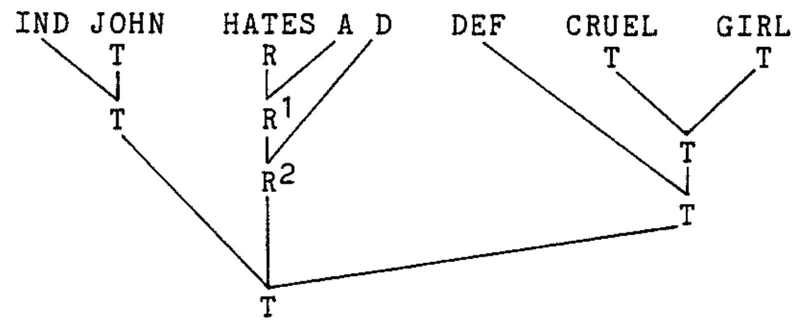
(15')


(restrictive)

(15'')


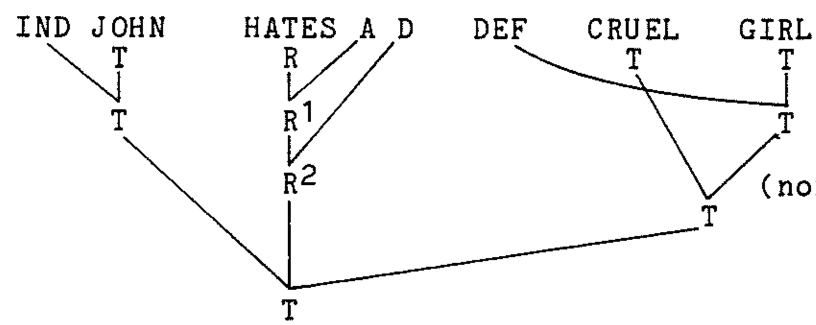
(non-restrictive)

(16')



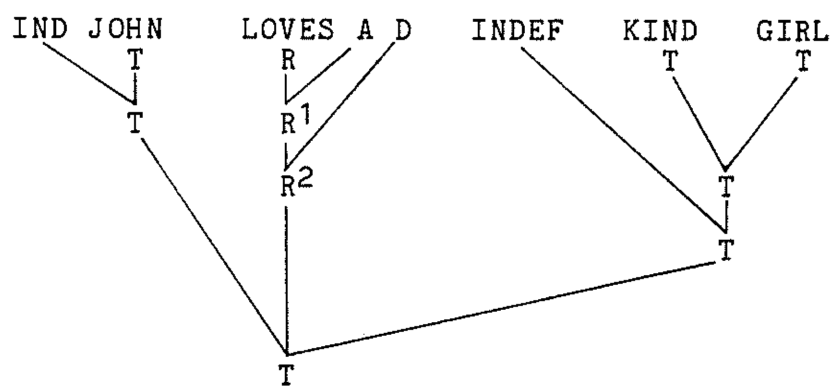
(restrictive)

(16'')



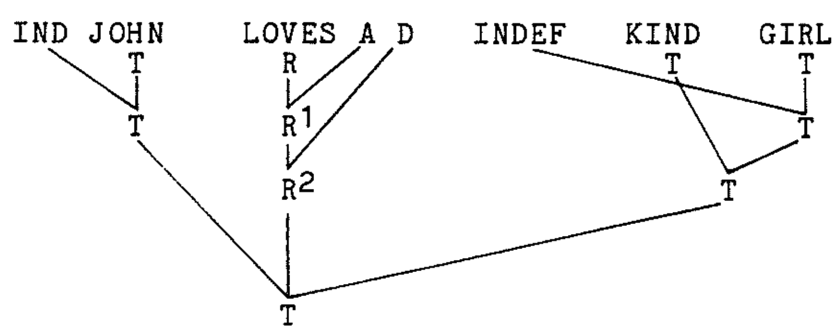
(non-restrictive)

(17')



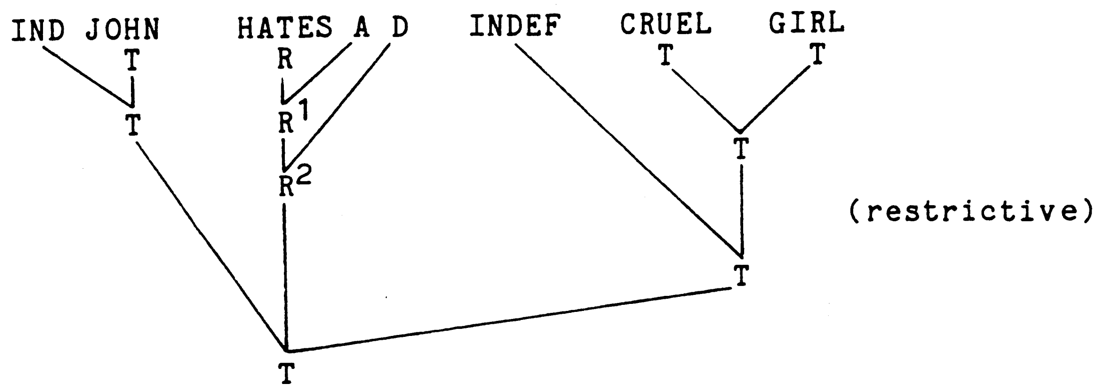
(restrictive)

(17'')

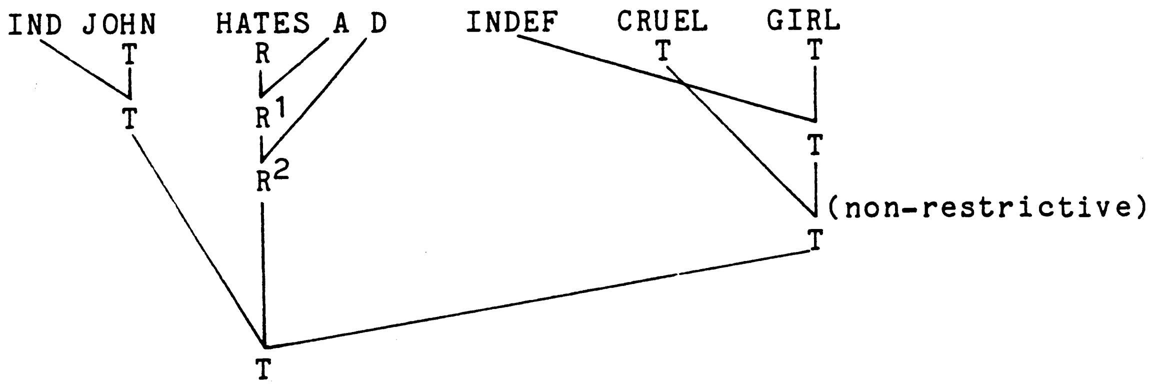


(non-restrictive)

(18')



(18'')



The readings (15'),(15'') of (15); (16'),(16'') of (16); (17'),(17'') of (17); and (18'),(18'') of (18) are called *ordinary restrictive relative phrasal readings*; (15'), (16'), (17'), and (18') are said to be *short-scoped*, while (15''), (16''), (17''), and (18'') are said to be *long-scoped*.

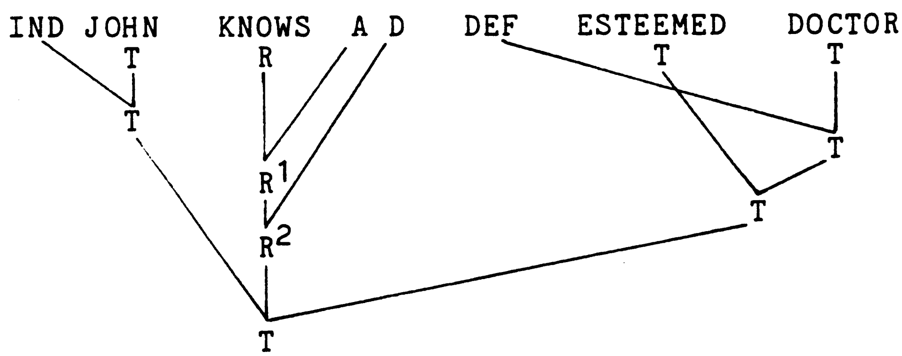
A final note: while the situations with clausal and phrasal readings are wholly analogous insofar as the pattern of entailments is concerned, there are some differences as regards *normality*: that is, while each of the readings r(1), r'(1), ..., r(8), r'(8) has been a normal reading of its respective sentence, this has not been the case for the readings r'(15) of (15), r'(16) of (16), r'(17) of (17), nor r'(18) of (18), though it has been the case for the readings r(15) of (15), r(16) of (16), r(17) of (17), and r(18) of (18). The reason for this is that the adjectives "good" and "cruel" in the positions they occupy relative to "loves" and "hates" in (15), (16), (17), and (18) signal restrictive relatives, insofar as John's loving or hating the girl is suggestively dependent on the girl's being kind or cruel. When such dependence is not suggested by the lexical meanings of words, or, in the extreme case, where total independence is suggested, then the non-restrictive reading is at least possible (i.e., weakly normal) or even dominant.

Consider the following example:

(19) John knows the esteemed doctor.

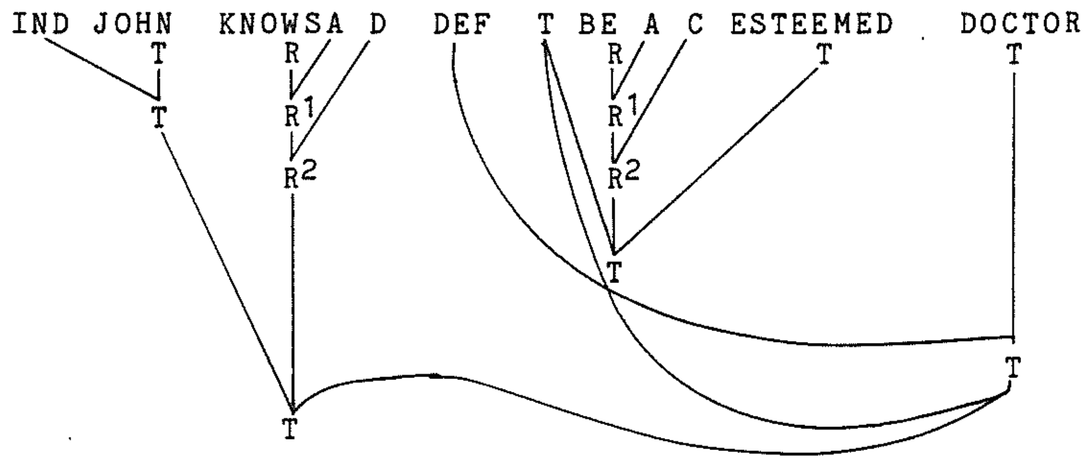
The doctor's being esteemed is independent of John knowing him, which has the following non-restrictive relative phrasal reading:

(19')



But even here, the more dominant normal reading would seem to be the following non-restrictive relative clausal reading:

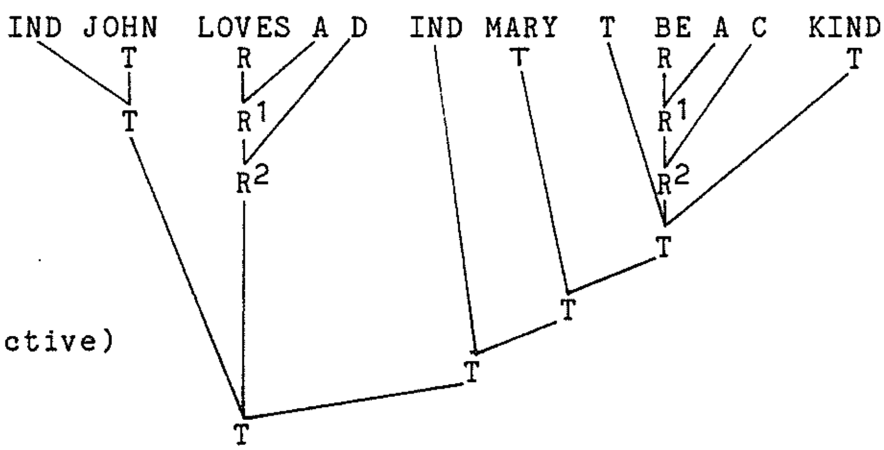
(19'')



Let us consider several examples involving relative clauses and phrases on an <u>individual-expression</u>:

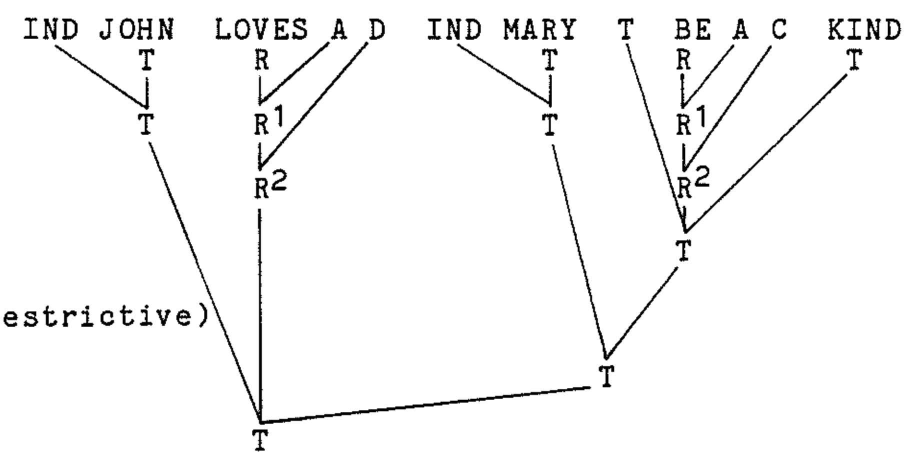
(20) John loves Mary who is kind

(20')

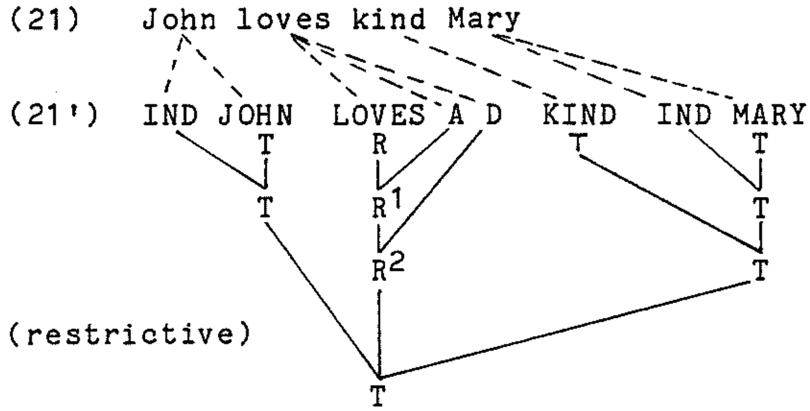


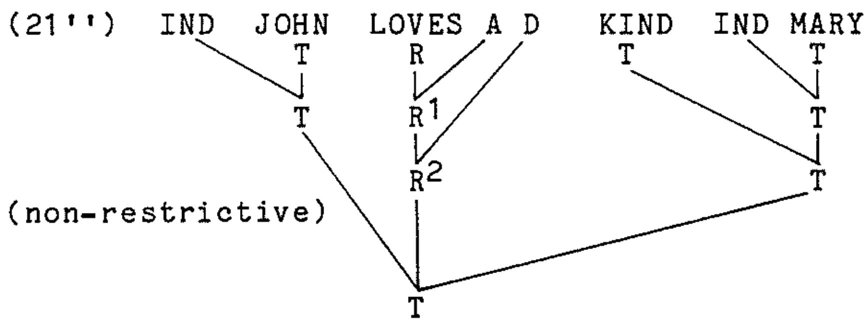
(restrictive)

(20'')



(non-restrictive)





The difference between the restrictive relative clausal reading (20) of (20') and the non-restrictive relative clausal reading (20'') of (20') is that (20') distinguishes those "Marys" that are kind, and asserts that John loves (at least) some particular one of these Marys, whereas (20'') distinguishes (at least) some particular Mary and asserts that John loves (at least) that particular Mary.

Entirely analogous remarks apply to the restrictive relative phrasal reading (21') of (21) and the non-restrictive relative phrasal reading (21'') of (21). Indeed, (20) is equivalent to (21) under the restrictive relative readings (20') and (21') and under the non-restrictive relative readings of (20) and (21), respectively.

It is a byproduct of our treatment of relative clauses that restrictive relative clauses and phrases can be formed only on thing-expressions containing an initial determiner. Since, in particular, we do not employ determiners to modify relation-expressions or modifiers (a consequence of the fact that we are restricting our treatment to TR-languages), all relative clauses and phrases on relation-expressions and modifiers are non-restrictive.

3.6 <u>Ergative Readings</u>

Consider the sentences:

(1) John threw the ball

(2) John moved the ball

(3) John threw

(4) John moved

(5) The ball was thrown

(6) The ball was moved

(7) The ball threw

(8) The ball moved

(1) intuitively entails (3) and (5) but not (7), while (2) intuitively entails (6) and (8) but not (4).

There appear to be two ways within our framework in which this asymmetry might be explained relative to the surface string (2):

(a) There is an implicit lexical morpheme realized in (2), which has the meaning of "cause" and which has the grammatical function of a transitive verb whose agent is that which causes, i.e., the denotation of "John," and whose direct object is that which is caused, i.e., the denotation of "moved the ball," which here is to be construed as an *event*, namely that denoted by "the ball moved." In this sense (2) is intuitively equivalent to "John caused the ball to move" or "John caused that the ball move(d)."

(b) While, as in (a), "John" still denotes the agent, the action is not that denoted by an implicit lexical morpheme, but rather is that denoted by several implicit logical morphemes, which have the grammatical effect of converting the phrase "moved the ball" in (2) into an intransitive verb whose agent is the denotation of "John." That is to say, these logical morphemes convert the event denoted by "moved the ball," i.e., "the ball moved" into a 1-place relation whose only argument is the denotation of "John." In this sense, (2) would mean, very roughly, that John engaged in an action, namely the action of moving the ball, rather than, say, an action that produced, i.e, caused, moving the ball. That is, under (b), what John did was move the ball; whereas, under (a), what John did was to cause "move the ball," i.e., cause the ball to move. Another way to put it is to say that, under (a), John produced an event, whereas, under (b), John acted an event.

There is yet a further distinction in possible ways of construing "moved the ball" in (2) which cross-cut distinctions (a) and (b), namely, the distinction between

(i) That sense of "the ball moves" where the denotation of "the ball" is the agent of the denotation of "moves," and

(ii) That sense of the ball moves -- more compactly carried by "the ball is moved" -- wherein the denotation of "the ball" is the direct object of the denotation of "moves."

The intended distinction between senses (i) and (ii) of "the ball moves" can perhaps be better appreciated by considering the sentence:

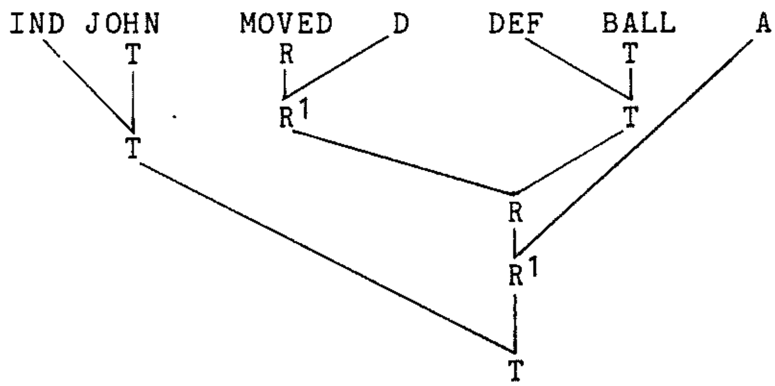
(9) John hurried the waiter

where sense (i) would appear to be dominant, which can be contrasted with the sentence (2) where sense (ii) appears to be dominant.

The alternative among the four above, namely (a)(i), (a)(ii), (b)(i), and (b)(ii), that appears most correct in construing sentence (2) is the last, namely (b)(ii). Under this alternative, (2) would be construed as affirming that John engaged in the action of the ball being moved. This alternative corresponds to what will shortly be defined as an <u>ergative reading</u>.

My reasons for the choice (b)(ii) are, briefly, the following: First, I prefer (b) to (a) because (b) avoids the introduction of a special lexical ("causative") representational morpheme that is not explicitly marked in the surface string (1); second, I prefer alternative (ii) to (i), because, when formalized in conjunction with (b), it alone provides a reading (2') for (2):

(2')

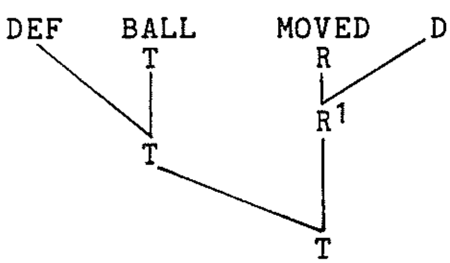


under which (2) entails (6):

(6) The ball was moved

under the reading (6') of (6):

(6')



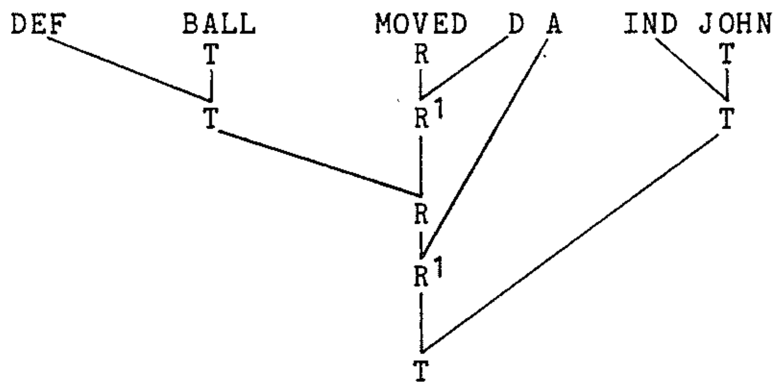
We call (2') *the ergative reading* of (2), and define the notion of *ergative reading* in a general way later in this section.

We note the following:

(10) The ball was moved by John

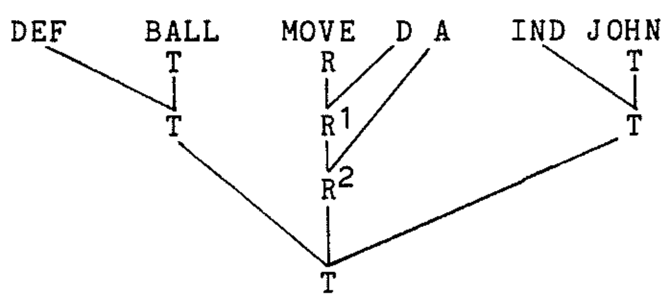
like (2), can also be construed in senses corresponding to (a)(i), (a),(ii), (b)(i), and (b)(ii).

If (10) is construed ergatively, then (10) receives the ergative reading (10'):

(10') 

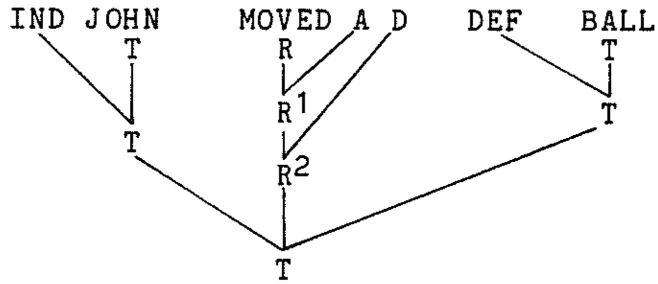
That is, under the ergative readings (2') and (10') of (2) and (10), respectively, and under the (dominant) passive reading (6') of (6), (2) and (10) inter-entail each other and both entail (6).

Note further that if (10) is assigned the *passive reading* (10'') below rather than the ergative reading (10') above:

(10'') 

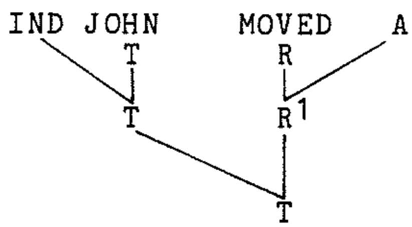
then (10) also entails (6) under this reading of (10) as well, and the reading (6') of (6). However, the ergative reading (10') of (10) and the passive reading (10'') of (10) are not equivalent. Nor is the active reading (2'') (below) of (2) equivalent to the ergative reading (2') of (2):

(2'')



In particular, (2) entails (4) under the active reading (2'') of (2) and under the following dominant normal (active) reading of (4):

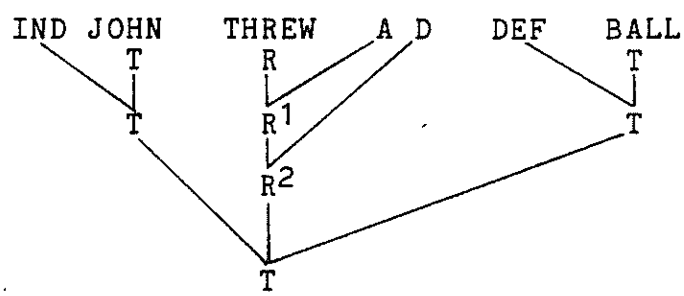
(4')

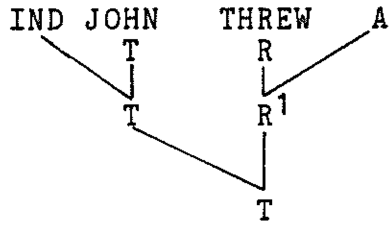


whereas the ergative reading (2') of (2) does not.

By way of contrast, (1) entails (3) under the following active readings (1'),(3') of (1) and (3) respectively:

(1')

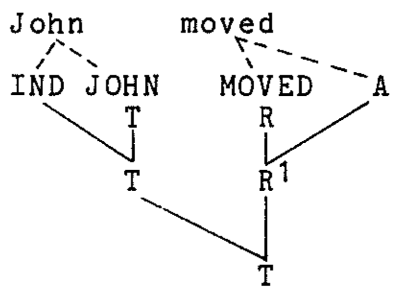


(3')


which appears consistent with intuition.

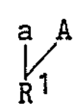
Analogously, (2) entails (4) under the active readings (2'') of (2) and (4') of (4):

(4)

(4')


which is not consistent with intuition. The difference between the above two cases is that in the former case, the active readings (1') and (3') of (1) and (3) respectively, were both normal, whereas in the latter case, while the active reading (4') of (4) is a normal reading of (1), the active reading (2'') of (2) is not a normal reading of (2). The normal reading of (2), rather, is the ergative reading (2') given earlier.

The following characterization of ergative reading reflects the above considerations. An ergative reading of a word-string e is a sentential reading whose syntactic representation component e' is such that the major relation expression in e' has the form



where a is a sentence with major relation-expression r^n, where r^n is in turn of the form $\underset{R^1}{b\;D}$, for some modifier b.

We have, in passing, also referred to "active readings" and "passive readings." These can be characterized more exactly as follows:

An <u>active reading</u> of a word-string e is a sentential reading whose syntactic representation component e' is such that the case morpheme A in the major relation-expression r^m of e precedes the case morpheme D in r^m relative to the relative-place ordering on the major thing-expressions of e'. A <u>passive reading</u> of a word-string e is a sentential reading whose syntactic representation component e' is such that the case morpheme D in the major relation-expression r^m of e precedes the case morpheme A in r^m relative to the relative-place ordering on the major thing-expressions of e'.

3.7 Intensional Readings

We maintain the option of utilizing a "possible-worlds" treatment of intensionality by the use of the set R of binary relations occurring as the third term of a semantic theory <F,V,R>, as we had for the logical modality relations in Section 2.3.2.4 of Chapter 2. On the other hand, our approach permits us to treat certain types of intensionality in ways other than by "possible-worlds" interpretations. In this section, we explore such an alternative.

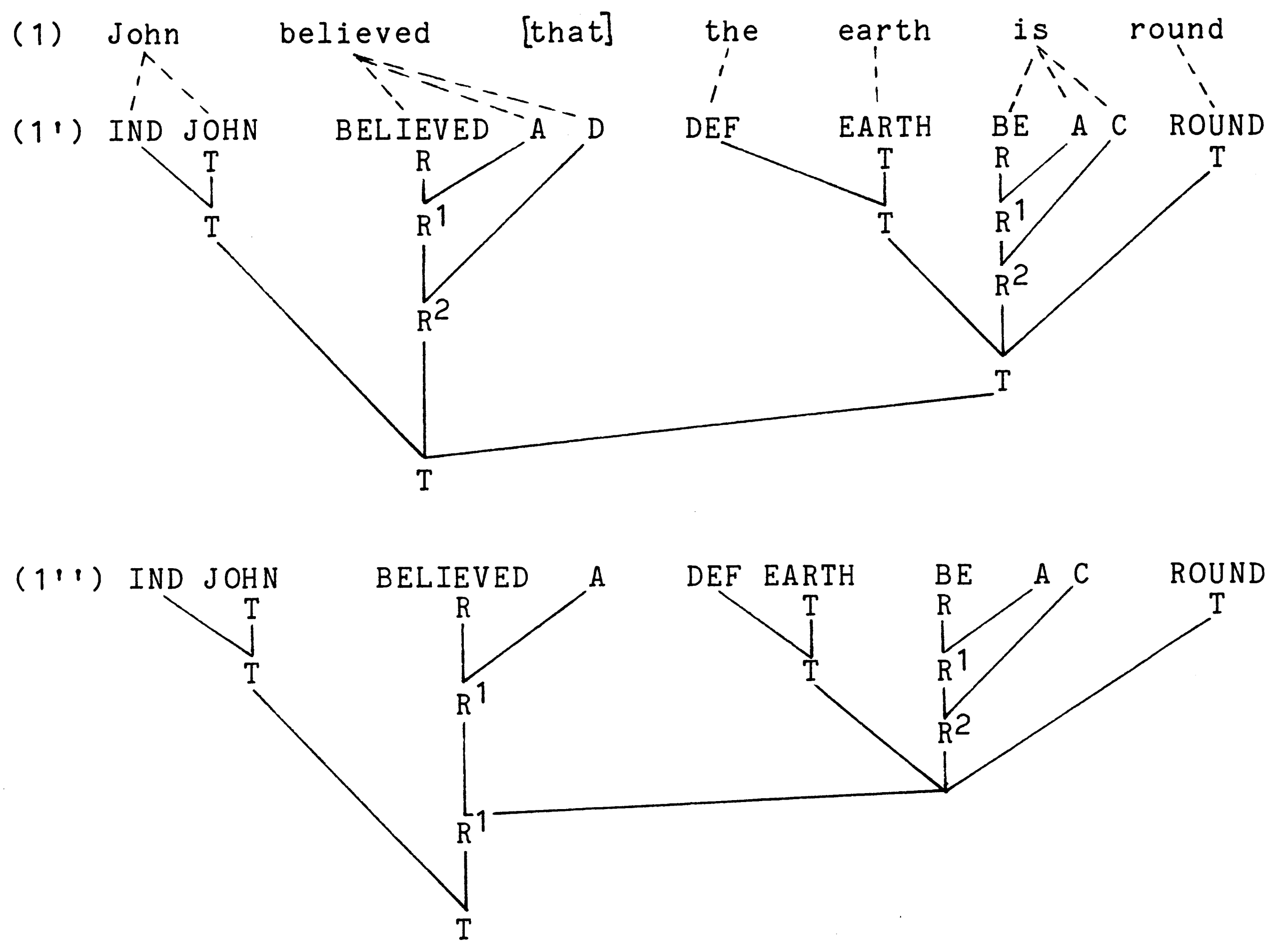
Broadly speaking, we intensionalize an occurrence of an expression by semantically *underspecifying* the structure of the denotation of that expression (at that occurrence). Such underspecification prevents the drawing of undesired entailments.

Within our framework, a reading <e',i> of an occurrence of a natural-language word-string e within a containing word-string b is *an intensional reading of e in b* just in case e' is a modifier of SYN_L^{TR}, and there is an occurrence of e within a containing word-string d such that in the syntactic component of the dominant normal reading of d, e is syntactically represented as a thing or relation expression. A reading of e in b that is not intensional is called *extensional*.

Let me motivate this definition: Recall that the dominant normal reading of a word-string is that reading which has the highest degree of normality with respect to usual contexts of utterance. Thus, by the above definitions, an intensional reading of a word-string e within a containing expression b is one in which, while the usual denotation of e is a thing or a relation, its denotation in b is neither a thing or relation, but rather a way of qualifying (i.e., modifying) a thing or relation.

Formally, the unintended entailments of a word-string b under an intensional reading of a word-string e within b, i.e., those entailments of b that would be induced by a reading of b in which e were syntactically represented by a thing or relation expression, are effectively blocked by representing e as a modifier within b.

Consider the readings of (1) whose respective syntactic components are (1') and (1''):



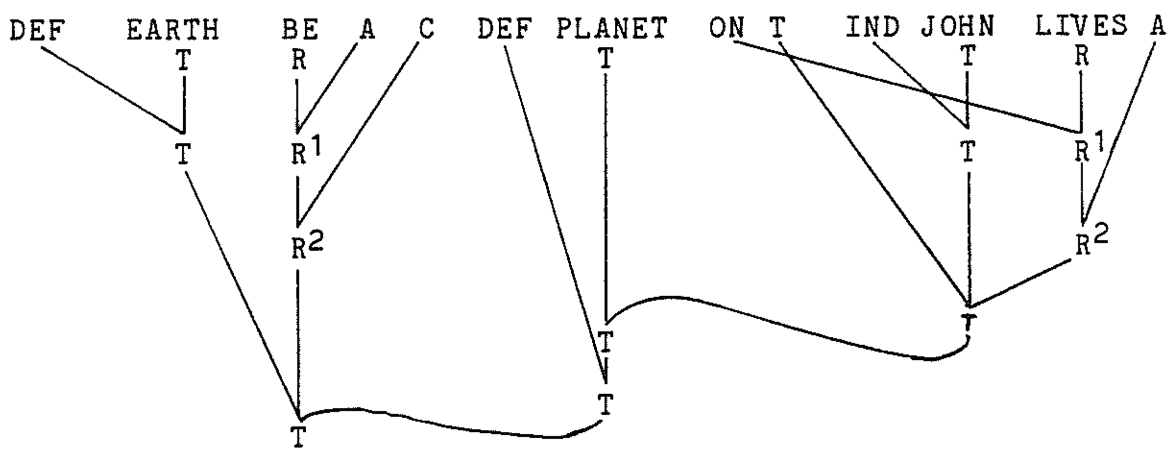
(1') contains an <u>extensional</u> reading of "the earth is round" in (1), while (1'') contains an <u>intensional</u> reading of "the earth is round" in (1). Accordingly, we call (1') the *extensional reading* of (1), and we call (1'') the *intensional reading* of (1). That is, the reading (1') of (1) means, roughly, that *John* is the agent of the action *believed* and that the entity that is believed is the event *the earth is round.*[112] Thus (1') interprets *believed* as a 2-place relation between a believer and an event. On the other hand, the reading (1'') of (1) means roughly that *John* is the agent of the action *believed the earth is round*, that is, John is the agent of the action of holding an earth-is-round belief. Thus, under the extensional reading (1') of (1), the word-string "the earth is round" is interpreted as (i.e., denotes) a thing, whereas under the intensional reading (1'') of (1), this word-string that it is interpreted as (i.e., denotes) a modifier on a relation.

Consider now the identity:

(2) The earth is the planet on which John lives

and the following reading of (2):

(2')



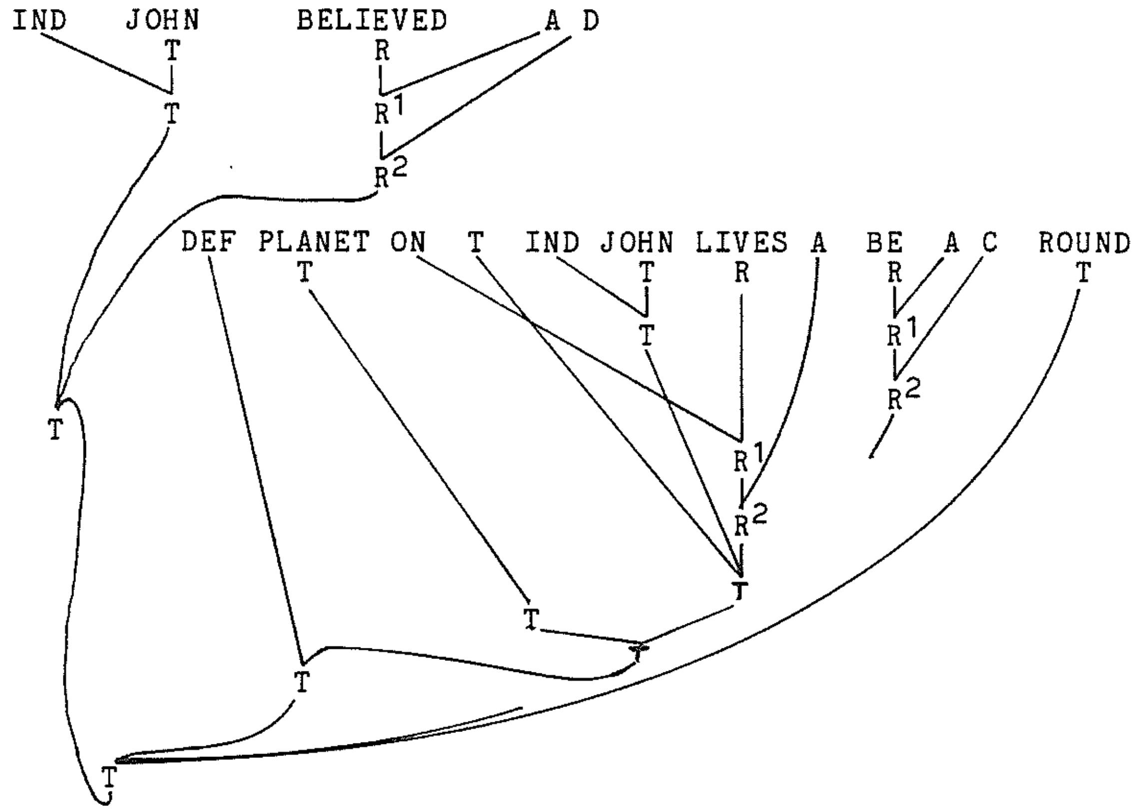
Note 112. Note that in this treatment the entity that is believed is an event and not a "mental" counterpart of an event, such as a concept or a proposition.

Consider, further, the sentence

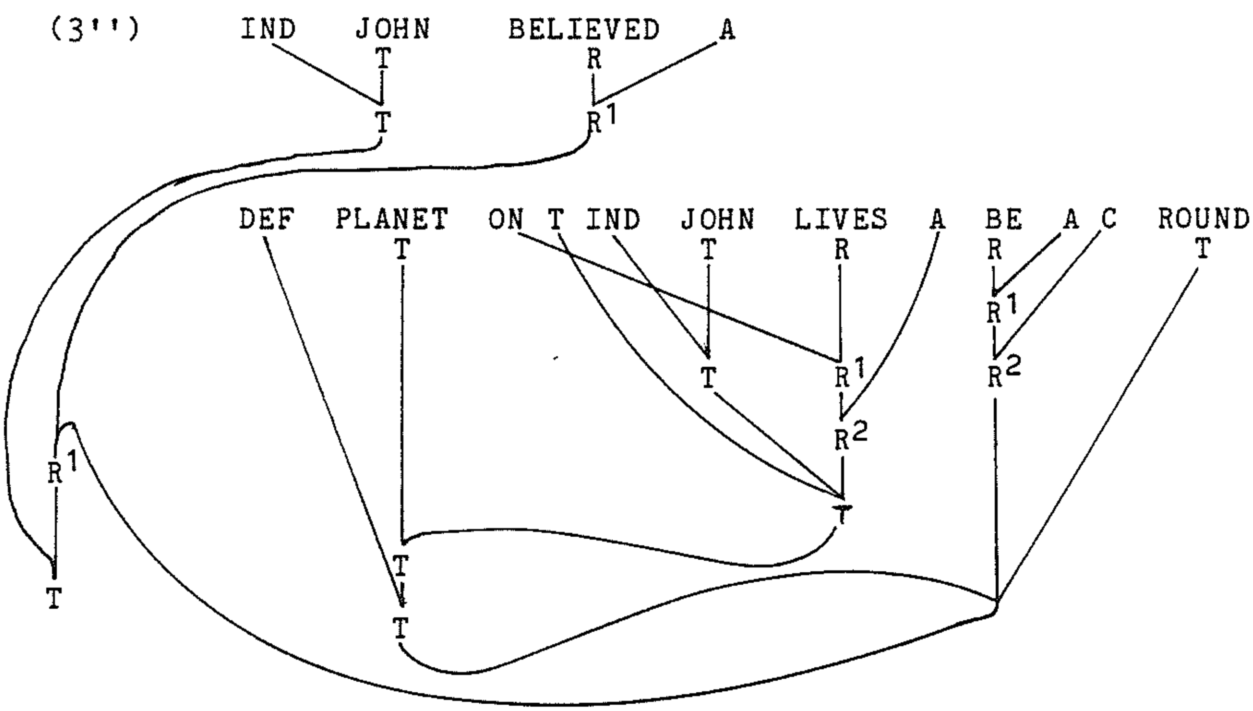
(3) John believed that the planet on which John lives is round

and the following two readings of (3):

(3')

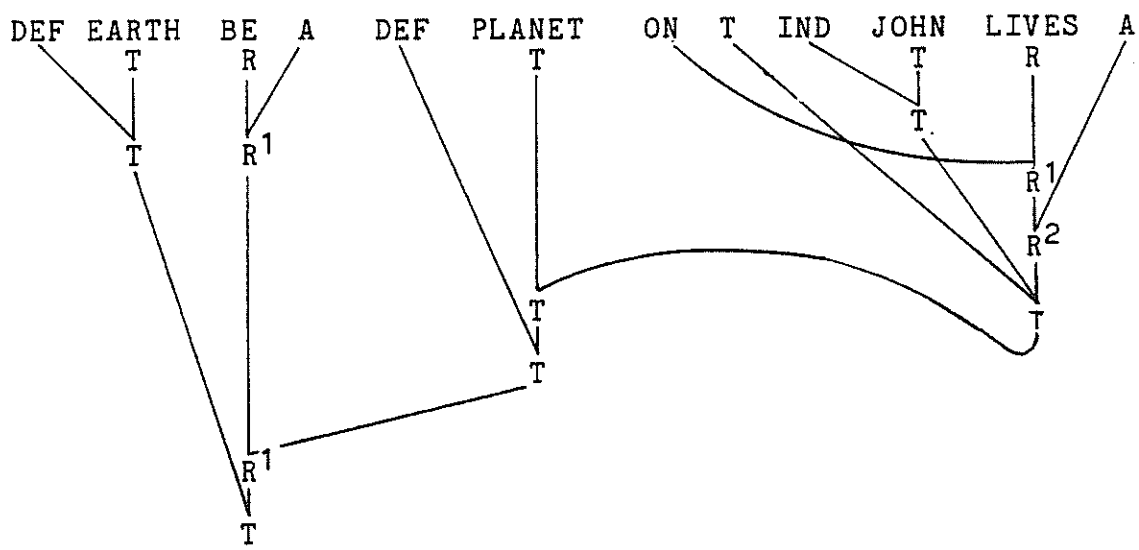


(3'')



Wholly analogous to the above considerations regarding the readings (1') and (1'') of (1), (3') contains an extensional reading and (3'') contains an intensional reading of "The planet on which John lives is round" in (3). We can express this more naturally, though less accurately, by saying that (3') is an extensional reading of (3) and that (3'') is an intensional reading of (3). For ease of description, we will use this latter means of expression, and speak of the containing sentence as having a given extensional or intensional reading. Now the logical semantic axioms LSA (1) - LSA (31) have the consequence that (1) and (2) together entail (3) under the extensional readings (1'), (2'), and (3') of (1), (2), (3) respectively, while (1) and (2) do not together entail (3) under the intensional readings (1'') and (3'') of (1) and (3) respectively and under the extensional reading (2') of (2) displayed. Nor would the latter entailment follow if we were (somewhat perversely) to replace the extensional reading (2) of "The planet on which John lives" in (2) by an intensional one thereby obtaining:

(2'')



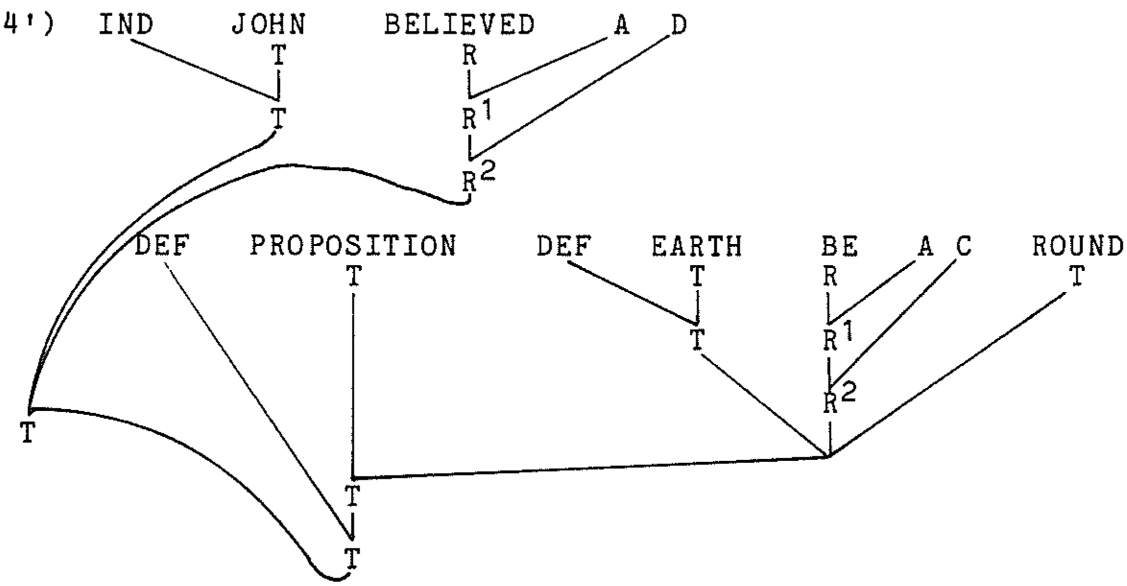
The reason that the latter intensional-reading-based entailments (appropriately) fail is that the set-theoretic structure of the denotations of key expressions in (1'') and (3'') is <u>under-specified</u> within INT_L^{TR}.

Let us consider a further version of (1):

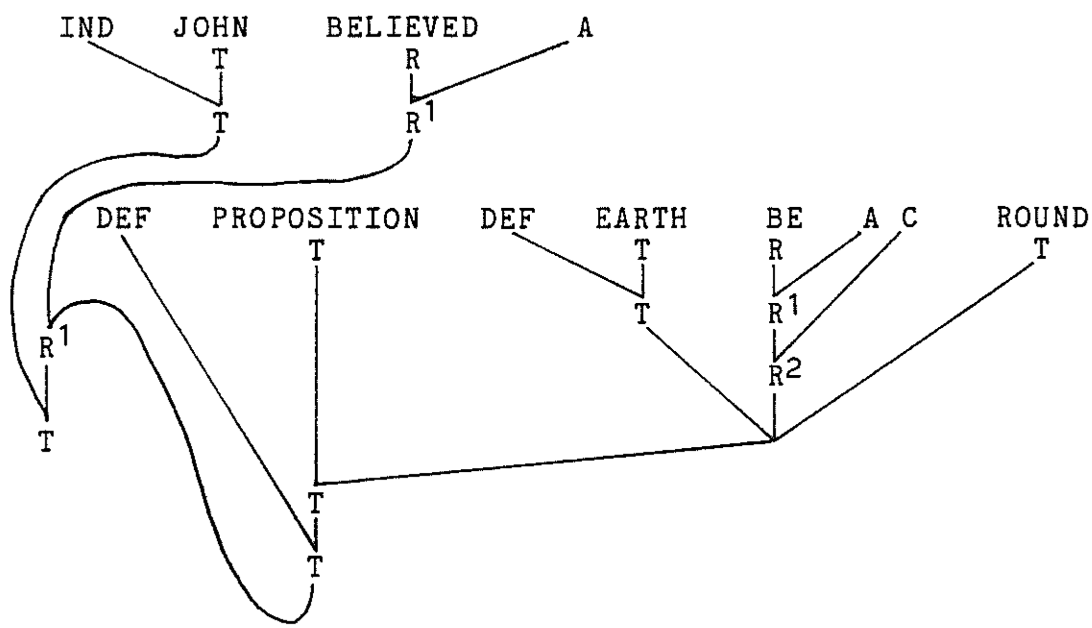
(4) John believed the proposition that the earth is round

and consider extensional and intensional readings of (4'), (4'') respectively of (4):

(4')



(4'')

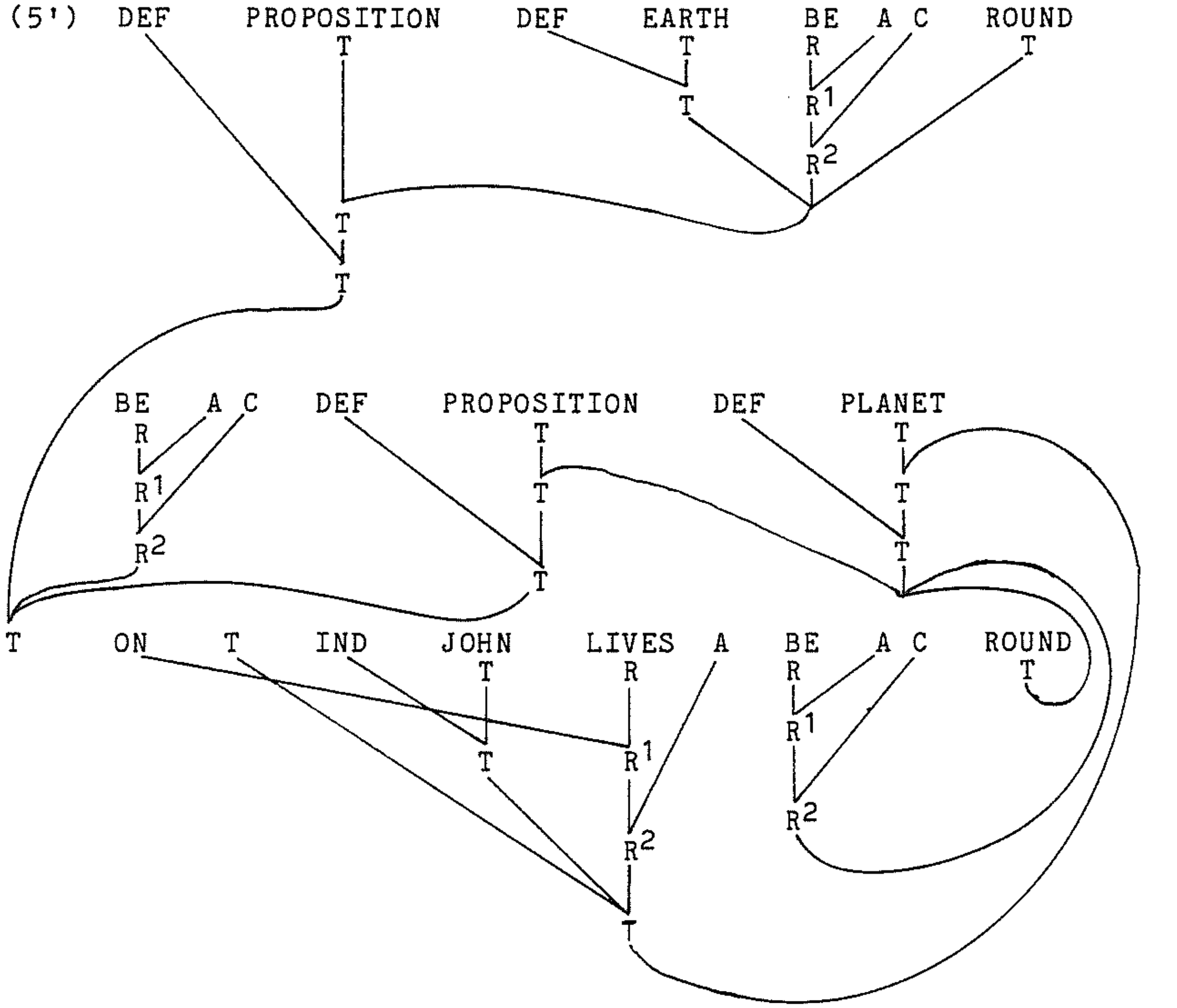


Now consider the identity:

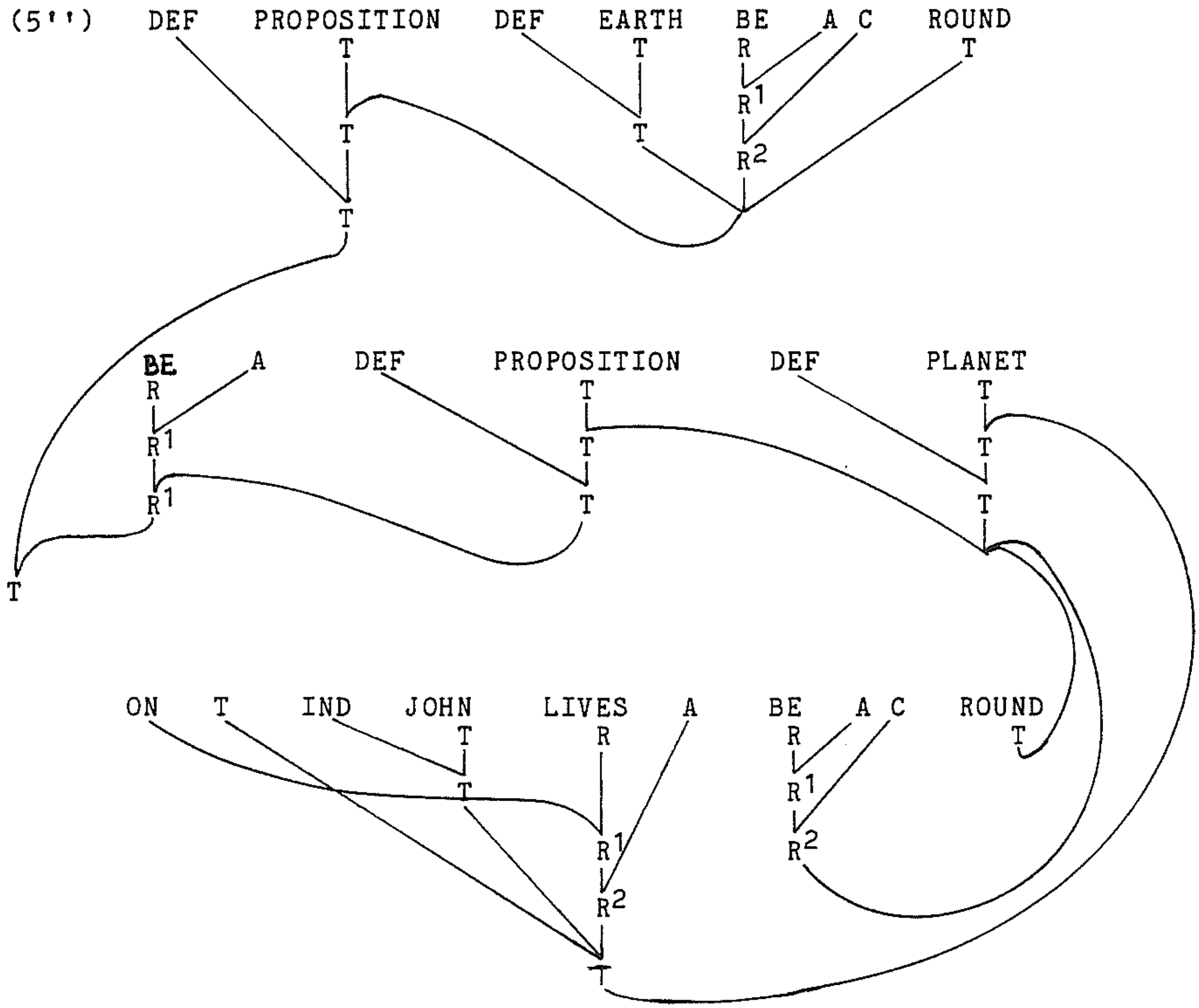
(5) The proposition that the earth is round is the proposition that the planet on which John lives is round

together with an extensional reading (5'), and an intensional reading (5'') of (5):

(5')



(5'')

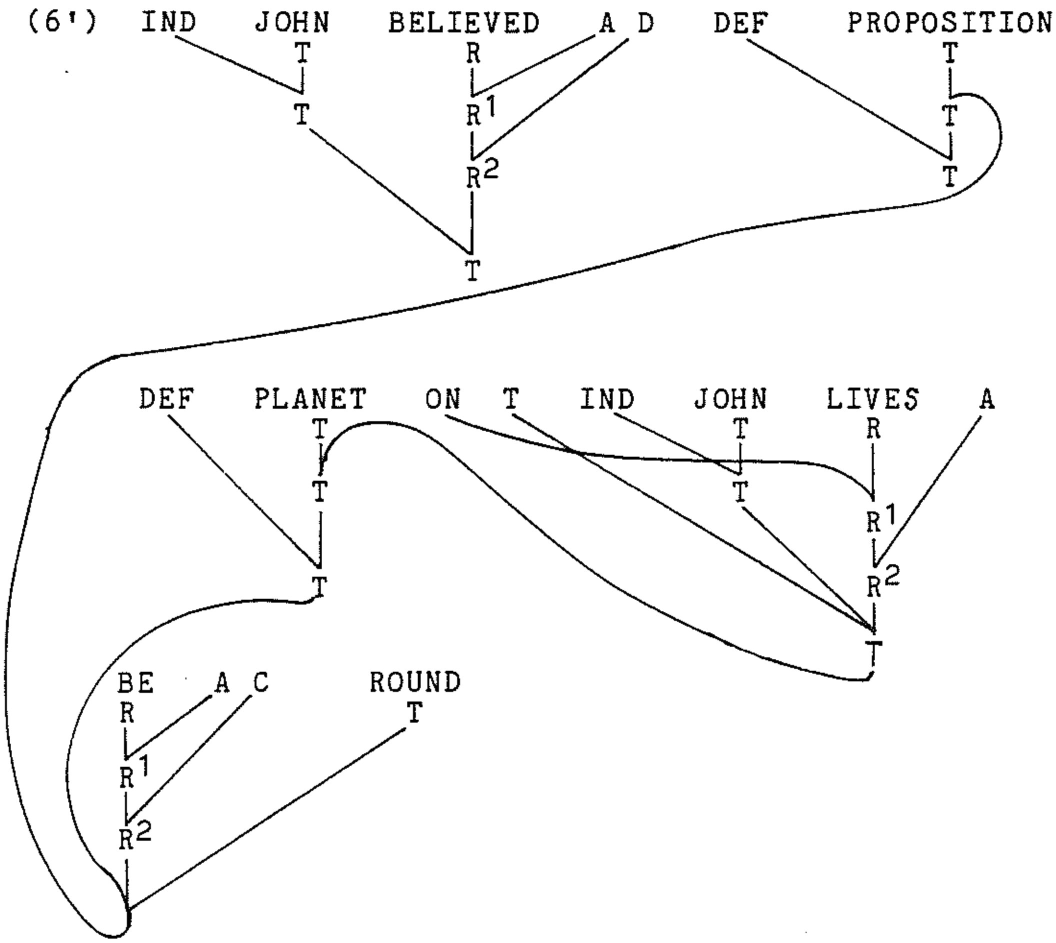

DEF
PROPOSITION
DEF
EARTH
BE
A
C
ROUND
T
R
R1
R2
BE
A
DEF
PROPOSITION
DEF
PLANET
ON
T
IND
JOHN
LIVES
A
BE
A
C
ROUND

Consider finally

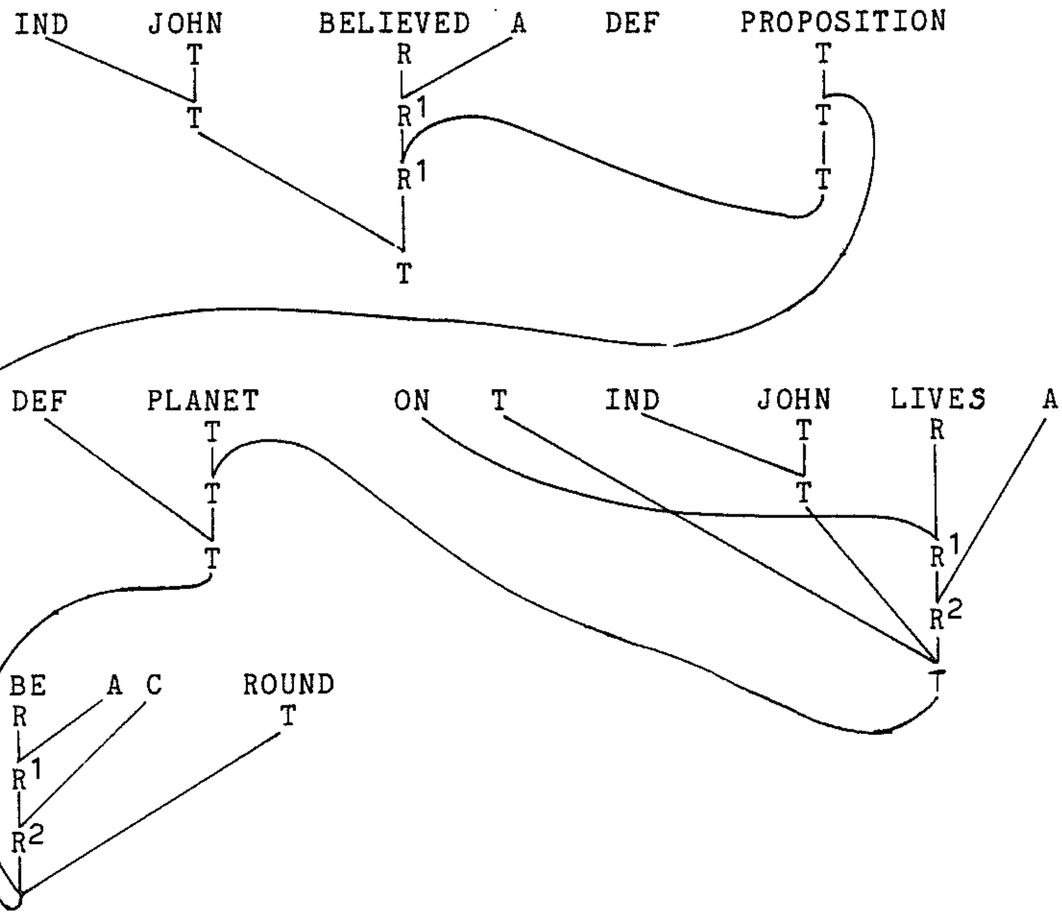
(6) John believed the proposition that the planet on which John lives is round.

an intensional reading (6'), and extensional reading (6'') of (6):

(6')



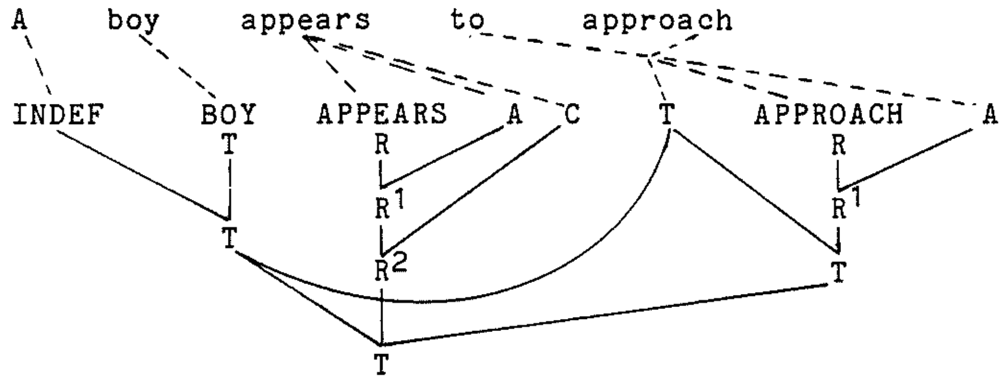
(6'')



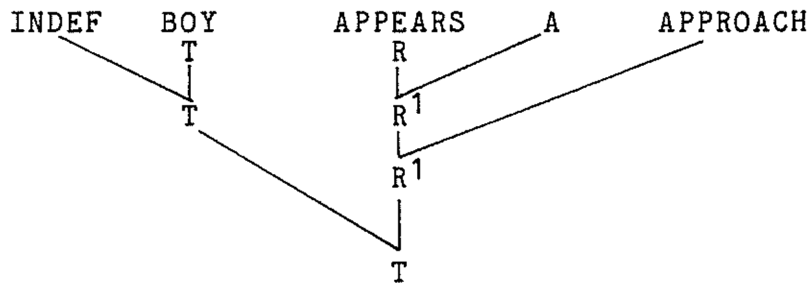
Now, analogous to the situation with (1), (2), (3) we have that (4) and (5) together entail (6) under their respective extensional readings (4'),(5'), and (6'), but not under their intensional readings (4'') and (6''), whether we use the extensional reading (5') or the intensional reading (5'') of (5).

Let us consider an example of another kind:

(7) (7')



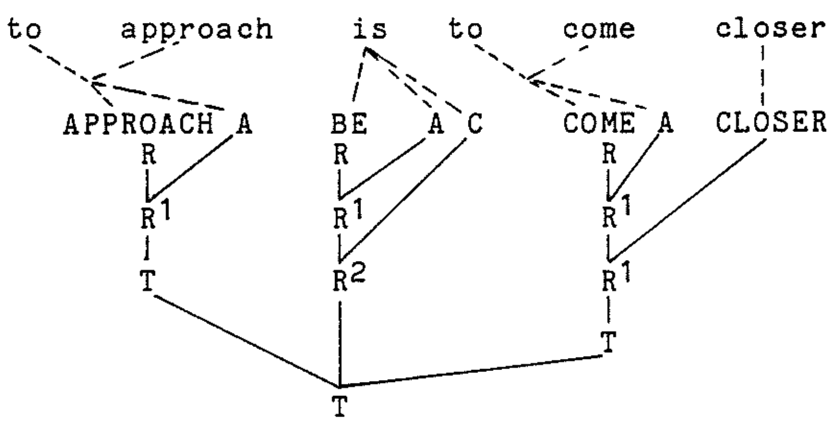
(7'')



(7') comprises an extensional reading of "to approach" in (7) while (7'') comprises an intensional reading of "to approach" in (7).

We next state an identity (8), and its dominant normal reading (8').

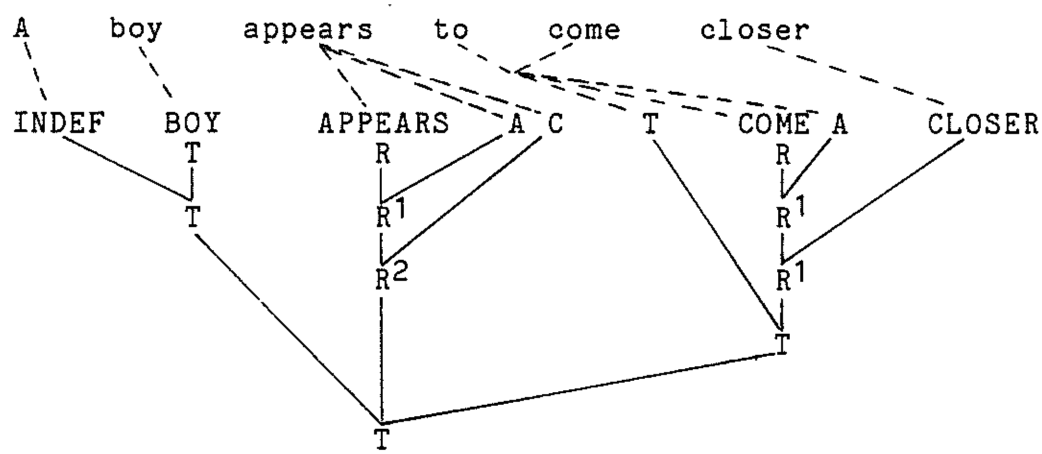
(8) (8')



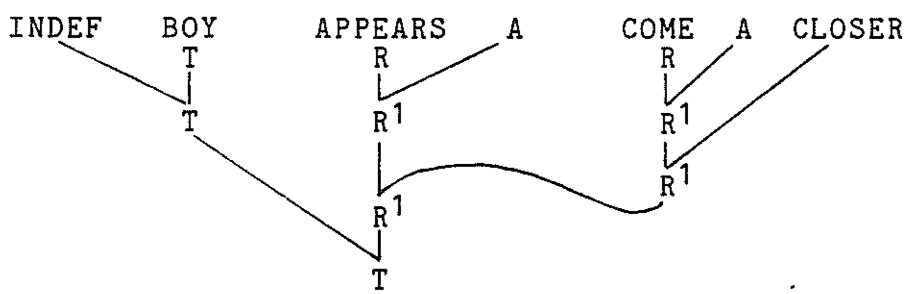
In analogy to our earlier procedure, we can consider the possible consequence (9), a reading (9') of (9) which comprises an extensional reading of "to come closer" in (9) and a reading (9'') of (9) which comprises an intensional reading of "to come closer" in (9):

(9)

(9')



(9'')

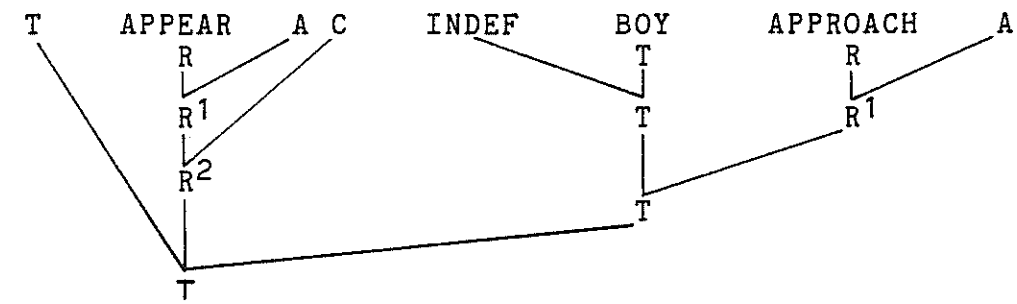


The semantic axioms of Chapter 2 have the consequence that (7') and (8') together entail (9'), but do *not* have the consequence that (7'') and (8') together entail (9''). That is, (7) and (8) together entail (9) under the extensional readings (7'), (8') and (9') of (7), (8), and (9) respectively, but (7) and (8) together do *not* entail (9) under the intensional reading (7'') of (7), the reading (8') of (8), and under either the extensional reading (9') of (9) or the intensional reading (9'') of (9).

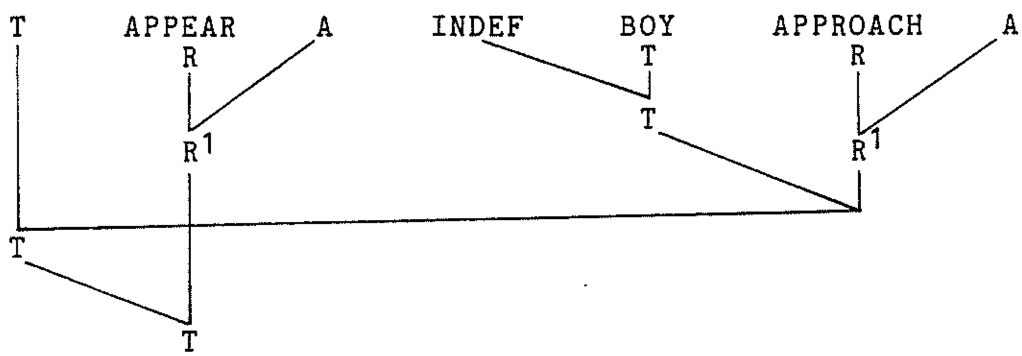
It is also of interest to note an example where a whole sentence is intensionalized within a containing sentence:

It appears that a boy approaches

has, among its normal extensional readings:



and among its normal intensional readings:



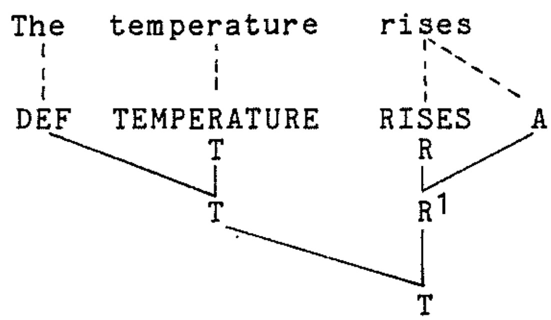
The extensional reading identifies "it" and "the boy approaches," asserting, very roughly, that the former appears to be the latter. The intensional reading asserts, very roughly, that a boy-approaches-thing appears.

In the above examples, the intensional readings of the identity statements (2), (5), and (8) did not play a significant role in blocking the incorrect entailment. We now consider an example[113] where the intensional reading of the identity statement <u>does</u> play a significant role in blocking the incorrect entailment.

Consider the sentence (10) and its extensional[114] readings.

(10)

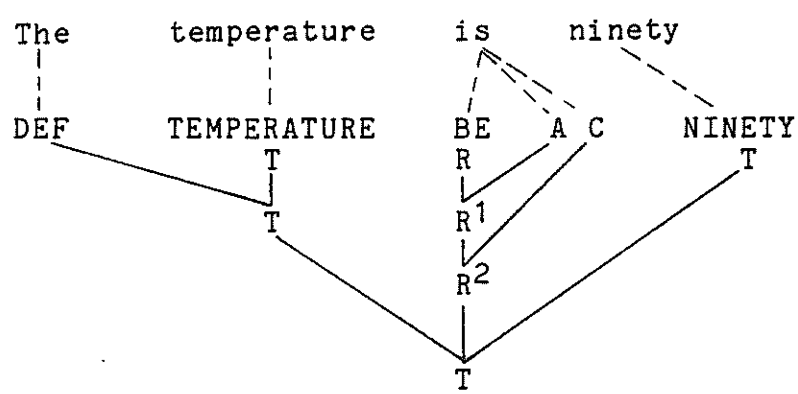
(10')



Consider now the identity (11) along with a reading (11') that extensionalizes "ninety" and a reading (11'') that intensionalizes "ninety."

(11)

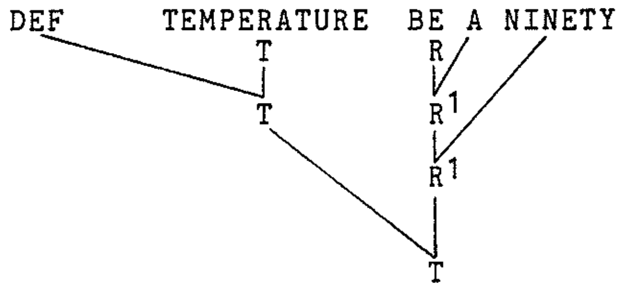
(11')



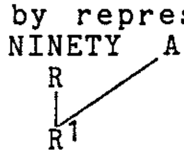
Note 113. Deriving from Partee [5] and discussed in Montague [3].

Note 114. There is no reasonable--i.e. normal--intensional reading of any part of (10).

(11'')[114.1]



Note 114.1. Recourse to intensionality is not essential here to block the unwanted entailment; for example, the same effect could be achieved by representing "ninety" in (11'') as a one-place relation:



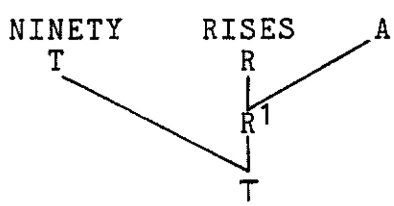
The difference in readings (11') and (11'') of (11) can be very roughly indicated as follows: (11') asserts that the entity *the-temperature* is identical to something that has the characteristic of being (i.e., measuring at) ninety, whereas (11'') asserts that the entity *the-temperature* "ninetys," i.e., undergoes the action of assuming the characteristic of being "ninety."

Consider finally a possible conclusion of (10) and (11), namely:

(12) Ninety rises

and an extensional "reading" (12') of (12):[115]

(12')



which asserts, very roughly, that something that has the characteristic of being (i.e., measuring at) ninety also rises.

Now (10) and (11) together entail (12) only under the extensional reading (11') of the key identity (11); that is, under the semantic axioms of Chapter 2, (10') and (11') entail (12'), but (10') and (11") do not entail (12').

Let us summarize these results pertaining to examples (10) - (12): The logical semantic axioms of Chapter 2 yield the following two consequences:

Note 115. There is no normal intensional reading of (12)

First, (10) and (11) do not entail (12) under the extensional reading (10') of (10), the intensional reading (11'') of (11), and the extensional reading (12') of (12).

Second, (10) and (11) do entail (12) under their respective extensional readings (10'), (11') and (12').

Let us briefly consider a second example[116] which uses the indefinite article "a":

(13) A price rises

(14) A price is a number

(15) A number rises

To bar the invalid entailment of (15) from (13) and (14), we can employ a precisely parallel treatment for (13), (14), and (15) as we did for (10), (11), and (12) earlier where we had chosen an intensional (11'') rather than extensional (11') reading of the key identity (11).

Let us consider a further type of example:

(16) John is a skillful mathematician

(17) All mathematicians are musicians

(18) John is a skillful musician

The dominant sense of (16) is that John is skillful-as-a-mathematician, and the dominant sense of (18) is that John is skillful-as-a-musician. And, under these senses, (16) and (17) together do not intuitively entail (18). The way to bar the entailment of (18) from (16) and (17) is to intensionalize the

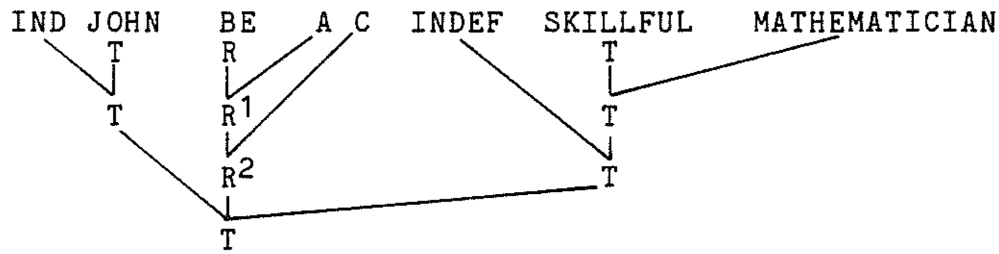
---

Note 116. Deriving from Montague.

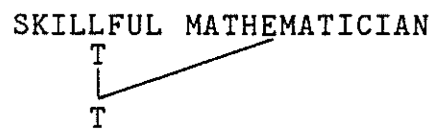
phrases "skillful mathematician" and "skillful musician" in (17) and (18) respectively.

For example, an intensional reading of (16) would be: (The situation with (18) is completely analogous)

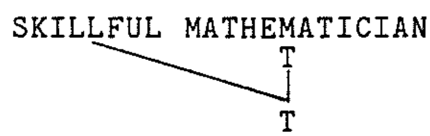
(16')



It should be clear in the above that the intuitive sense of the $SYN^{TR}_{English}$ expression



is skillful-as-a-mathematician. Moreover, the corresponding extensional version of this phrase is



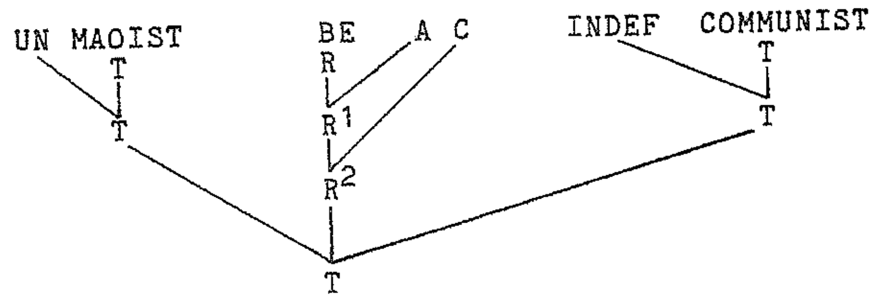
We next give an example of a one-premise argument involving intensions that reflects one of the immediately preceding considerations:

(19) Every Maoist is a Communist

(20) Every alleged Maoist is an alleged Communist

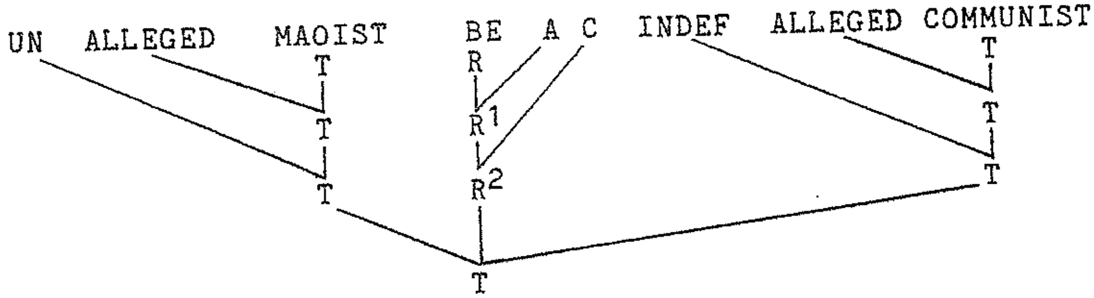
The dominant normal reading of (19) is the extensional:

(19')



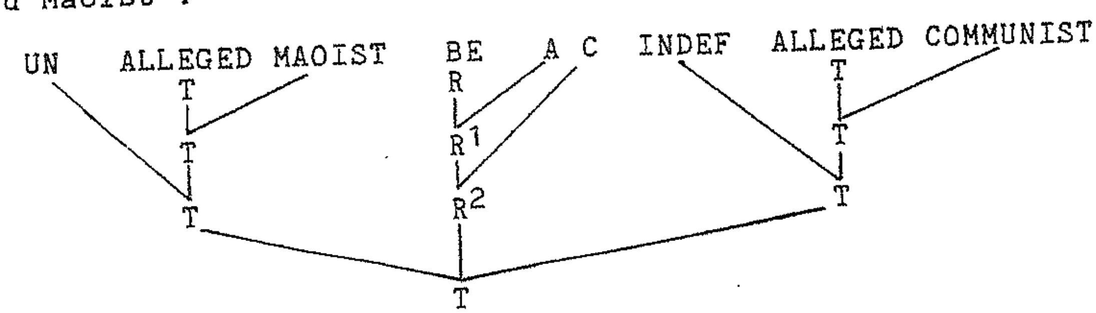
The dominant normal reading of (20) that extensionalizes the phrases "alleged Communist" and "alleged Maoist" is as follows:

(20')



Now, by the semantic axioms of Chapter 2, (19') entails (20'); that is (19) entails (20) under the extensional readings (19') of (19) and (20') of (20). This entailment is incorrect. The solution, as before, is to intensionalize parts of (20) which bar this entailment. Specifically, we employ the following intensional reading of (20) which is the dominant normal reading of (20) that intensionalizes the phrases "alleged Communist" and "alleged Maoist":

(20'')



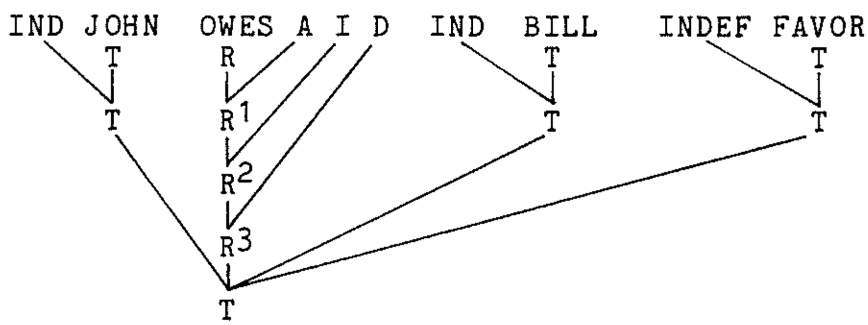
The above phrasings have the intuitive sense of alleged-to-be-a-Maoist and alleged-to be-a-Communist. The semantic axioms of Chapter 2 do not permit the entailment of (20'') from (19'); that is, they do not permit the entailment of (20) from (19) under their respective readings (19') and (20'').

As an example of an intensional construction of a further kind, let us consider the example

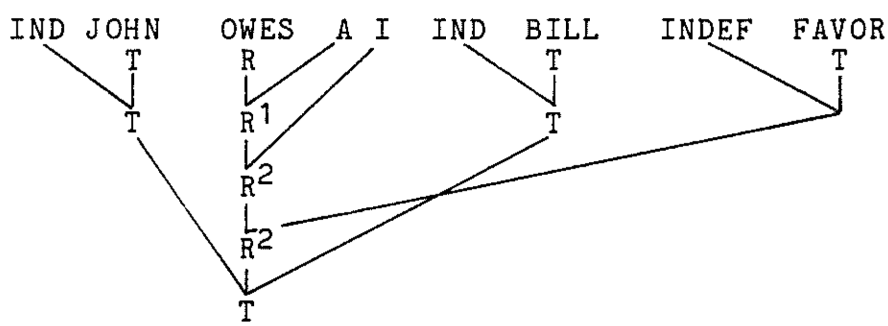
(21) John owes Bill a favor

There are various possible normal readings of (21):

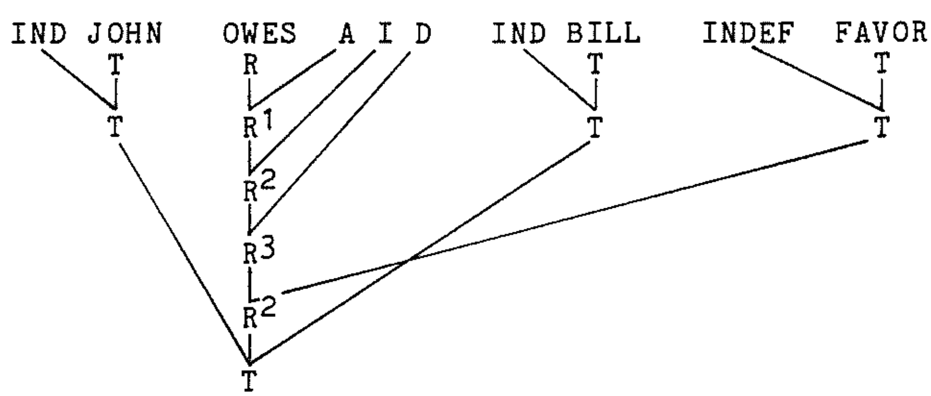
(21')



(21'')



(21''')



Now (21') and (21''') are equivalent under the semantic theory of Chapter 2, whereas (21'') is not equivalent to (21') under that semantic theory. Thus (21') and (21'') comprise distinct normal readings (relative to the semantic theory of Chapter 2) of the English word-string (21). The differences are, briefly, as follows: (21') is such that there is a normal reading of:

(22) there is a favor such that it is owed to Bill by John.

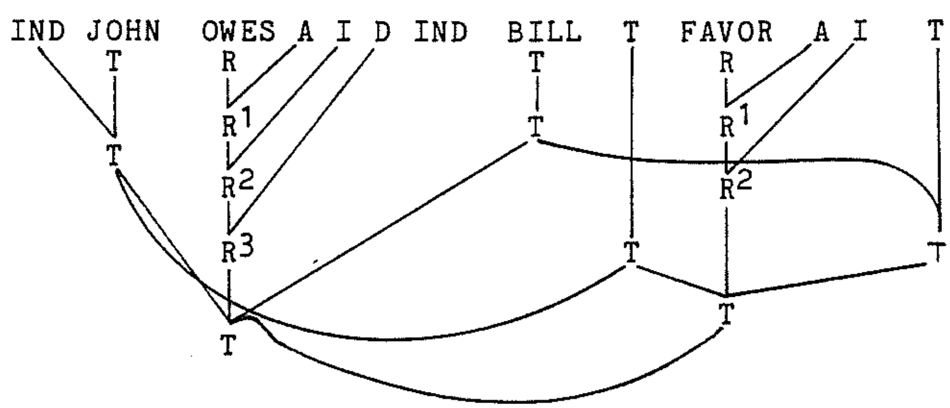
which is equivalent to a normal reading of (21) whose syntactic representation is (21'), whereas there is _no_ normal reading of (22) that is equivalent to a normal reading of (21) whose syntactic representation is (21''). Another difference is that (21') is such that there is a normal reading of

(23) John owes Bill something

which is such that, under it, (21) entails (23) under the dominant normal reading of (23) and under a normal reading of (21) whose syntactic representation is (21'), whereas this is true of neither (22') nor (23').

As another kind of normal reading of (21), consider:

(21'''')



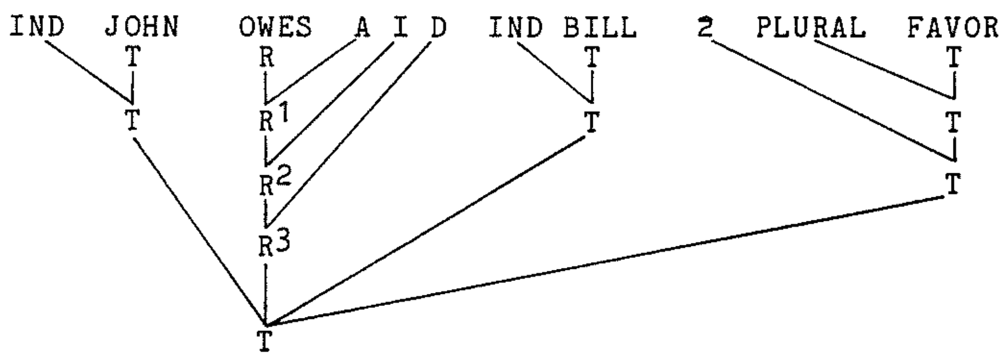
There is no grammatical English word-string w for which (21'''') would be the syntactic representation component of a normal reading of w. We can, nevertheless, attempt to approximate the meaning of a suitable w by the ungrammatical but understandable English word-string

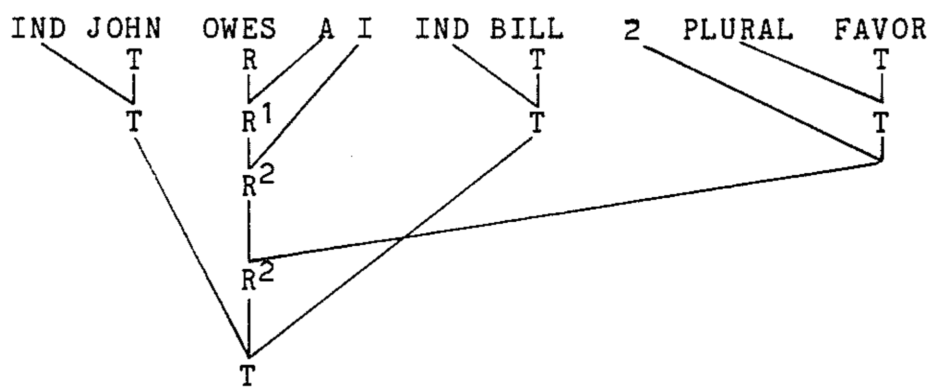
(21_a''') John owes Bill that he (John) favors Bill

where "favors" is to be interpreted (as shown in 21'''') as a relation between that which renders (i.e., John) and that which is rendered to (i.e., Bill) which relation has the meaning carried by the phrase "doing a favor for."

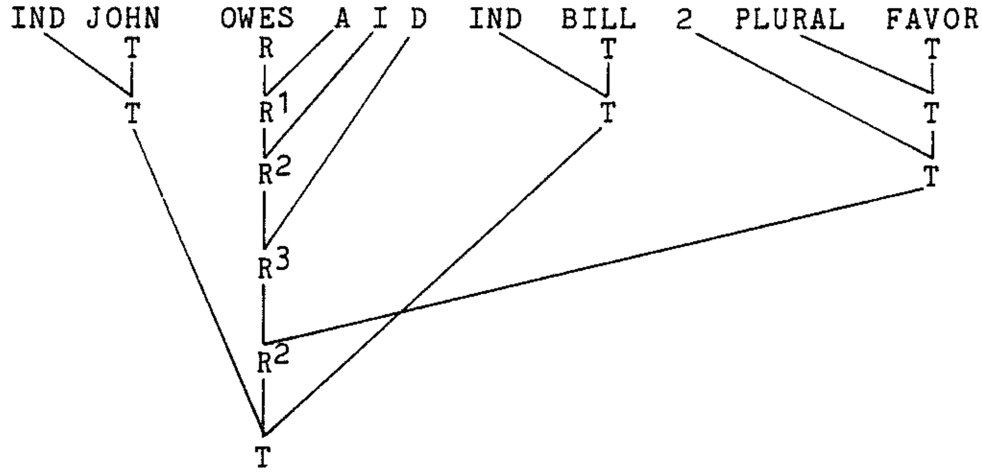
To get another perspective on (21) and its readings (21'), (21''), and (21'''), consider:

(23) John owes Bill two favors

(23')


(23'')


(23''')



Analogous remarks hold for the (23'), (23''), and (23''') as held for (21'), (21'') and (21'''). I would like only to make some remarks concerning some relationships of the first three to the last three:

(23) entails (21) under the readings (23') or (23''') of (23) and the readings (21') or (21''') of (21), hence (23) would entail (22) under some normal reading of (22) and either of the readings (23') or (23'') of (23). On the other hand (23) would not entail (21) under the reading (23'') of (23) and the reading (21'') of 21).

3.8 Branching Qauntifier Readings

Hintikka [5] has identified certain sorts of English sentences which cannot be adequately expressed in the predicate calculus unless quantifier strings are regarded as *partially ordered* rather than as linearly ordered.

The question arises as to whether we need to extend logical semantic axiom (8.1), which defines the denotations of SYN_L^{TR}-sentences, in terms of the informal-predicate-logic notation of the semantic metalanguage of SYN_L^{TR}, to include also branching quantifiers. We note that even if axiom (8.1) needed to be extended, the notation of SYN_L^{TR} would not itself need to be changed - since the order of quantifiers in a sentence of SYN_L^{TR} is wholly given by the indexes on the major thing-expression of that sentence, and these indexes could be specified to include also partial orderings of them, hence partial orderings of the thing-expressions to which they are applied.

The purpose of this section is to indicate the way that cases requiring branching quantifiers within a predicate calculus formulation could be handled within our own system without an analogous extension of our semantic metalanguage to a branching quantifier version of the informal predicate calculus. We refer to those readings that would formalize the cases that Hintikka shows to require branching quantifiers as *stratified independent readings*, and contrast them with some other closely related readings that could possibly be regarded as alternative normal readings of these cases.

The examples in the literature of English sentences requiring branching quantifiers in order to express their dominant reading are of the form[117]

(1) Some friend of every man and some friend of every woman know each other.

The dominant normal reading of (1) is that reading to the effect that (i) there is a collection x of collections of friends of men, which is such that, for every man m there is a collection m^o comprised of some friends of m, which is an element of x, (ii) there is a collection z of collections of friends of women, which is such that for every woman w there is a collection w^o comprised of some friends of w, which is an element of z, and (iii) for every man m and for every woman w all persons in m^o and all persons in w^o know each other. Since in this reading, one "stratifies" the collection of friends of men (women) into subcollections of persons who are all friends of given men (women), we call this reading the stratified independent reading of (1).

There is also a second "independent" reading of (1) which, while normal, is not dominant, namely that, for some collection of persons which includes at least one friend of every man and, for some collection of persons which includes at least one friend of every woman, all persons in the first collection and all persons in the second collection know each other. We distinguish

---

Note 117. I believe that examples of this form are due to Hintikka, but may be due partly to Henkin.

this independent reading from the preceding one by referring to it as the *unstratified independent reading of (1)*.

These independent readings of (1) have the characteristic that, under each of them, the sentence (1) is equivalent to the sentence (1a):

(1a) Some friend of every woman and some friend of every man know each other

We contrast these two independent readings of (1) and (1a) with that *dependent reading of (1)* whereby every man has a friend who knows and is known by some friend of every woman , and that *dependent reading of (1a)* whereby every woman has a friend who knows and is known by some friend of every man. It is clear that (1) and (1a) would not be equivalent under their respective dependent readings.

We can formulate reasonable stratified normal readings of sentences like (1) by employing the differentiated relative in application to a thing-expression with an unindexed thing-label[118], in conjunction with the binary logical morpheme S-CONJ[119], and formulate unstratified normal readings of sentences like (1) by the application of the differentiated relative to a thing-expression with an indexed thing-label[120], in conjunction with the use of the binary logical morpheme CONJ[121]. The semantic import of our treatment of unstratified independent readings is, in part, that the quantifiers of the semantic metalanguage are

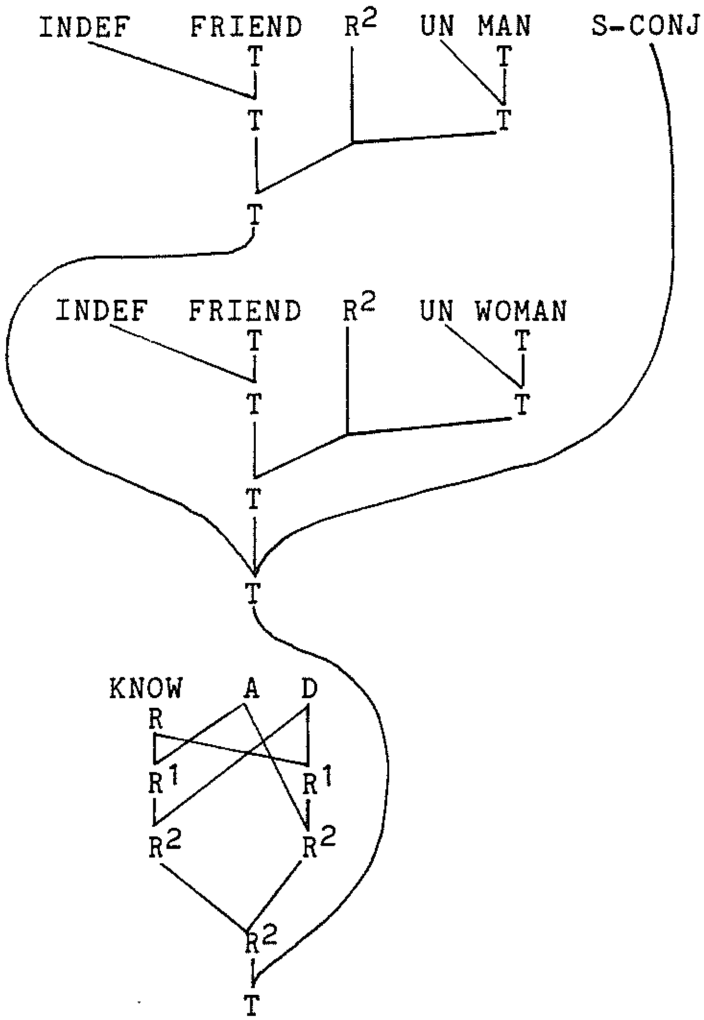
---

Note 118. Whose interpretation is given by L.S.A. (14.22).
Note 119. Whose interpretation is given by L.S.A. (14.23)
Note 120. Whose interpretation is given by L.S.A. (14.21).
Note 121. Whose interpretation is given by L.S.A. (14.17).

thereby permitted to range over <u>ordered n-tuples</u> of elements of the domain of discourse; n = 2 for (1).

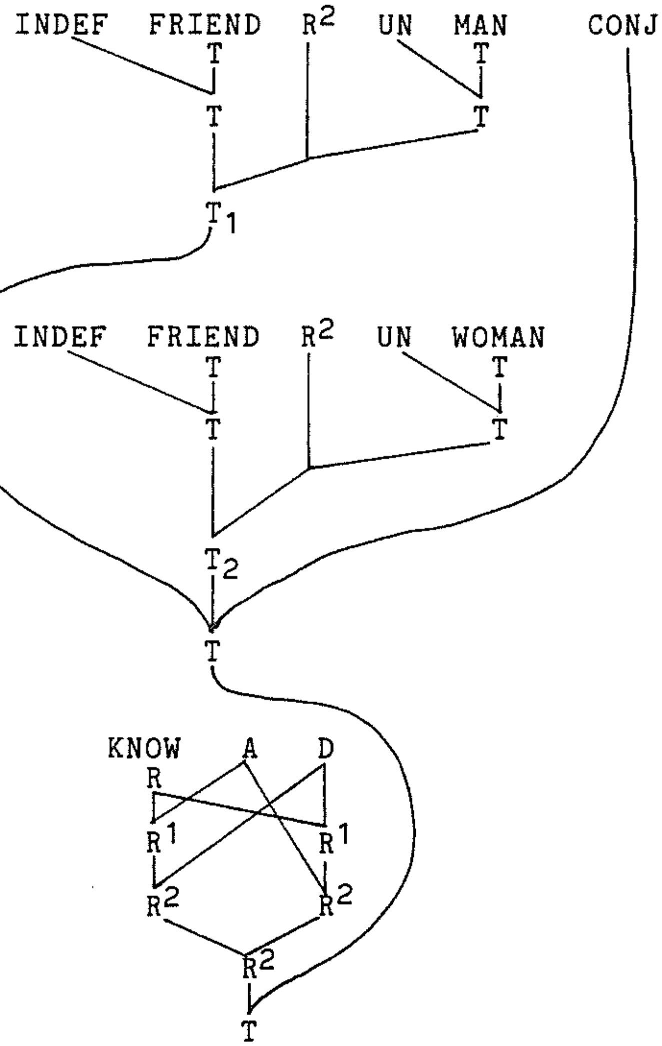
Let us consider a syntactic representation (1') of (1) which, in conjunction with any semantic theory satisfying the axioms of Chapter 2, comprises a reading of (1) that appears to adequately capture the <u>stratified</u> <u>independent</u> way of understanding the noun phrases of (1), which is, moreover, the dominant normal reading of (1):

(1')



In contrast to (1') above, (1'') below is a syntactic representation which, in conjunction with any semantic theory satisfying the axioms of Chapter 2, comprises a reading of (1) that appears to adequately capture the unstratified independent way of understanding the noun phrases of (1):

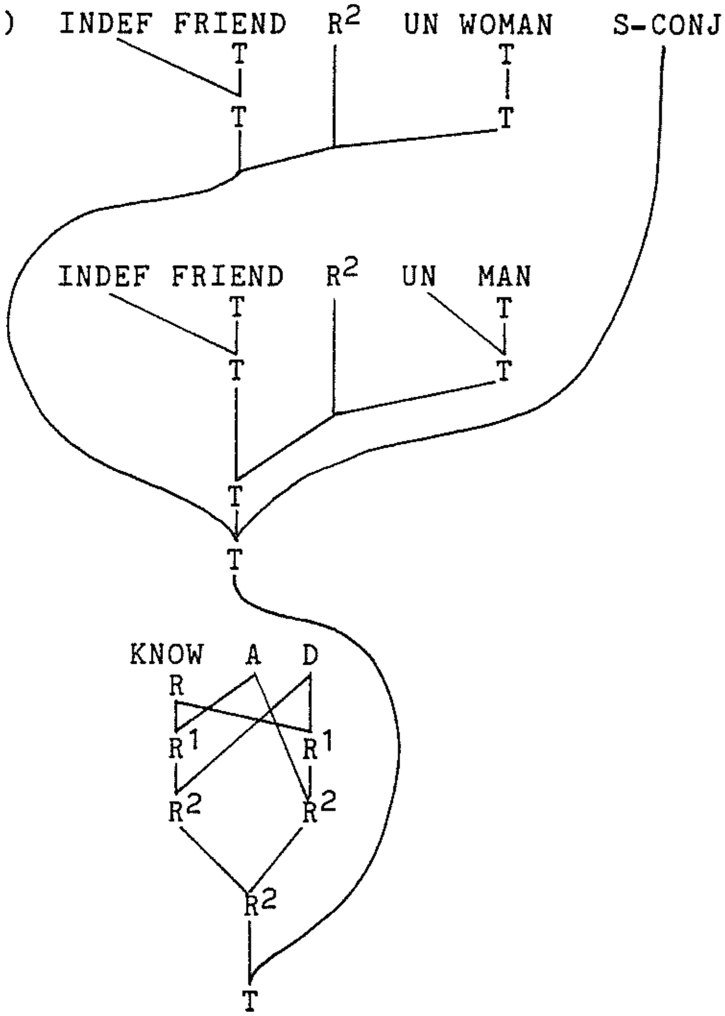
(1'')



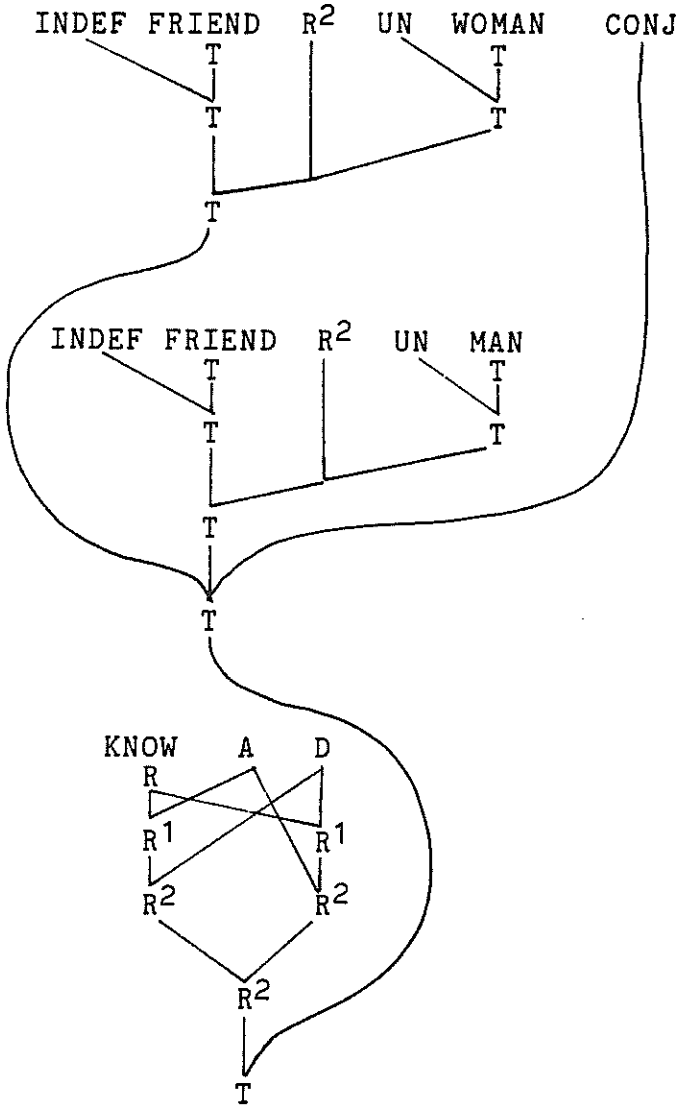
It is clear that the stratified independent reading (1') of (1) is the dominant normal reading of (1); indeed (1'') would appear to be, by my ear, at best only marginally normal. That is to say, the word-string (1) usually signals the reading (1') rather than the reading (1'').

To continue with this line of examples, consider the following syntactic representations (1a') and 1a'') of the sentence (1a):

(1a')



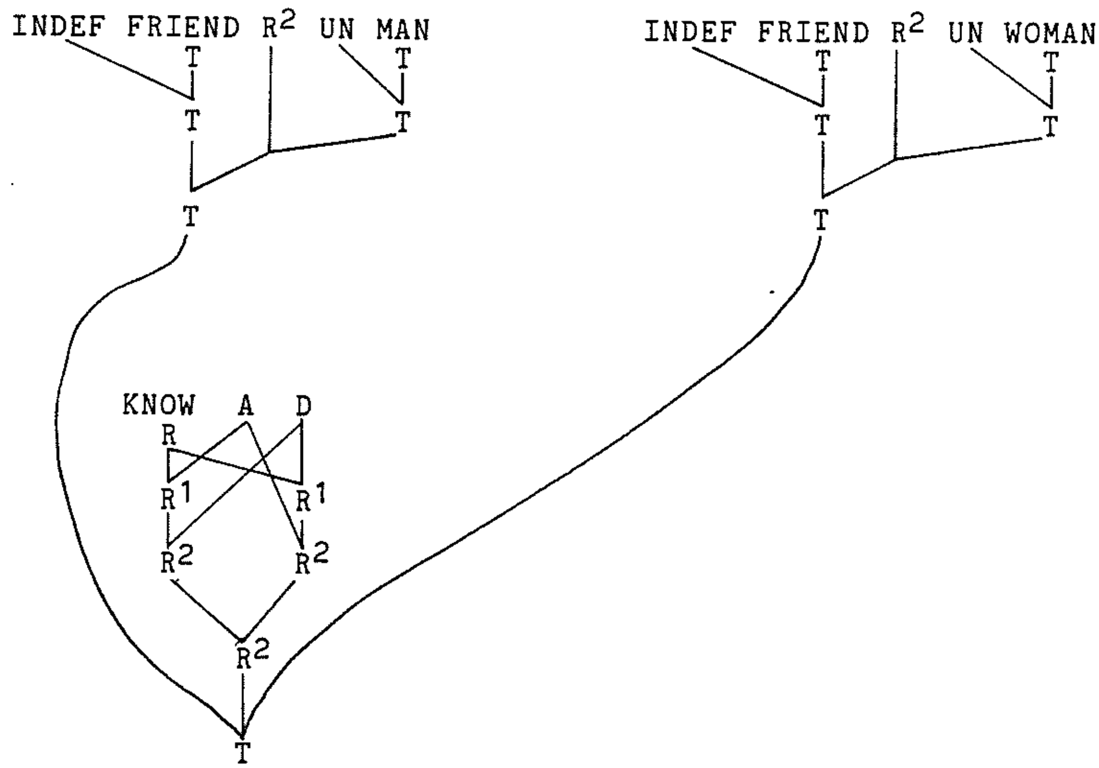
(1a'')


INDEF FRIEND R2 UN WOMAN CONJ
INDEF FRIEND R2 UN MAN
KNOW A D
T
R
R1
R2

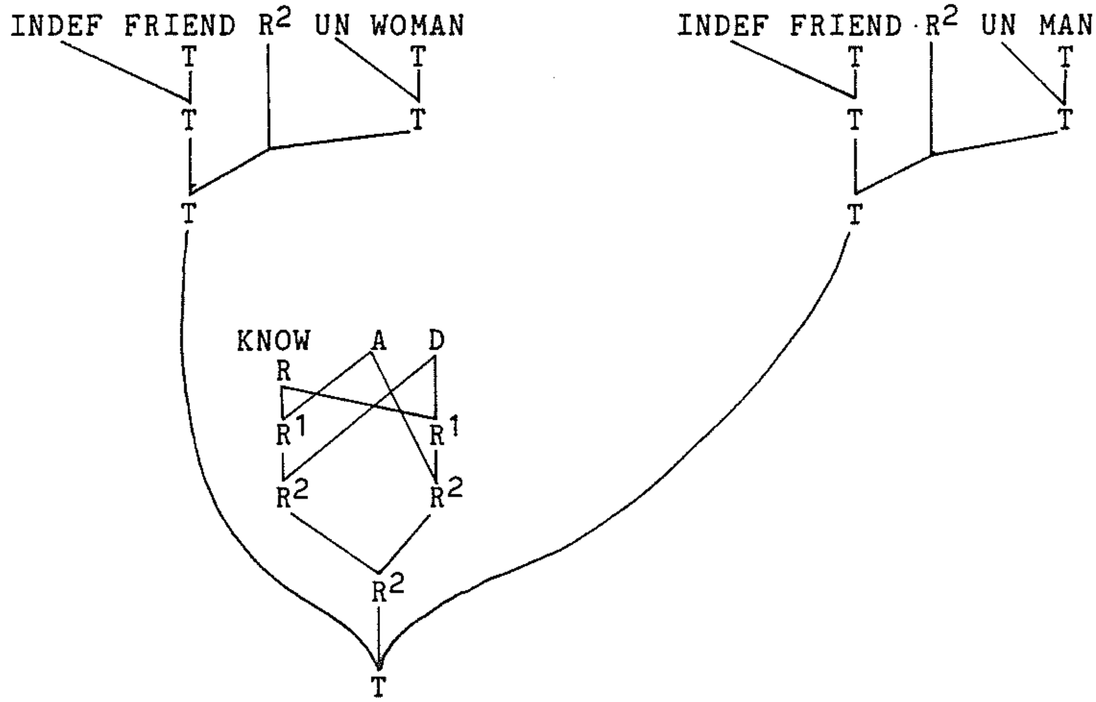
In conjunction with any semantic theory satisfying the axioms of Chapter 2, (1a') captures the *stratified independent* way of understanding the noun phrases of (1a), while (1a'') captures the *unstratified independent* way of understanding those noun phrases. It can be shown that (1) is equivalent to (1a) under the "dependent" readings (1'') of (1) and (1a'') of (1a), but that (1) is not equivalent to (1a) under the "independent" readings (1') of (1) and (1a') of (1a). Moreover, it can also be shown that (1) under the reading (1'') entails (1) under the reading (1'), but that the converse fails.

In contrast to the above two independent readings of (1) and (1a), we can represent the dependent readings of (1) and (1a) respectively by:

(1''')



(1a''')



Readings (1''') and (1a''') are not strong normal readings of (1) and (1a) respectively. (1''') and (1a''') would, however, be the dominant normal readings of the following English sentences[121.1]:

(1.1) Some friend of every man knows some friend of every woman

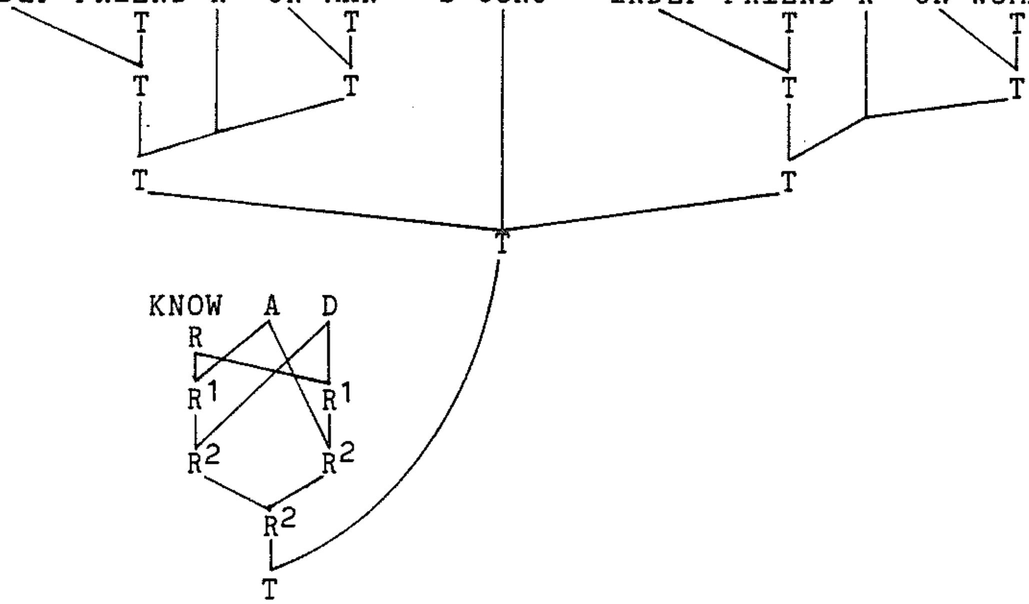
(1.2) Some friend of every woman knows some friend of every man

For explicitness, we exhibit the denotations of the two independent readings of (1); first the denotations f(1') of the stratified independent reading (1') of (1), then the denotation f(1'') of the unstratified independent reading (1'') of (1).

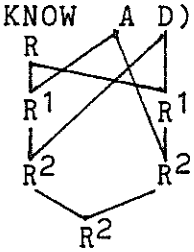
Note 121.1. If furthermore, we were to interchange the relation-expression and the second major thing-expression in (1''') and (1a'''), then these would be <u>homologous</u> normal readings as well.

f(1') =

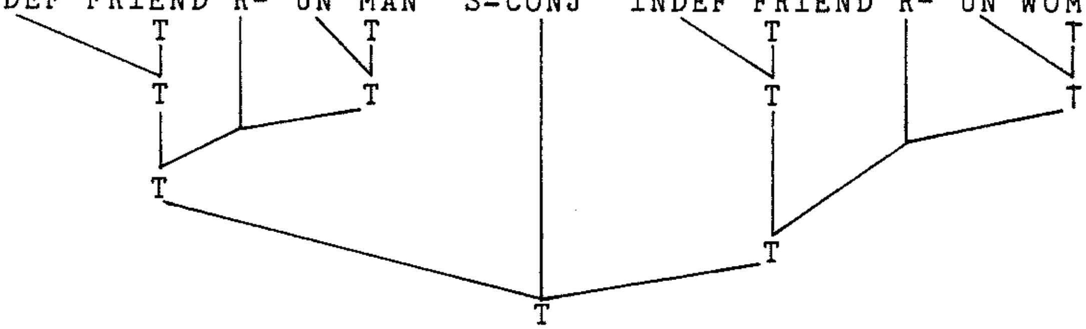
f(INDEF FRIEND R^2 UN MAN S-CONJ INDEF FRIEND R^2 UN WOMAN)



= {<f(KNOW A D), K> : K is a minimal set such that

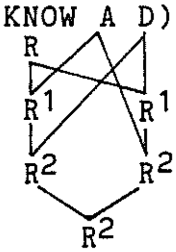


(1)K⊆f(INDEF FRIEND R^2 UN MAN S-CONJ INDEF FRIEND R^2 UN WOMAN)

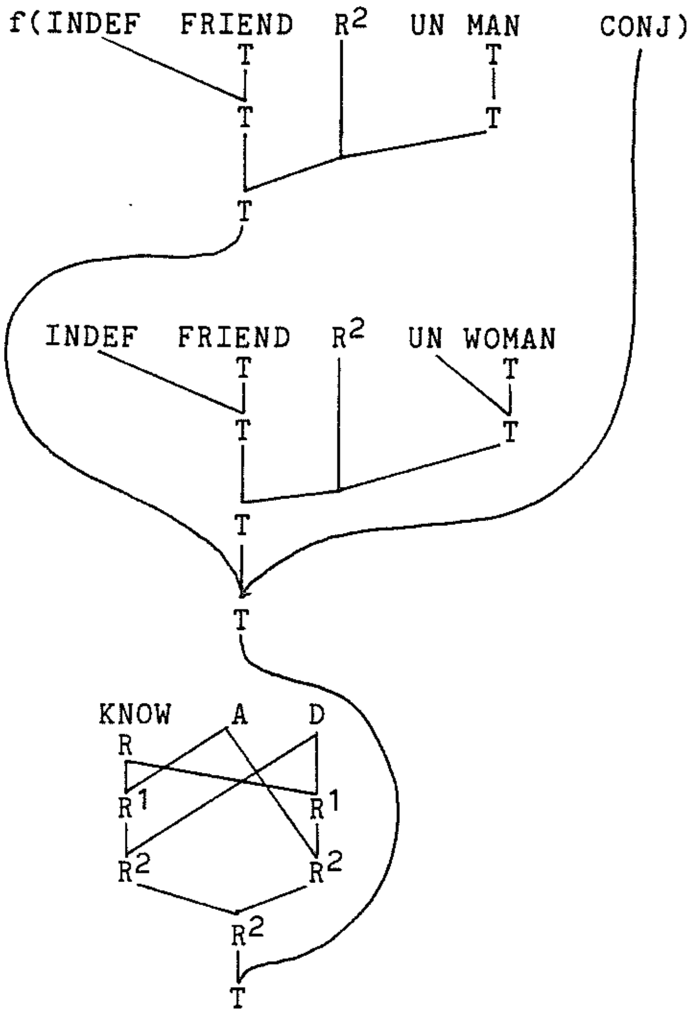


and

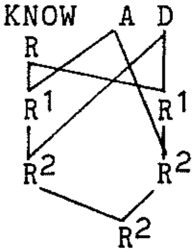
(2) for some x∈K, and for all y, if y∈x, then y∈f(KNOW A D)))}



f(1'') =



= {<f(KNOW A D , K> : K is a minimal set such that



(1) K ⊆ f(INDEF FRIEND R^2 UN MAN CONJ INDEF FRIEND R^2 UN WOMAN)}